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On Construction of Modified Class of Estimators for Population Variance using Auxiliary Attribute

Chandni Kumari, Ratan Kumar Thakur

***Abstract:** In this paper, an improved estimator for population variance has been proposed to improvise the log-type estimators proposed by Kumari et al. (2019). The properties of proposed estimators are derived up to the first order of approximation. The proposed estimator found to be better than the existing estimators in the sense of mean squraed error and percent relative efficiency. A numerical study is included to support the use of the suggested classes of estimators.*

***Keywords:** Population Variance Estimators in The Sense of Mean Squared Error*

I. INTRODUCTION

In sample surveys, auxiliary information always plays a vital role in better estimation of the parameter under investigation. This information can be used to improve the precision of the estimators. The suitable utilization of this auxiliary information can reduce the MSE of the sample mean, thus resulting in more efficient estimators. This paper, is an attempt to extend the powerful Searls approach to the traditional estimators using auxiliary information regard- ing to variables in simple random sampling. Many authors like, Singh et al. (1973), Das and Tripathi (1978), Sisodia and Dwivedi (1981), Isaki (1983), Bahl and Tuteja (1991), Prasad and Singh (1992), Swain (1994), Garcia and Cebrian (1996), Upadhaya and Singh (2001), Kalidar and Cingi (2006a, 2006b); Gupta and Shabbir (2006, 2007), Yadav and Kadilar (2013, 2014) had proposed an improved ratio estimators using Searls type estimators. Recently, Bhushan et al. (2017) among others; Kumari et al. (2019) have made the use of logarithmic relationship between the auxiliary attribute and study variable as logarithm func- tion which is very common in various branches of science as well as non-science disciplines. In this paper, some improved logarithmic estimators are proposed for improving the efficiency of the Kumari (2019) estimators as these classes of estimators are expected to improve the mean squared error. The proposed estimators would work considerably well in case when the study variable is logarithmically related to the auxiliary attribute.

Consider a finite population U = $U_1, U_2, \ldots, U_N$ of size N from which a sample of size n is drawn according to simple random sampling without replacement (SRSWOR). Let y_i and f_i denotes the values of the study and auxiliary attribute for the ith unit $(i=1,2,\ldots,N)$, of the population. Further, let $\bar{y}$ and $\bar{f}$ be the sample means and

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* Correspondence Author

Chandni Kumari*, Department of Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow (U.P), India.

Ratan Kumar Thakur, Department of Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow (U.P), India.

$s_y^2=\sum_{i=1}^{n}(y_i-\bar{y})^2/(n-1)$ and $s_f^2=\sum_{i=1}^{n}(f_i-\bar{f})^2/(n-1)$ be the sample variance of the study and auxiliary attribute respectively

II. ESTIMATORS AVAILABLE IN LITERATURE

2.1. Conventional Variance Estimator

$$t_0 = s_y^2$$

The bias and variance of t_0 to the first order of approfimation, are given a

$$B(t_0) = 0$$
$$V(t_0) = S_y^4 I b_{2y}^*$$

2.2. Isaki Ratio Estimator

$$t_1 = s_y^2\left(\frac{S_f^2}{s_f^2}\right)$$

The bias and MSE of t_1 to the first degree of approfimation are given as

$$B(t_1) = S_y^2 I[b_{2f}^* - I_{22yf}^*]$$
$$V(t_1) = S_y^4 I\left[b_{2y}^* + b_{2f}^* - 2I_{22yf}^*\right]$$

2.3. Conventional Product Estimator

$$t_2 = s_y^2\left(\frac{s_f^2}{S_f^2}\right)$$

The bias and MSE of t_2 upto the first order of approfimation are given as

$$B(t_2) = S_y^4 I I_{22}^*$$
$$V(t_2) = S_y^4 I\left[b_{2y}^* + b_{2f}^* + 2I_{22yf}^*\right]$$

2.4. Isaki Regression Estimator

Isaki (1983) suggested the following regression estimator for population variance

$$t_3 = s_y^2 + b(S_f^2 - s_f^2)$$

where b is a sample regression coefficient whose population regression coefficient is β.

The bias and MSE of t_3 to the first degree of approfimation are given as

$$
\begin{aligned}
B(t_3) &= 0 \\
MSE(t_3) &= S_y^4 I \left[b_{2y}^* - \frac{I_{22yf}^{*2}}{b_{2f}^*} \right]
\end{aligned}
$$

2.5. Singh et al. Estimator

Singh et al. (1973) Considered the Following Estimator

$$t_4 = \alpha_4 \, s_y^2$$

Where α_4 is a Searl (1964) constant. The optimum value of Searls constant is $\alpha_4 = n/(n+b_{2y}^*)$ for which the mean squared error is minimum.

$$MSE(t_4)_{opt} = S_y^4 I \left[\frac{n b_{2y}^*}{n + b_{2y}^*} \right]$$

2.6. Das and Tripathi Estimator

$$t_5 = s_y^2 \left[\frac{S_f^2}{S_f^2 + \alpha_5 (s_f^2 - S_f^2)} \right]$$

Where α_5 is a constant. The bias and MSE of t_5 to the first degree of approfimation is given as

$$
\begin{aligned}
B(t_5) &= S_y^2 I [\alpha_5^2 b_{2y}^* - \alpha_5 I_{22yf}^*] \\
V(t_5) &= S_y^4 I \left[b_{2y}^* + \alpha_5^2 b_{2f}^* - 2\alpha_5 I_{22yf}^* \right]
\end{aligned}
$$

The MSE of t_5 is optimum for $\alpha_5 = I_{22}^* / b_{2f}^*$ and is given by

$$MSE(t_5)_{opt} = S_y^4 I \left[b_{2y}^* - \frac{I_{22}^{*2}}{b_{2f}^*} \right]$$

2.7. Prasad and Singh Estimator

Prasad and Singh (1992) introduced the following estimator

$$t_6 = \alpha_6 \left(s_y^2 \frac{S_f^2}{s_f^2} \right)$$

where α_6 is a Searls content.

The bias and MSE of t_6 to the first degree of approfimation is given as

$$
\begin{aligned}
B(t_6) &= S_y^2 I [\alpha_6 (n + b_{2f}^* | - I_{22yf}^*) - n] \\
V(t_6) &= S_y^4 I \left[\alpha_6^2 (n + b_{2y}^* + 3b_{2f}^* - 4I_{22yf}^*) - 2\alpha_6 (n + b_{2f}^* - I_{22yf}^*) - n \right]
\end{aligned}
$$

The MSE of t_6 is optimum for $\alpha_6 = (n + b_{2f}^* - I_{22yf}^*)/(n + b_{2y}^* + 3b_{2f}^* - 4I_{22yf}^*)$

and is given as

$$MSE(t_6)_{opt} = S_y^4 I \left[n - \frac{(n + b_{2f}^* - I_{22yf}^*)^2}{n + b_{2y}^* + 3b_{2f}^* - 4I_{22yf}^*} \right]$$

2.8. Garcia and Cebrian estimator

Garcia and Cebrian (1996) introduced the following estimator

$$t_7 = s_y^2 \left(\frac{S_f^2}{s_f^2} \right)^{\alpha_7}$$

where α_7 is a Searls contant.

The bias and MSE of t_7 to the first degree of approfimation is given as

$$
\begin{aligned}
B(t_7) &= S_y^2 I [\alpha_7 \frac{(\alpha_5 + 1)}{2} b_{2f}^* - \alpha_7 I_{22yf}^*] \\
V(t_7) &= S_y^4 I \left[b_{2y}^* + \alpha_7^2 b_{2f}^* - 2\alpha_7 I_{22yf}^* \right]
\end{aligned}
$$

The MSE of t_7 is optimum for $\alpha_7 = I_{22}^* / b_{2f}^*$ and is given as

$$MSE(t_7)_{opt} = S_y^4 I \left[b_{2y}^* - \frac{I_{22yf}^{*2}}{b_{2f}^*} \right]$$

2.9. Upadhaya and Singh Estimator

Upadhaya and Singh (2001) suggested following estimator

$$t_8 = s_y^2 + \alpha_6 (S_f^2 - s_f^2)$$

where α_8 is a constant. The MSE of t_8 to the first degree of approfimation is given as

$$V(t_8) = S_y^4 I \left[b_{2y}^* + \alpha_8^2 \frac{S_f^4}{S_y^4} b_{2f}^* - 2\alpha_8 \frac{S_f^2}{S_y^2} I_{22yf}^* \right]$$

The MSE of t_8 is optimum for $\alpha_8 = \dfrac{S_y^2 I_{22yf}^*}{S_f^2 b_{2f}^*}$ and is given as

$$MSE(t_8)_{opt} = S_y^4 I \left[b_{2y}^* - \frac{I_{22yf}^{*2}}{b_{2f}^*} \right]$$

2.10. Shabbir and Gupta (2006) estimator

Sabbir and Gupta (2006) Proposed the Following Estimator

$$t_9 = \lambda t_m$$

where λ is a Searls (1964) contant whose value is to be determined later. Here t_m is a combination of Singh et al. (1973), Prasad and Singh (1992) and is defined as

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$$t_m = K_1 s_y^2 + K_2 s_y^2 \left(\frac{S_f^2}{s_f^2} \right)$$

where K_1 and K_2 are the weights such that $K_1 + K_2 = 1$ The bias and MSE of t_9 to the first degree of approfimation is given as

$$\begin{aligned} B(t_9) &= S_y^2 I \left[\lambda \left\{ n + K_2 (b_{2f}^* - I_{22yf}^*) \right\} - n \right] \\ V(t_9) &= S_y^4 I \left[\lambda^2 \left(n + b_{2y}^* + K_2^2 b_{2f}^* + 2K_2 b_{2f}^* - 4K_2 I_{22yf}^* \right) - 2\lambda \left(n + K_2 b_{2f}^* - K_2 I_{22yf}^* \right) + n \right] \end{aligned}$$

The MSE of t_9 is optimum for

$$\lambda = \frac{n + K_2 (b_{2f}^* - I_{22yf}^*)}{n + b_{2y}^* + K_2^2 b_{2f}^* + 2K_2 b_{2f}^* - 4K_2 I_{22yf}^*} \text{ and}$$

$$K_2 = \frac{I_{22}^*}{b_{2f}^*} \text{ is given as}$$

$$MSE(t_9)_{opt} = S_y^4 I \left[n - \frac{\left\{ n + I_{22yf}^* - \frac{I_{22yf}^{*2}}{b_{2f}^*} \right\}^2}{n + b_{2y}^* + 2I_{22yf}^* - 3\frac{I_{22yf}^{*2}}{b_{2f}^*}} \right]$$

2.11. Shabbir and Gupta (2007) estimator

$$t_{10} = k_1 s_y^2 + k_2 (S_f^2 - s_f^2) efp \left(\frac{S_f^2 - s_f^2}{S_f^2 + s_f^2} \right)$$

where k_1 and k_2 are suitably chosen coantants.

Situation 1. $k_1 + k_2 = 1$

The bias and MSE of t_{10} the first degree of approfimation are given as

$$\begin{aligned} B(t_{10}) &= (k_1 - 1) \left(S_y^2 - \frac{1}{2} S_f^2 I b_{2f}^* \right) + k_1 S_y^2 I \left[\frac{3}{8} b_{2f}^* - \frac{1}{2} I_{22yf}^* \right] \\ V(t_{10}) &= S_y^4 I \left[k_1^2 \left\{ n + b_{2y}^* + \frac{S_f^4}{S_y^4} b_{2f}^* + \frac{1}{4} b_{2f}^* - I_{22yf}^* + 2\frac{S_f^2}{S_y^2} \left(I_{22yf}^* - \frac{1}{2} b_{2f}^* \right) \right\} \right] \\ &\quad -2S_y^4 I k_1 \left[\left\{ n + \frac{S_f^4}{S_y^4} + \frac{S_f^2}{S_y^2} \left(I_{22yf}^* - \frac{1}{2} b_{2f}^* \right) b_{2f}^* - K_2 I_{22yf}^* \right\} + n + \frac{S_f^4}{S_y^4} b_{2f}^* \right] \end{aligned}$$

The optimum value of k_1 which minimizes MSE (t_{10}) is given as

$$k_1 = \frac{A_1 + A_3}{A_1 + A_2 + 2A_3}, \text{ where } A_1 = n + \frac{S_f^4}{S_y^4} b_{2f}^*,\ A_2 = b_{2y}^* + \frac{1}{4} b_{2f}^* - I_{22yf}^*,\ A_3 = \frac{S_f^2}{S_y^2} \left(I_{22yf}^* - \frac{1}{2} b_{2f}^* \right)$$

$$MSE(t_{10})_{opt} = S_y^4 I \left[A_1 - \frac{(A_1 + A_3)^2}{A_1 + A_2 + 2A_3} \right]$$

Situation 2. Unconstrained choice of k_1 and k_2

The bias and MSE of t_{10}^* the first degree of approfimation are given as

$$B(t_{10}^*) = (k_1 - 1)S_y^2 + \frac{1}{2}K_2 S_f^2 I b_{2f}^* + k_1 S_y^2 I \left[\frac{3}{8} b_{2f}^* - \frac{1}{2} I_{22yf}^* \right]$$

$$MSE(t_{10}^*)_{opt} = S_y^4 I \left[\frac{Var(\hat{S}_{reg})}{1 + \frac{Var(\hat{S}_{reg})}{S_y^4}} \right]$$

2.12. Kalidar and Cingi estimator

Kadilar and Cingi (2006) suggested the following ratio type estimator

$$t_{11} = \omega_1 s_y^2 + \omega_2 \left(s_y^2 \frac{S_f^2}{s_f^2} \right) \tau$$
$$\text{where } \omega_1 + \omega_2 = 1$$

The optimum bias and MSE of t_{11} are given as

$$B(t_{11}) = S_y^2 \left[(\omega_1^* - 1) + \omega_1^* \tau I b_{2f}^* \right]$$
$$MSE(t_{11}) = S_y^4 I \left[\lambda^2 \left(n + b_{2y}^* + K_2^2 b_{2f}^* + 2K_2 b_{2f}^* - 4K_2 I_{22yf}^* \right) - 2\lambda \left(n + K_2 b_{2f}^* - K_2 I_{22yf}^* \right) + n \right]$$

The MSE of t_{11} is optimum for $\lambda = \frac{n + K_2(b_{2f}^* - I_{22yf}^*)}{n + b_{2y}^* + K_2^2 b_{2f}^* + 2K_2 b_{2f}^* - 4K_2 I_{22yf}^*}$ and $K_2 = \frac{I_{22yf}^*}{b_{2f}^*}$ is given by

$$MSE(t_{11})_{opt} = S_y^4 I \left[n - \frac{\left\{ n + I_{22yf}^* - \frac{I_{22}^{*2}}{b_{2f}^*} \right\}^2}{n + b_{2y}^* + 2I_{22yf}^* - 3\frac{I_{22}^{*2}}{b_{2f}^*}} \right]$$

2.13. Yadav and Kalidar (2013) Estimator

Yadav and Kadilar (2013) introduced the following estimator

$$t_{12} = s_y^2 efp \left[1 - \frac{a_{12} s_f^2}{S_f^2 + (a_{12} - 1)s_f^2} \right]$$

where a_{12} is a constant. The optimum MSE of t_{12} to the first degree of approfimation is given as

$$MSE(t_{12})_{opt} \;=\; S_y^4 I \left\{ b_{2y}^* - \frac{I_{22yf}^{*2}}{b_{2f}^*} \right\}$$

2.14. Yadav and Kadilar (2014) estimator

Yadav and Kadilar (2014) introduced the following ratio-product-ratio estimator

$$t_{\alpha,\beta_{12}} = \alpha \left[\frac{(1-\beta)s_f^2 + \beta S_f^2}{\beta s_f^2 + (1-\beta)S_f^2} \right] s_y^2 + (1-\alpha) \left[\frac{\beta s_f^2 + (1-\beta)S_f^2}{(1-\beta)s_f^2 + \beta S_f^2} \right] s_y 2$$

where a and β are real constant. The optimum MSE of $t_{a,\beta_{12}}$ to the first degree of approfima-tion is given as

$$MSE(t_{12}^*)_{opt} \;=\; S_y^4 I b_{2y}^*$$

III. THE PROPOSED ESTIMATORS

We propose the following new classes of log-type estimators for the population variance S_y^2 as:

$$T_1^* \;=\; w_1 \, s_y^2 \left[1 + log \left(\frac{S_f^2}{s_f^2} \right) \right]^{a_1} \qquad (3.1)$$

$$T_2^* \;=\; w_2 \, s_y^2 \left[1 + a_2 \, log \left(\frac{S_f^2}{s_f^2} \right) \right] \qquad (3.2)$$

$$T_3^* \;=\; w_3 \, s_y^2 \left[1 + log \left(\frac{S_f^{*2}}{s_f^{*2}} \right) \right]^{a_3} \qquad (3.3)$$

$$T_4^* \;=\; w_4 \, s_y^2 \left[1 + a_4 \, log \left(\frac{S_f^{*2}}{s_f^{*2}} \right) \right] \qquad (3.4)$$

where

$$s_f^{*2} \;=\; a s_f^2 + b$$
$$S_f^{*2} \;=\; a S_f^2 + b$$

such that $a(\neq 0), b$ are either real numbers or functions of the known parameters of the auxiliary attribute f such as the standard deviations S_f, coefficient of variation C_f, coefficient of kurtosis b_{2f}, coefficient of skewness b_{1f}, and correlation coefficient ρ of the population. If $a_i = 0$, then the proposed estimator becomes the usual per unit variance estimator s_y^2. If $a_i = +1$, then the proposed estimator works as ratio type estimator and if $a_i = -1$, then the proposed estimator works as product type estimator having efficiency conditions equivalent to that of generalized product and ratio estimators respectively.

IV. BIAS AND MSE OF PROPOSED ESTIMATORS

In this paper, the biases and the mean square error(s) of all the estimators are considered up to the terms of order n^{-1}.

Theorem 1. The bias and MSE of T_1^* are given by

$$Bias\,(T_1^*) = S_y^2 \left[w_1 \left\{ 1 - a_1 I I_{22yf}^* + a_1 I b_{2f}^* + \frac{a_1^2}{2} I b_{2f}^* \right\} - 1 \right]$$

$$MSE\,(T_1^*) = S_y^4 \, w_1^2 \left[1 + I(b_{2y}^* + 2a_1^2 b_{2f}^* - 4a_1 b_{2f}^* + 2a_1 b_{2f}^*) \right] - 2S_y^4 w_1 \left[1 + I(a_1 b_{2f}^* + \frac{a_1^2 b_{2f}^*}{2} - a_1 I_{22}^*) \right] + 1$$

Proof. Consider the estimator,

$$
\begin{aligned}
T_1^* &= w_1\, s_y^2 \left[1 + log\left(\frac{S_f^2}{s_f^2}\right)\right]^{a_1} \\
&= w_1 S_y^2\,(1+\epsilon_0)\,[1 - log\,(1+\epsilon_1)]^{a_1} \\
&= w_1\, S_y^2\,(1+\epsilon_0)\left[1 - a_1\left(\epsilon_1 - \frac{\epsilon_1^2}{2}\right) + \left(\frac{a_1\,(a_1+1)}{2}\right)\left(\epsilon_1 - \frac{\epsilon_1^2}{2}\right)^2\right] \\
&= (w_1 - 1)\, S_y^2 + w_1 S_y^2\left[\epsilon_0 - a_1\left(\epsilon_1 - \frac{\epsilon_1^2}{2}\right) + \frac{a_1\,(a_1+1)}{2}\epsilon_1^2 - a_1\epsilon_0\epsilon_1\right]
\end{aligned}
$$

Squaring on both the sides and then taking expectation over the proposed estimator, we have

$$
\begin{aligned}
&= S_y^4\,(w_1-1)^2 + w_1^2\, S_y^4\,\left[E(\epsilon_0)^2 + a_1^2\, E(\epsilon_1)^2 + 2\, a_1\, E(\epsilon_0\,\epsilon_1)\right] \\
&+\; 2\, S_y^4 \quad w_1\;(w_1-1)\left[a_1 E(\epsilon_1)^2 - a_1 E(\epsilon_0\epsilon_1) + \frac{a_1^2}{2}\, E(\epsilon_1^2)\right]
\end{aligned}
$$

Using the results from Sukhatme and Sukhatme, we have

$E(\epsilon_0) = 0 = E(\epsilon_1)$, $E(\epsilon_0)^2 = Ib_{2y}^*$, $E(\epsilon_1)^2 = Ib_{2f}^*$, $E(\epsilon_0\epsilon_1) = II_{22}^*$ where $b_{2f}^* = b_{2f} - 1$, $b_{2y}^* = b_{2y} - 1$ and $I_{22}^* = I_{22} - 1$; $I_{pq} = m_{pq}/m_{20}^{\frac{p}{2}}\, m_{02}^{\frac{q}{2}}$, $\quad m_{pq} = \sum_{i=1}^{N}(Y_i - \bar{Y})^p (Y_i - \bar{Y})^q/N$, $I = 1/N$, $b_{2y} = m_{40}/m_{20}^2$, $\quad b_{2f} = m_{04}/m_{02}^2$ are the coefficient of kurtosis of y and f respectively.

Now, substituting the above results in (4:1), we get,

$$
\begin{aligned}
MSE\,(T_1^*) = S_y^4\, w_1^2\,\left[1 + I(b_{2y}^* + 2a_1^2 b_{2f}^* - 4a_1 I_{22yf}^* + 2a_1 b_{2f}^*\right] \\
- 2S_y^4 w_1\left[1 + I(a_1 b_{2f}^* + \frac{a_1^2 b_{2f}^*}{2} - a_1 I_{22yf}^*)\right] + 1
\end{aligned}
$$

Corollary 2. The optimum value of a_1 and w_1 are

$$a_1 = \frac{I_{22}^*}{b_{2f}}$$

$$w_1 = \frac{B_1}{A_1}$$

where

$$
\begin{aligned}
A_1 &= 1 + I(b_{2y}^* + 2a_1^2 b_{2f}^* - 4a_1 I_{22yf}^* + 2a_1 b_{2f}^* \\
B_1 &= 1 + I(a_1 b_{2f}^* + \frac{a_1^2 b_{2f}^*}{2} - a_1 I_{22yf}^*)
\end{aligned}
$$

respectively. Also, the minimum mean squared error of T_1^* is

$$MSE\,(T_1^*)_{opt} = S_y^4\, I\left[1 - \frac{B_1^2}{A_1}\right]$$

Proof. Obvious, using Theorem 1.

Theorem 3. The bias and MSE of T_2^* are given by

$$\begin{aligned} Bias\left(T_2^*\right) &= S_y^2\left[w_2\left\{1-a_2 I I_{22yf}^* + \frac{a_2^2}{2} I b_{2f}^*\right\}-1\right] \\ MSE\left(T_2^*\right) &= S_y^4\, w_2^2\left[1+I(b_{2y}^* + a_2^2 b_{2f}^* - 4a_2 I_{22yf}^* + a_2 b_{2f}^*\right] \\ &\quad -2S_y^4 w_2\left[1+I\left(\frac{a_2 b_{2f}^*}{2} - a_2 I_{22yf}^*\right)\right]+1 \end{aligned}$$

respectively.

Proof. Trivial and similar to derivation of Theorem 1, hence omitted.

Corollary 4. The optimum value of a_2 and w_2 are

$$a_2 = \frac{I_{22}^*}{b_{2f}^*} \quad and \quad w_2 = \frac{B_2}{A_2}$$

where

$$\begin{aligned} A_2 &= 1+I(b_{2y}^* + a_2^2 b_{2f}^* - 4a_2 I_{22yf}^* + a_2 b_{2f}^*) \\ B_2 &= 1+I\left(\frac{a_2 b_{2f}^*}{2} - a_2 I_{22yf}^*\right) \end{aligned}$$

Also, the minimum mean squared error of T_2^* is

$$MSE\left(T_2^*\right)_{opt} = S_y^4\left[1-\frac{B_2^2}{A_2}\right]$$

Theorem 5. The bias and MSE of T_3^* are given by

$$\begin{aligned} Bias\left(T_3^*\right) &= S_y^2\left[w_1\left\{1-a_3\eta I I_{22yf}^* + a_3\eta^2 I b_{2f}^* + \frac{a_3^2}{2} I\eta^2 b_{2f}^*\right\}-1\right] \\ MSE\left(T_3^*\right) &= S_y^4\, w_1^2\left[1+I(b_{2y}^* + 2a_3^2\eta^2 b_{2f}^* - 4a_3\eta b_{2f}^* + 2a_3\eta^2 b_{2f}^*)\right] \\ &\quad -2S_y^4 w_1\left[1+I(a_3\eta^2 b_{2f}^* + \frac{a_3^2}{2}\eta^2 b_{2f}^* - a_3\eta I_{22yf}^*)\right]+1 \end{aligned}$$

respectively.

Proof. Trivial and similar to derivation of Theorem 1, hence omitted.

Corollary 6. The optimum value of a_3 and w_3 is

$$a_3 = \frac{I_{22yf}^*}{b_{2f}^*} \quad and \quad w_3 = \frac{B_3}{A_3}$$

where

$$\begin{aligned} A_3 &= 1+I(b_{2y}^* + 2a_3^2\eta^2 b_{2f}^* - 4a_3\eta b_{2f}^* + 2a_3\eta^2 b_{2f}^*) \\ B_3 &= 1+I(a_3\eta^2 b_{2f}^* + \frac{a_3^2}{2}\eta^2 b_{2f}^* - a_3\eta I_{22yf}^*) \end{aligned}$$

Also, the minimum mean squared error of T_3^* is

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$$MSE\left(T_3^*\right)_{opt} = S_y^4\left[1 - \frac{B_3^2}{A_3}\right]$$

Theorem 7. The bias and MSE of T_4^* are given by

$$\begin{aligned}
Bias\left(T_4^*\right) &= S_y^2\left[w_4\left\{1 - a_4\eta I I^*_{22yf} + \frac{a_4^2}{2} I\eta^2 b^*_{2f}\right\} - 1\right] \\
MSE\left(T_4^*\right) &= S_y^4\, w_4^2\left[1 + I(b^*_{2y} + a_4^2\eta^2 b^*_{2f} - 4a_4\eta I^*_{22yf} + a_4\eta^2 b^*_{2f}\right] \\
&\quad -2S_y^4 w_2\left[1 + I\left(\frac{a_4\eta^2 b^*_{2f}}{2} - a_4\eta I^*_{22yf}\right)\right] + 1
\end{aligned}$$

respectively.

Proof. Trivial and similar to derivation of Theorem 1, hence omitted.

Corollary 8. The optimum value of a_4 and w_4 is

$$a_4 = \frac{I^*_{22}}{b^*_{2f}} \quad and \quad w_4 = \frac{B_4}{A_4}$$

where

$$\begin{aligned}
A_4 &= 1 + I(b^*_{2y} + a_4^2\eta^2 b^*_{2f} - 4a_4\eta I^*_{22yf} + a_4\eta^2 b^*_{2f} \\
B_4 &= 1 + I\left(\frac{a_4\eta^2 b^*_{2f}}{2} - a_4\eta I^*_{22yf}\right) + 1
\end{aligned}$$

Also, the minimum mean squared error of T_3^* is

$$MSE\left(T_4^*\right)_{opt} = S_y^4\left[1 - \frac{B_4^2}{A_4}\right]$$

V. COMPARISON OF ESTIMATORS

In this section, we compare the proposed classes of estimators with some important estimators. The comparison will be in terms of their MSEs up to the order of n^{-1} Let us define

$$C_1 = b^*_{2y} + b^*_{2f} - 2I^*_{22},\ C_2 = b^*_{2y} + b^*_{2f} - 2I^*_{22},\ D = b^*_{2y}b^*_{2f} - I^*_{22},\ E = \left[\frac{nb^*_{2y}}{n + b^*_{2y}}\right],\ F =$$

$$\left[n - \frac{(n + b^*_{2f} - I^*_{22})^2}{n + b^*_{2y} + 3b^*_{2f} - 4I^*_{22}}\right], G = \left[n - \frac{\left\{n + I^*_{22} - \frac{I^{*2}_{22}}{b^*_{2f}}\right\}^2}{n + b^*_{2y} + 2I^*_{22} - 3\frac{I^{*2}_{22}}{b^*_{2f}}}\right], H = \left[A_1 - \frac{(A_1 + A_3)^2}{A_1 + A_2 + 2A_3}\right].$$

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$$
\begin{aligned}
MSE(t_0) &> MSE(T_1^*)_{opt} \quad \text{if} \quad b_{2y}^* + \frac{B^2}{A} - n > 0 \\
MSE(t_1) &> MSE(T_1^*)_{opt} \quad \text{if} \quad C_1 - n + \frac{B^2}{A} > 0 \\
MSE(t_2) &> MSE(T_1^*)_{opt} \quad \text{if} \quad C_2 - n + \frac{B^2}{A} > 0 \\
MSE(t_3) &> MSE(T_1^*)_{opt} \quad \text{if} \quad D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0 \\
MSE(t_4) &> MSE(T_1^*)_{opt} \quad \text{if} \quad E - n - \frac{B^2}{A} > 0 \\
MSE(t_5) &> MSE(T_1^*)_{opt} \quad \text{if} \quad D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0 \\
MSE(t_6) &> MSE(T_1^*)_{opt} \quad \text{if} \quad F - n + \frac{B^2}{A} > 0 \\
MSE(t_7) &> MSE(T_1^*)_{opt} \quad \text{if} \quad D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0 \\
MSE(t_8) &> MSE(T_1^*)_{opt} \quad \text{if} \quad D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0 \\
MSE(t_9) &> MSE(T_1^*)_{opt} \quad \text{if} \quad G - n + \frac{B^2}{A} > 0 \\
MSE(t_{10}) &> MSE(T_1^*)_{opt} \quad \text{if} \quad H - n + \frac{B^2}{A} > 0 \\
MSE(t_{11}) &> MSE(T_1^*)_{opt} \quad \text{if} \quad G - n + \frac{B^2}{A} > 0 \\
MSE(t_{12}) &> MSE(T_1^*)_{opt} \quad \text{if} \quad D - \left(n - \frac{B^2}{A}\right) b_{2f}^* > 0
\end{aligned}
\tag{5.-12}
$$

VI. EMPIRICAL STUDY

To compare the efficiency of the suggested class of estimator numerically, we considered nine natural data sets. The description of the population is given below.

Population 1. (Cochran (1977), Pg. no. 107)

y : number of persons per block

f : number of rooms per block

$S_y^2 = 214.69$, $S_f^2 = 56.76$, $b_{2y}^* = 1.2387$, $b_{2f}^* = 1.3523$, $I_{22}^* = 0.5432$, $C_f = 0.1450$, $\bar{f} = 58.8$, $\rho = 0.6515$, $n = 10$.

Population 2. (Cochran (1977), Pg. no. 203)

y : actual weight of peaches on each tree

f : eye estimate of weight of peaches on each tree.

$S_y^2 = 99.81$, $S_f^2 = 85.09$, $b_{2y}^* = 0.9249$, $b_{2f}^* = 1.2932$, $I_{22}^* = 1.1149$, $C_f = 0.1621$, $\bar{f} = 56.9$, $\rho = 0.9937$, $n = 10$.

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Population 3. (Sukhatme P. V. (1970), Pg. no. 185)
y : wheat acreage in 1937
f : wheat acreage in 1936
$S_y^2 = 26456.99$, $S_f^2 = 22355.76$, $b_{2y}^* = 2.1842$, $b_{2f}^* = 1.2030$, $I_{22}^* = 1.5597$, $C_f = 0.5625$, $\bar{f} = 265.8$, $\rho = 0.977$, $n = 10$.

Population 4. (Singh D and Chaudhary F. S., Pg. no. 107). y : number of boats landing at a particular centre
f : catch of fish in quintals.
$S_y^2 = 201324.4$, $S_f^2 = 396.8889$, $b_{2y}^* = 0.9462$, $b_{2f}^* = 0.6078$, $I_{22}^* = 0.6333$, $C_f = 0.7288$, $\bar{X} = 27.3333$, $\rho = 0.9308$, $n=4$.

Population 5. (Singh D and Chaudhary F. S., Pg. no. 141).
y :number of bearing lime trees
f :area under lime (in acres)
$S_y^2 = 6564586.45$, $S_f^2 = 1092.1024$, $b_{2y}^* = 12.2574$, $b_{2f}^* = 4.5788$, $I_{22}^* = 6.7126$, $C_f = 1.4273$, $\bar{X} = 22.6209$, $\rho = 0.9021$, $n=9$.

Population 6. (Choudhary F. S. and Singh D., Pg. no. 176).
y : number of cows in milk enumerated
f : number of cows in milk in the previous year.

$S_y^2 = 332721.2079$, $S_f^2 = 281472.7868$, $b_{2y}^* = 6.2079$, $b_{2f}^* = 5.0043$, $I_{22}^* = 4.9528$, $C_f = 0.8276$, $\bar{X} = 641.05$, $\rho = 0.8933$, $n=8$.

Population 7. (Singh S., Pg. no. 324-325). y : approximate duration of sleep (in minutes)
f : age in years of the persons.
$S_y^2 = 3582.579$, $S_f^2 = 85.2367$, $b_{2y}^* = 1.6678$, $b_{2f}^* = 1.2389$, $I_{22}^* = 0.9961$, $C_f = 0.1349$, $\bar{X} = 67.2667$, $\rho = -0.8552$, $n=9$.

Population 8. (Singh S., Pg. no. 1114). y : approfimate duration of sleep (in minutes)
f : age in years of the persons.
$S_y^2 = 0.0073$, $S_f^2 = 0.0063$, $b_{2y}^* = 2.6323$, $b_{2f}^* = 2.4016$, $I_{22}^* = 1.8351$, $C_f = 1.2352$, $\bar{X} = 0.1831$, $\rho = 0.7789$, $n=11$.

By using the above data set, the percent relative efficiency of the different estimator are given in Table 2.

In the above table, the relative efficiency of the proposed estimator is much better as compared to other estimators for all the data sets given here.

Table 1: PRE of the Estimators with Respect to t_0

Est.	Pop 1	Pop 2	Pop 3	Pop 4	Pop 5	Pop 6	Pop 7	Pop 8
t_0	100	100	100	100	100	100	100	100
t_1	121.38	320.81	815.60	329.16	359.35	475.29	182.39	193.03
t_2	33.68	19.48	33.56	33.54	40.50	29.39	34.04	30.24
t_3	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.00
t_4	112.38	639.14	121.42	30.91	236.19	177.59	118.53	123.93
t_5	121.38	109.23	1347.98	330.96	507.23	475.29	192.40	214.00
t_6	112.95	639.14	818.13	348.83	381.77	558.96	212.28	242.02
t_7	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.00
t_8	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.03
t_9	143.14	749.18	1434.48	349.88	528.70	559.08	220.35	258.35
t_{10}	121.38	489.41	306.84	227.25	319.46	457.77	188.82	228.88
t_{10}^*	82.34	648.39	1369.81	354.05	643.42	552.89	210.93	237.93
t_{11}	82.53	322.84	815.55	329.05	358.77	269.13	182.54	102.02
t_{12}	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.00
$t_{\alpha,\beta_{12}}$	100	100	100	100	100	100	100	100
t_{13}	121.38	639.14	1347.98	330.39	507.23	475.29	192.40	214.00
T_1^*	147.02	1163.41	1863.95	439.47	991.93	2120.43	247.66	311.67

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VII. CONCLUSION

The present study extends the idea of Bhushan and Kumari (2019) regarding the effective use of auxiliary information if the relationship between the study variable and the auxiliary attribute is of logarithmic type. Further, the efficiency of the proposed estimators are compared with some conventional estimators and some recent estimators of Singh et al. (1973), Das and Tripathi (1978), Sisodia and Dwivedi (1981), Isaki (1983), Bahl and Tuteja (1991), Prasad and Singh (1992), Swain (1994), Garcia and Cebrian (1996), Upadhaya and Singh (2001), Kalidar and Cingi (2006a, 2006b); Gupta and Shabbir (2006, 2007), Yadav and Kadilar (2013, 2014). The proposed estimator is most efficient than all the estimators. This study is also supported through an empirical study and the result of this study is quite encouraging.

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AUTHOR DETAILS

Dr. Chandni Kumari received the PhD degree in Statistics from Babasaheb Bhimrao Ambedkar University, Lucknow, India. Her research interests are in the areas of applied statistics. In particular, she has keen interest on Sample Surveys. She has published many research articles in reputed international and national journals.

Mr. Ratan Kumar Thakur is perusing Ph.D. in the Department of Statistics from Babasaheb Bhimrao Ambedkar University, Lucknow, India. His main area of research interests are Applied statistics, Neural Networks, Linear Regression, Time Series Forecasting, Sample Survey.