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Chiral Structure of Particles Bound by Magnetic Forces

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Authors' contributions

This work was carried out in collaboration between both authors. Author HPM derived the formalism, performed the full analysis, interpreted the results and prepared the first draft of the manuscript. Author SG checked all results by independent programming. Both authors read and approved the final manuscript.

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Abstract

Aims/ Objectives: Particle bound states exist only as microscopic systems in form of atomic and subatomic particles. An interesting class of these objects are particles bound by magnetic forces, which exhibit the particular property of chirality (handedness, which is not parity symmetric). These particles are discussed in quantum field theory based on a QED like Lagrangian with fermion and boson fields, in which about ten boundary conditions can be defined. With four (but effectively two) adjustable parameters only, this leads to a stringent test of the special mathematical structure of the underlying field theory.

A first kind of these particles are leptons, e, μ, τ and neutrinos. With an additional quantum condition the radii of charged leptons can be deduced. Other systems of magnetic binding may

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be found in atoms, a first example being weakly bound H-atoms, which may be the origin of gravitation.

Keywords: Particle bound by magnetic forces described in quantum field theory based on a QED type Lagrangian with fermion and boson fields; Solutions for charged leptons and weakly bound atoms, which violate parity symmetry.

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1 Introduction

Bound or stationary systems belong to the most interesting and basic objects in physics, since they are stable over long periods of time. Their average potential and kinetic energies are related by the virial theorem, leading to a well-defined frequency or mass. A large fraction of the mass of the universe is composed of particle bound states in the vacuum in form of atoms and subatomic particle, like hadrons and leptons.

For these systems binding of elementary fermions is not sufficient; accompanying bosons (not only an attractive boson-exchange interaction) are essential to equilibrate the momentum of fermions, see ref. [1]. A special class of these particles are those bound by magnetic forces, which are schematically shown in Fig. 1. Since magnetic forces arise from the motion of charge, two charged fermion components are needed, which rotate with relative velocity (v/c) to each other, see the green flashes in Fig. 1. The momentum of these fermions has to be compensated by two boson components, indicated by yellow flashes.

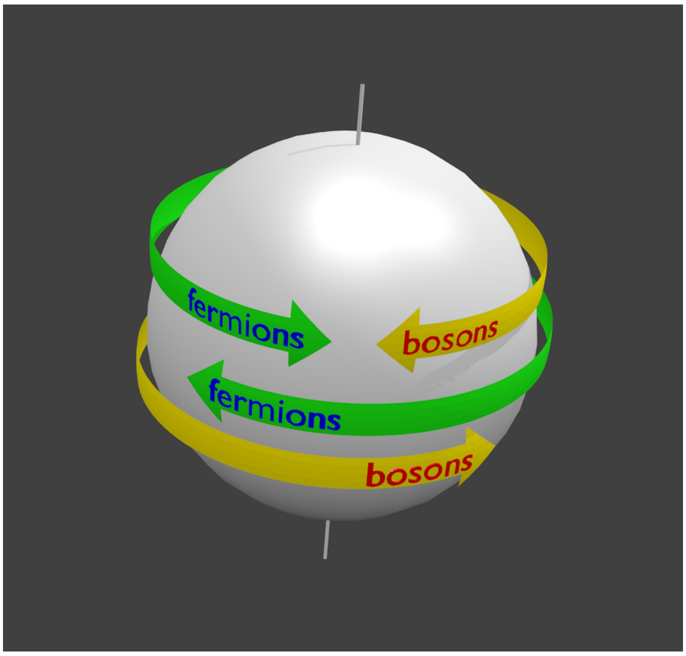

Fig. 1. Schematic view of a particle bound by magnetic forces. The green flashes show the motion of two different fermion components; the rotation of these fermions is counterbalanced by bosons, shown by yellow flashes

A particularity of these bound states arises from the vector-structure of magnetic forces, in which the directions of spin, relative motion and attraction (and therefore binding) are perpendicular to each other, see the right hand rule of the Lorentz force [2]. The axis of spin is perpendicular to the rotation axis, so attraction exists only for one direction of rotation (the opposite rotation would give repulsion of the fermions). Therefore, these bound states are not symmetric under parity transformations, which is called chirality or handedness [3]. Leptons and their antiparticles are known to have this property, for leptons only a left-handed type exists, whereas antileptons are right-handed. But so far, leptons have been treated as point particles, a real bound state description, in which the chiral structure is understood, has not been performed.

2 Theoretical Description

For the description of these systems a quantum field theory similar to that in ref. [1] has been used, based on a Lagrangian similar to QED, but with fermions dressed by boson fields, which may be written in the form

$$\mathcal{L} = \frac{1}{\tilde{m}^2}(\bar{\Psi} D_\nu)\ i\gamma^\mu D_\mu\ (D^\nu \Psi)\ -\ \frac{1}{4} F_{\mu\nu}F^{\mu\nu}\ , \tag{2.1}$$

where $\tilde{m}$ is the mass parameter and Ψ are charged fermion fields, $\Psi = \Psi^+$ and $\bar{\Psi} = \Psi^-$. Vector boson fields A_μ with coupling g to fermions are contained in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$. The second term of the Lagrangian represents the Maxwell term of electromagnetism with field strength tensors $F^{\mu\nu}$ given by $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, which gives rise to both electric and magnetic coupling.

The three covariant derivatives D_μ in the Lagrangian (2.1) lead to a theory, which includes higher order boson and fermion fields. However, in the past this type of Lagrangian has not been studied in detail, because in standard (divergent) gauge theory the necessary $1/\tilde{m}^2$ factor gives rise to uncontrolled divergences. Further, Lagrangians with higher order fermion fields can lead to nonphysical solutions [4]. None of these arguments are important for the present case: the Lagrangian (2.1) leads to a finite theory due to a constrained normalization of boson fields, therefore a $1/\tilde{m}^2$ factor is acceptable. Further, it gives rise to a real bound state formalism with negative binding energies, in which nonphysical solutions can be excluded by strict geometrical and other basic constraints.

By inserting $D^\mu = \partial^\mu - igA^\mu$ and $D_\nu D^\nu = \partial_\nu\partial^\nu - ig(A_\nu\partial^\nu + \partial_\nu A^\nu) - g^2 A_\nu A^\nu$, the first part of the Lagrangian gives rise to a number of terms, which contain boson and fermion fields and/or their derivatives. For stationary solutions only two terms of the Lagrangian contribute

$$\mathcal{L}_{2g} = \frac{-ig^2}{\tilde{m}^2}\ (\bar{\Psi} A_\nu)\ \gamma^\mu \partial_\mu\ (A^\nu \Psi) \tag{2.2}$$

and

$$\mathcal{L}_{3g} = \frac{-g^3}{\tilde{m}^2}\ (\bar{\Psi} A_\nu)\ \gamma^\mu A_\mu\ (A^\nu \Psi)\ . \tag{2.3}$$

From the Lagrangians (2.2) and (2.3) fermion-antifermion matrix elements have been derived, see the details in ref. [1], which can be written by

$$\mathcal{M}_{2g} = \frac{\alpha^2}{\tilde{m}^5}\ \bar{\psi}(p')\ A_\nu(q_4')A^\mu(q_3')\ \gamma_\mu\gamma^\rho\ \partial A_\rho(q_2')\partial A^\sigma(q_1')\ \psi(p) \tag{2.4}$$

and

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^5}\ \bar{\psi}(p')\ A_\nu(q_4')A^\mu(q_3')\ \gamma_\mu\gamma^\rho A_\rho(q_2)A^\sigma(q_1)\ A_\sigma(q_2')A^\tau(q_1')\ \psi(p)\ , \tag{2.5}$$

in which $\psi(p)$ is a fermion wave function $\psi(p) = \frac{1}{\tilde{m}^{3/2}}\Psi(p_1)\Psi(p_2)$. These matrix elements can be further simplified by assuming also boson (quasi) wave functions $W_\mu^\nu(q') = \frac{1}{\tilde{m}}A_\mu(q_j')A^\nu(q_i')$ of scalar

($\mu = \nu$) and vector ($\mu \neq \nu$) structure. In a fundamental description of bound states as discussed here, a reduction from four to three dimensions is natural due to stability of the system in time; the three-dimensional momenta can then be Fourier transformed to r-space. By reducing the above matrix elements to three dimensions one gets boson wave functions of scalar and vector structure $w_s(q')$ and $w_v(q')$ and a boson-exchange interaction $v_v(q)$. This leads to fermion matrix elements

$$\mathcal{M}_{2g} = \frac{\alpha^2}{2\tilde{m}^3}\ \bar{\psi}(p')\ w_s(q')\ \partial^2 w_s(q')\ \psi(p) \tag{2.6}$$

and

$$\mathcal{M}_{3g} = \frac{\alpha^2}{\tilde{m}^2}\ \bar{\psi}(p')\ w_{s,v}(q') v_v(q) w_{s,v}(q')\ \psi(p)\ , \tag{2.7}$$

but also to a boson matrix element

$$\mathcal{M}^g = \frac{\alpha^2}{\tilde{m}^2}\ w_{s,v}(q')\ v_v(q)\ w_{s,v}(q')\ . \tag{2.8}$$

By going from fermion-antifermion (q^+q^-) matrix elements to $(q^+q^-)^n q^\pm$ structures, which can be bound by magnetic forces, two fermion wave functions are needed, which rotate with relative velocity (v/c). The corresponding matrix elements can be combined to simpler ones, which are of the same forms as those above with additional (v/c) factors. Fourier transformed to r-space, this leads to matrix elements in the form

$$\mathcal{M}^f_{ng}(r) = \bar{\psi}_{s,v}(r)\ V_{ng}(r)\ \psi_{s,v}(r)\ (v/c)^2\ , \tag{2.9}$$

with fermion wave functions $\psi_{s,v}(r)$ of scalar and vector structure and two potentials $V_{2g}(r)$ and $V_{3g}(r)$ of the form

$$V_{2g}(r) = \frac{\alpha^2(\hbar c)^2(2s+1)}{8\tilde{m}} \Big(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\Big)\frac{1}{w_s(r)} + E_o \tag{2.10}$$

with s=0 for scalar and s=1 for vector states, and

$$V_{3g}(r) = \frac{\alpha^3\hbar c}{\tilde{m}} \int dr'\ , w_{s,v}(r')\ v_v(r-r')\ w_{s,v}(r')\ , \tag{2.11}$$

where $w_{s,v}(r)$ are boson (quasi) wave functions of scalar and vector structure and $v_v(r) \sim -\hbar c\, w_v(r)$ a boson-exchange interaction. $\psi^2_{s,v}(r)$ and $w^2_{s,v}(r)$ can be interpreted as densities. It should be emphasized that in the present case of magnetic binding the interaction $v_v(r)$ is of vector structure, which is attractive only for rotation in one particular direction, e.g. left-handed. Bound states for which attraction is obtained for right-handed rotation are of different nature. Further, the boson matrix element is of the form

$$\mathcal{M}^g(r) = \frac{\alpha^3\hbar c}{\tilde{m}}\ w_{s,v}(r)\ v_v(r)\ w_{s,v}(r)\ (v/c)\ . \tag{2.12}$$

Bound states can be generated for different angular momentum L. In the present study we consider only $L = 0$, leading to bound states of scalar and vector character of the form $\psi_s(\vec{r}\,) = \psi_s(r) Y_o(\theta, \Phi)$ and $\psi_v(\vec{r}\,) = \psi_v(r) Y^2_1(\theta, \Phi)$, respectively.

A coupling to the vacuum is made by assuming $E_o = 0$. This implies that the vacuum is the lowest state with energy $E_{vac} = 0$, consequently all elementary fermions (quantons) have to be massless. The potential $V_{2g}(r)$ is important for a dynamical stabilization of the system: created fermion-antifermion pairs are locked during overlapping boson fields and form a stable system, which cannot decay. As shown below, $V_{2g}(r)$ shows a quite linear rise towards larger radii, very

with dimension 1/fm.

similar to the empirical "confinement" potential required in hadron potential models [5].

For a bound state of fermions and bosons the radial form of the fermion and boson wave functions should be similar

$$\psi_{s,v}(r) \sim w_{s,v}(r) \ . \tag{2.13}$$

Fermion vector states have normally angular distributions of dipole form $P_1^2(cos\theta)$, but for a free particle different orientations in space cancel out. However, due to rotation of the system the direction of momentum is fixed, leading to a wave function in q-space of dipole character $\psi_v(q) = 4\pi \int r^2 dr\ j_1{}^2$
$(qr)\psi_v(r)$.

Orthogonality of the fermion wave functions leads to

$$\int r^2 dr\ \psi_s(r)\psi_v(r) = \int r^2 dr\ w_s(r)w_v(r) =< r_{w_s,w_v} >= 0 \ . \tag{2.14}$$

This condition requires that the wave functions are finite (with finite radial moments). Condition (2.14) is satisfied for

$$w_v(r) = w_{v_o}\ [w_s(r) + \beta R\ \frac{dw_s(r)}{dr}] \ , \tag{2.15}$$

where w_{v_o} is obtained from the normalization of the density $w_v^2(r)$ with $2\pi \int rdr\ w_v^2(r) = 1$ and $\beta R = -\int r^2 dr\ w_s(r)/\int r^2 dr\ [dw_s(r)/dr]$. Because of the derivative structure, $w_v(r)$ has a smaller root mean square radius than $w_s(r)$. A natural condition requires therefore that the interaction for this state takes place inside the volume of the scalar state, leading to the geometrical boundary condition

$$|V_{3g}^v(r)| \simeq c\ w_s^2(r) \ . \tag{2.16}$$

The conditions (2.14) and (2.16) are satisfied by assuming the radial part of $w_s(r)$ by

$$w_s(r) \simeq w_{s_o}\ exp\{-(r/b)^\kappa\} \ , \tag{2.17}$$

where w_{s_o} is fixed by the density normalization $2\pi \int rdr\ w_s^2(r) = 1$.

The binding energies are given by $E_f^{ng} = 4\pi[\int r^2 dr\ \psi^2(r)V_{ng}(r) - \frac{1}{2}\int r^3 dr\ \psi^2(r)\frac{d}{dr}V_{ng}(r)]$ and $E_g = 2\pi[\int rdr\ w^2(r)v_v(r) - \frac{1}{2}\int r^2 dr\ w^2(r)\frac{d}{dr}v_v(r)]$. The masses (due to binding) are defined by the sum of absolute binding energies $M_{s,v} = |E_{2g}^{s,v}| + |E_{3g}^{s,v}|$, and the total mass of the system is given by $M_{tot} = M_{s,v} + m_1 + m_2$, where m_1 and m_2 are the participating fermion masses. As in ref. [1] for all calculations natural units are used.

There are boundary conditions, which allow to determine all four parameters of the model. The first one is momentum conservation, which implies that the average recoil momenta for bosons $< q_g^2 >_{rec}^{1/2}$ and fermions $< q_f^2 >_{rec}^{1/2}$ cancel each other

$$< q_g^2 >_{rec}^{1/2} + < q_f^2 >_{rec}^{1/2} = 0 \ . \tag{2.18}$$

With normalization $< q_g^0 >= \int qdq\ V_{3g}(q)$ and $< q_f^0 >= \int q^2 dq\ \psi(q)V_{3g}(q)$, this yields $< q_g^2 >_{rec} = \int q^3 dq\ V_{3g}(q)/ < q_g^0 >$ and $< q_f^2 >_{rec} = \int q^4 dq\ \psi(q)V_{3g}(q)/ < q_f^0 >$, where the Fourier transformed quantities are given by $(\psi, V_{3g})(q) = 4\pi \int r^2 dr\ j_o(qr)(\psi, V_{3g})(r)$.

Further, the coupling to the vacuum leads to energy-momentum conservation, which requires that the average momenta of the bound state are compensated by their binding energies

$$[< q_g^2 >^{1/2} + < q_f^2 >^{1/2}]\ (v/c) + E_g - x\ M_f = 0 \ , \tag{2.19}$$

where $x = \sqrt{2\tilde{m}/M_f}$ and (v/c) taken as positive. For basic systems energy-momentum conservation can be valid separately for bosons and fermions, this gives rise to four different constraints. For bosons this yields

$$< q_g^2 >^{1/2} (v/c) + E_g = 0 \tag{2.20}$$

and for fermions

$$< q_f^2 >^{1/2} (v/c) - x\ M_f = 0\ . \tag{2.21}$$

The momenta $< q_g^2 >=< q_g^2 >_{rec}$ and $< q_{f_s}^2 >=< q_{f_s}^2 >_{rec}$, but for vector particles $< q_{f_v}^2 >= \int q^4 dq\ \psi_v(q) V_{3g}^v(q) / < q_{f_v}^0 >$, where the Fourier transformed quantities are given by $(\psi_v, V_{3g}^v)(q) = 4\pi \int r^2 dr\ {j_1}^2(qr)(\psi_v, V_{3g}^v)(r)$.

A mass-radius condition derived from the structure of the potential $V_{2g}(r)$ reads

$$Rat_{2g} = \frac{(\hbar c)^2\ (v/c)^2}{\tilde{m}(M_s/2) < r_{w_s}^2 >} = 1\ . \tag{2.22}$$

Finally it is important to note that for magnetic binding the vector state with radial node is not stable. Nevertheless, energy-momentum conservation should be fulfilled also, because this state is part of the system.

3 Bound States of Elementary Fermions (Leptons)

In this case one can require further that the mass parameter $\tilde{m}$ is half of the generated bound state mass

$$\tilde{m} = \frac{1}{2} M_s = \frac{1}{2} (|E_{2g}^s| + |E_{3g}^s|)\ . \tag{3.1}$$

By the different boundary conditions (2.13), (2.16) and (2.18) - (3.1) the parameters of the model, κ, b, α and (v/c) are highly overconstrained. Even more, a value of $\kappa = 1.35 \pm 0.2$ is needed to get $V_{2g}(r)$ correct and $\alpha = 2.14$ is required to satisfy energy-momentum conservation. This leaves only two parameters, b and (v/c), by which all 10 boundary conditions have to be satisfied. As already stressed in ref. [1] the fulfillment of these conditions is far from trivial: the average momenta are related to the geometry only, whereas E_g and M_f are given by the eigenvalues in the boson and fermion potentials, which are of quite different structure. Nevertheless, for a reasonable determination of the parameters one further condition is needed, a quantum condition on the average radius.

Solutions are discussed for charged leptons, electron, muon and tau-lepton. For the electron a radius smaller than 10^{-9} fm is estimated [6]. The extremely small size explains that the electron could be assumed as point particle in weak interaction theory.

Resulting densities and potentials for a magnetically bound system with a boson root mean square (rms) radius of 2 10^{-10} fm is shown in Fig. 2. In the upper part the potential $V_{2g}(r)$ is shown, which has the typical characteristics of a confinement potential, established empirically in bound state calculations of hadrons [5]. This special potential form represents an inherent property of all bound states of relativistic particles, see refs. [1, 7]. In the second part of the figure the potentials $V_{3g}^{s,v}(r)$ are given together with the boson density $w_s^2(r)$. This shows that the geometrical boundary condition (2.16) between the density $w_s^2(r)$ and the potential $V_{3g}^v(r)$ is fulfilled. Finally, the Fourier transformed boson density and potentials are shown in the lower part, indicating very similar features of the system in r- and q-space. The resulting parameters, masses and radii are given in the upper part of table 1; further, average momenta and binding energies are given in the lower part, which indicate that momentum matching and energy-momentum conservation is fulfilled, even for the vector state, which is not stable. The given errors arise mainly from the spacing and cut-off in radius and momentum. These have been simply estimated by changing the momentum cut-off by ± 10 % at a value of about 7 times $< q_g^2 >_s^{1/2}$.

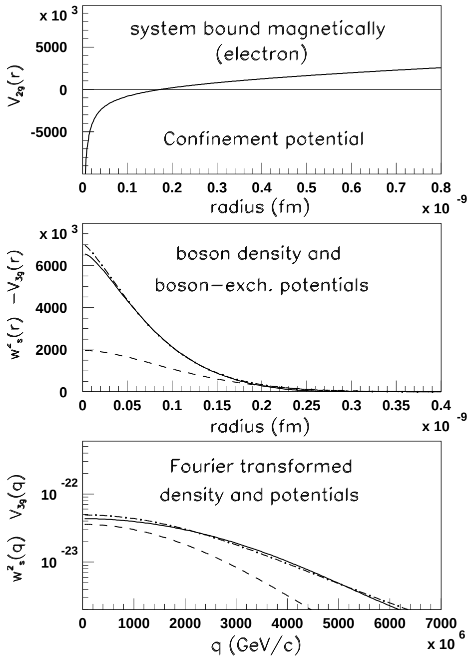

Fig. 2. Radial dependence of a system bound magnetically with a root mean square radius $< r^2_{w_s} >^{1/2}$ = 1.9 10^{-10} fm (electron). Upper part: Confinement potential $V_{2g}(r)$. Middle part: Boson density $w^2_s(r)$ (dot-dashed line) and boson-exchange potentials $|V^{s,v}_{3g}(r)|$ given by dashed and solid lines, respectively. Lower part: Fourier transformed density (dot-dashed line) and potentials (dashed and solid lines).

For other systems, μ and τ, quite similar features of the densities and potentials are expected, which are just scaled by a different radius. However, for a reasonable estimate of their properties a quantum condition on the radius of these systems is needed.

For light atoms such a condition has been established [8] by assuming that the different states are higher harmonics of the strongest bound state. Similarly, the different leptons may be assumed as higher members of a fundamental state (with the same spin and charge $J^{ch} = 1/2^{\pm}$), which is in this case of the magnetic component [9] of the proton or antiproton $p^{\pm}_M$ (giving rise to the magnetic form factor). Characterizing $p^{\pm}_M$ by quantum number $n = 1$ and the different leptons by $n > 1$, their rms-radii should follow the radial form of the density $w^2_s(r)$ or the potential $V^s_{3g}(r)$.

Using this picture the results are shown in Fig. 3. By adjusting the radial fall-off of $w^2_s(r)$ and $V^s_{3g}(r)$ between the rms-radius of p_M and the assumed electron radius of 2 10^{-10} fm, the dashed and solid lines are obtained, from which an estimate of the rms-radii of τ and μ is obtained, as given in Table 1. In the lower part resulting values of $< q^2_{g,f} >^{1/2}$ (v/c) are compared to E_g and M_f, which shows that in all cases (also for the nonstable vector states) momentum matching and energy-momentum conservation is fulfilled. It should be noted that the electron radius [6] is quite uncertain; from the estimates in Fig. 3 the radii of the other leptons have the same relative uncertainties.

Tablr 1. Solutions for different systems n with $\kappa = 1.35$ and $\alpha = 2.14$. All dimensional quantities in GeV or fm. Upper limit on the electron radius R_{elec} taken from ref. [6] and rms-radius of p_M from ref. [9].

syst.	n	b	(v/c)	$< q^2_{g_s} >^{1/2}$	$< r^2_{g_s} >^{1/2}$	R_{exp}	M_s	M_{exp}
e	4	$2.1\ 10^{-10}$	$2.45\ 10^{-13}$	$2.11\ 10^{9}$	$1.9\ 10^{-10}$	$< 10^{-9}$	$0.51\ 10^{-3}$	$0.51\ 10^{-3}$
μ	3	$3.9\ 10^{-6}$	$9.22\ 10^{-7}$	$1.14\ 10^{5}$	$3.5\ 10^{-6}$	–	0.105	0.105
τ	2	$9.6\ 10^{-3}$	$4.1\ 10^{-2}$	$4.68\ 10^{1}$	$8.5\ 10^{-3}$	–	1.97	1.97
p_M	1	0.44	0.29	0.93	0.56	∼0.74	0.94	0.94

system	s	$< q^2_g >^{1/2}$ (v/c)	E_g	$< q^2_f >^{1/2}$ (v/c)	M_f
e	0	$0.51 \pm 0.03\ 10^{-3}$	-0.52 10^{-3}	$0.50 \pm 0.05\ 10^{-3}$	$0.51\ 10^{-3}$
$(e$	1)	$0.76 \pm 0.1\ 10^{-3}$	-0.84 10^{-3}	$2.8 \pm 0.3\ 10^{-3}$	$2.5\ 10^{-3}$
μ	0	0.105 ± 0.01	-0.107	0.10 ± 0.02	0.105
$(\mu$	1)	0.155 ± 0.02	-0.17	0.42 ± 0.1	0.505
τ	0	1.98 ± 0.2	-1.96	1.98 ± 0.5	1.97
$(\tau$	1)	3.06 ± 0.3	-3.12	8.5 ± 2.5	9.10

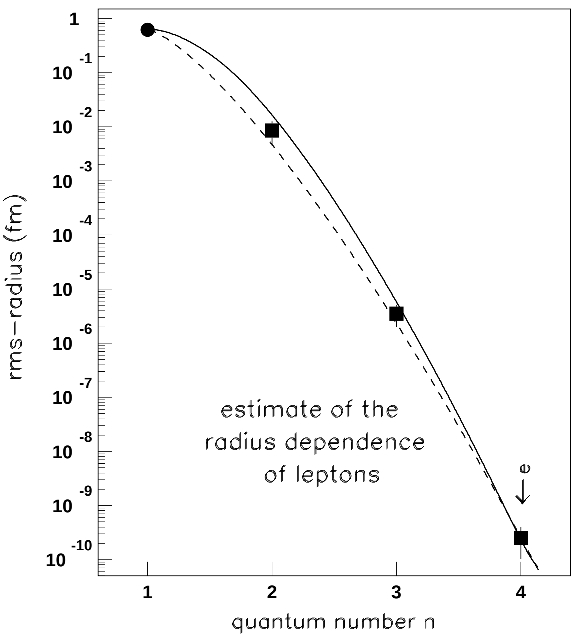

Fig. 3. Estimates of fermion root mean square radii of μ and τ leptons, given by solid squares, from a quantum condition demanding that the radii of the different leptons follow the radial density $w^2_s(r)$ (dashed line) or potential $|V^s_{3g}(r)|$ (solid line) of the basic proton (antiproton) system. The rms-radius of the magnetic part of the proton binding potential is given by solid point, the lines are adjusted to give an electron rms-radius of 2 10^{-10} fm (solid square).

The chiral structure is manifested in the handedness of these bound states. For leptons of $(q^+q^-)^n q^-$ structure ($q^\pm$ are charged massless quantons) the motion is dominated by negative charge, leading to left-handedness; for $(q^+q^-)^n q^+$ antileptons the motion is dominated by positive charge, giving rise to right-handedness.

A similar description should be possible for neutral leptons (neutrinos). But different from charged leptons, only the elementary fermion spins (different for the neutral quantons q^o and $\bar{q}^o$) can give rise to a tiny binding energy.

4 Bound States of Atoms (H-H)

Another type of magnetic bound state may exist in form of weakly bound atoms. Since in this case the mass parameter is given by the reduced mass $\tilde{m} = m_1\ m_2/(m_1 + m_2)$ and not by the generated mass (as for binding of elementary fermions), there are no ambiguities in the extraction of b. One solution has been found with a mass parameter $\tilde{m} = 0.469$ GeV (which corresponds to a system of two hydrogen atoms), slope parameter $b = 2\ 10^6$ fm and $(v/c)^2 = 8.9\ 10^{-31}$, leading to an extremely small binding energy of about 5 10^{-36} eV. The small binding indicates that this type of binding can play a role only in large macroscopic systems of more than 10^{40} H-atoms.

The detailed features of density and potentials are very similar to those of the electron in Fig. 2 by changing the radial scale by about 16 orders of magnitude. Resulting parameters and extracted radii, momenta and energies are given in Table 2. For the scalar state the average momenta of bosons and fermions are 0.22 10^{-6} GeV, indicating that momentum matching is obtained. Multiplied with (v/c) this yields about 6.7 10^{-23} GeV for bosons and fermions, in agreement with the boson and fermion energies, as shown in the lower part of Table 2. However, for the unstable vector state energy-momentum conservation is not fulfilled for bosons and fermions separately, but still for the sum of boson and fermion contributions, respecting total energy-momentum conservation (2.19).

The obtained binding of H-atoms may be related to gravitation, since large amounts of hydrogen atoms exist in the universe. To inspect this possibility, an equivalent first order coupling constant may be defined by relating the radial integral of the potential $V^s_{3g}(r)$ to that of a gravitational potential $V_{gr}(r) = \alpha_{gr}\ \frac{\hbar c}{r}$. This yields

$$\alpha_{gr} = \frac{\int V^s_{3g}(r)\ dr}{\int \frac{\hbar c}{r}\ dr}\ . \tag{4.1}$$

The deduced coupling constant $\alpha_{gr} = 5.9\ 10^{-39}$ may be compared to Newton's gravitational constant $G_N = 6.707\ 10^{-39}(\hbar c)GeV^{-2}$, obtained from a gravitation potential of the form $V_{grav}(r) = G_N(m_1 m_2)/r$. Using m_i=0.94 GeV this yields $G_N/(m_1 m_2) = 5.91\ 10^{-39}$, which is in quantitative agreement with the deduced value of α_{gr}. Therefore, gravitation is likely to be understood as magnetic binding of atoms.

Following this conjecture, magnetic binding of $> 10^{70}$ (hydrogen) atoms leads to large (gravitational) systems with masses and rotation profiles compatible to those observed for galaxies. Other characteristics of gravitation can be understood also, as the deflection of light by solar or galactic systems (gravitational lensing), which is explained by optical deflection on electromagnetic potentials, similar to the scattering of photons from electrons (Compton scattering). As a final point of interest, the present formalism gives rise to a natural solution of Bentley's paradox [10], since the interactions in the present approach (shown e.g. in Fig. 2) fall off much faster with distance than Newton's $1/r$ gravitational potential.

Tablr 2. Solution of an atomic systems bound magnetically, using $\kappa = 1.35$ and $\alpha = 2.14$. All dimensional quantities in GeV or fm.

system	$\tilde{m}$	b	(v/c)	$< q^2_{w_s} >^{1/2}$	$< r^2_{\psi_s} >^{1/2}$	M_s
$H-H$	0.469	2 10^6	3.0 10^{-16}	$0.22 \pm 0.01\ 10^{-6}$	1.8 10^6	4.7 10^{-45}

s	$< q^2_g >^{1/2}$ (v/c)	$< q^2_f >^{1/2}$ (v/c)	$\sum < q^2_{g,f} >^{1/2}$ (v/c)	E_g	xM_f	$xM_f - E_g$
0	6.7±0.1 10^{-23}	6.6±0.2 10^{-23}	13.5±0.2 10^{-23}	-6.8 10^{-23}	6.7 10^{-23}	13.5 10^{-23}
1	6.7±0.2 10^{-23}	8.9±0.5 10^{-23}	15.6±0.2 10^{-23}	-11.0 10^{-23}	4.3 10^{-23}	15.3 10^{-23}

5 Summary

Starting from quantum field theory a solution of particles has been constructed, which are bound by magnetic forces. The structure of this theory is similar to that for hadrons and atoms, but parity symmetry is broken due to the vector structure of magnetic forces. For elementary fermions this gives rise to bound states, which are of chiral structure, left-handed leptons e, μ and τ and right-handed antileptons $\bar{e}$, $\bar{\mu}$ and $\bar{\tau}$. A similar description may be possible for neutrinos, but for these objects only a tiny binding is possible arising entirely from the elementary fermion spins.

In addition, a magnetically bound system of hydrogen atoms has been found, which shows an equivalent first order coupling constant in agreement with Newton's gravitational constant G_N. This type of bound state may be the origin of gravitation.

The present bound state formalism has the most simple symmetry structure possible, based on massless elementary bosons and fermions only and requires no external parameters. Further, about 10 boundary conditions could be satisfied by one or two adjustable parameters. This may be taken as strong indication of a really fundamental theory, in which all free particles of nature in the hadronic, leptonic, atomic (and probably gravitational) sector can be understood.

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Competing Interests

Authors have declared that no competing interests exist.

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