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New Exact Traveling Wave Solutions to Burgers Equation

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

In this letter, we seek new traveling wave solutions to Burgers equation via a new approach of improved (G'/G) -expansion method. We handle the calculations with the aid of computer software Maple-13. As a result, many periodic and soliton like solutions have been achieved in terms of the hyperbolic functions, trigonometric functions, exp-functions and rational function solutions. The method is very simple for solving nonlinear evolution equations (NLEEs). Further, both two and three-dimensional plots of the obtained wave solutions are also given to imagine the dynamics of the equation.

Keywords: Burgers equation; new approach of improved (G'/G) -expansion; traveling wave solutions; exact solutions.

Mathematics subject classification: 35K99, 35P05, 35P99.

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1. INTRODUCTION

Nonlinear evolution equations (NLEEs) that models most phenomena have been the subject of study in various branches of Mathematical and Physical sciences such as physics, biology, chemistry, biochemistry, applied and pure mathematics, applied and pure physics etc. Recently, both mathematicians and physicist have devoted huge exertion to study exact solution of the NLEEs and many powerful methods have been established such as, the inverse scattering transform method [1], the complex hyperbolic function method [2,3], the ansatz method [4,5], the (G'/G) -expansion method [6-11], the modified simple equation method [12,13], the exp-functions method[14], the sine-cosine method [15], the Jacobi elliptic function expansion method [16,17], the F-expansion method[18,19], the Backlund transformation method [20], the Darboux transformation method [21], the homogeneous balance method [22-24], the Adomian decomposition method [25], the auxiliary equation method [26], the $\exp(-\varphi(\xi))$ -expansion method [27,28] and so on. Recently, Kheiri et. al. [29] found some traveling wave solutions of the Burgers equation using basic (G'/G) expansion method and found only three solutions a trigonometric, a hyperbolic and a rational function solution. They consider the Burger equations of the form:

$$u_t + uu_x = u_{xx} \tag{1}$$

Introduced by Burgers [30] as a model for turbulence, equation (1) and its inviscid counterpart $u_t + uu_x = 0$ are essential for their role in modeling a wide array of physical systems such as traffic flow, shallow water waves.

To the best of our knowledge, this equation is not solved via a new approach of improved (G'/G) -expansion method with the auxiliary equations $GG'' = AG^2 + BGG' + CG'^2$ considering both the positive and negative values of suffices in the considered solutions.

In this article, we study the traveling wave solutions of the Burgers equation via new approach of improved (G'/G) -expansion method with auxiliary equation $GG'' = AG^2 + BGG' + CG'^2$.

2. NEW APPROACH OF IMPROVED (G′/G)-EXPANSION METHOD

Let us consider a NLEE for $U(x,t)$ in the form

$$P(U, U_x, U_t, U_{xx}, U_{xt}, U_{tt}, \ldots\ldots) = 0 \tag{2}$$

Where P is a polynomial, which includes nonlinear terms and the highest order derivatives.The transformation

$$U(x,t) = u(\xi),\ \xi = x - wt, \tag{3}$$

permits us rising Eq.(2) to an ODE for $u = u(\xi)$

$$P(u, u', u'', \cdots\cdots\cdots) = 0 \tag{4}$$

Consider that the solution of ODE Eq.(3) can be uttered by a polynomial in (G'/G) as follows

$$u = \sum_{j=-m}^{m} a_j (G'(\xi)/G(\xi))^j \tag{5}$$

Where

$G(\xi)$ satisfies the ODE

$$GG'' = AG^2 + BGG' + CG'^2, \tag{6}$$

where A, B, C are real parameters and $C \neq 1$, then the solutions of ODE Eq.(6) are

Case-1: For the condition $B \neq 0,\ \Delta_1 = B^2 + 4A(1-C) \geq 0,$ we have

$$G'/G = \frac{B}{2(1-C)} + \frac{\sqrt{\Delta_1}}{2(1-C)}\left(\frac{C_1 \exp\left(\frac{\sqrt{\Delta_1}}{2}\xi\right) + C_2 \exp\left(-\frac{\sqrt{\Delta_1}}{2}\xi\right)}{C_1 \exp\left(\frac{\sqrt{\Delta_1}}{2}\xi\right) - C_2 \exp\left(-\frac{\sqrt{\Delta_1}}{2}\xi\right)}\right) \tag{7}$$

Case-2: For the condition $B \neq 0,\ \Delta_1 = B^2 + 4A(1-C) < 0,$ we have

$$G'/G = \frac{B}{2(1-C)} + \frac{\sqrt{-\Delta_1}}{2(1-C)}\left(\frac{iC_1\cos\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right) - C_2\sin\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right)}{iC_1\sin\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right)}\right) \tag{8}$$

Case-3: For the condition $B = 0,\ \Delta_2 = A(C-1) \geq 0,$ we have

$$G'/G = \frac{\sqrt{\Delta_2}}{(1-C)}\left(\frac{C_1\cos\left(\sqrt{\Delta_2}\xi\right) + C_2\sin\left(\sqrt{\Delta_2}\xi\right)}{C_1\sin\left(\sqrt{\Delta_2}\xi\right) - C_2\cos\left(\sqrt{\Delta_2}\xi\right)}\right) \tag{9}$$

Case-4: For the condition $B = 0,\ \Delta_2 = A(C-1) < 0,$ we have

$$G'/G = \frac{\sqrt{-\Delta_2}}{(1-C)}\left(\frac{iC_1\cosh\left(\sqrt{-\Delta_2}\xi\right) - C_2\sinh\left(\sqrt{-\Delta_2}\xi\right)}{iC_1\sinh\left(\sqrt{-\Delta_2}\xi\right) - C_2\cosh\left(\sqrt{-\Delta_2}\xi\right)}\right) \tag{10}$$

where C_1 and C_2 are arbitrary constants and $i = \sqrt{-1}$ while $C \neq 1$ $a_i, w, A, B, C;\ j = -m, \cdots\cdots, m$ are constants to be determined later, a_m and a_{-m} are not both zero simultaneously. The value of m can be fixed balancing the highest order derivatives and nonlinear terms in the Eq.(4). Using Eq.(5) and Eq. (6) into Eq.(4) and collecting all terms with the identical order of $(G'/G)^j;\ j = -m, \cdots\cdots, m$, and setting them to zero, yields a set of algebraic equations for unknowns $a_i, w, A, B, C;\ j = -m, \cdots\cdots, m$. Now, solving the algebraic equations for $a_i, w, A, B, C;\ j = -m, \cdots\cdots, m$ with the Maple software and putting in the general solutions of ODE (6), we gain the solutions of Eq. (1).

3. APPLICATION

Burgers' equation (1) is perhaps the simplest model that couples the nonlinear convective behavior of fluids with the dissipative viscous behavior. Introduced by Burgers [30] as a model for turbulence, equation (1) and its inviscid counterpart $u_t + uu_x = 0$ are essential for their role in modeling a wide array of physical systems such as traffic flow, shallow water waves.

In this section, we will use a new of approach improved (G'/G)-expansion method to find the exact traveling wave solutions of the Burgers equation (1). Inserting Eq. (3) into Eq. (1) we amend Eq. (1) into the following ODE:

$$-wu' + uu' - u'' = 0 \tag{11}$$

Integrating Eq. (11) one time with respect to traveling variable ξ yields

$$k - wu + u^2/2 - u' = 0 \tag{12}$$

Homogeneous balance gives $m = 1$ and thus the equation (1) has the following solution

$$u(\xi) = a_0 + a_1(G'/G) + a_{-1}(G/G') \tag{13}$$

Where $u(x,t) = u(\xi),\ \xi = x - wt$ and a_1, a_{-1} are not both zero simultaneously.

Inserting Eq.(13) and Eq.(6) into Eq.(12), let the coefficients of $(G'/G)^i,\ (i = \cdots\cdots -2,1,0,1,2\cdots\cdots)$ be zero, yields a set of algebraic equations as follows:

$a_1 + a_1^2/2 - a_1C = 0,\ \ a_0a_1 - wa_1 - a_1B = 0,$

$k + a_0^2/2 - wa_0 + a_{-1}a_1 - a_{-1} - a_1A + a_{-1}C = 0,$

$a_0a_{-1} - wa_{-1} + a_{-1}B = 0,$ and

$a_{-1}^2/2 + a_{-1}A = 0$

Solving these over determine set of equations for a_{-1}, a_0, a_1, w with the aid Maple 13, we achieve the following solutions:

Set-1: $a_{-1} = 0,\ a_0 = B \pm \sqrt{B^2 + 2k + 4A(1-C)},\ a_1 = 2C - 2,$
$w = \pm\sqrt{B^2 + 2k + 4A(1-C)}$ and

Set-2: $a_{-1} = -2A,\ a_0 = -B \pm \sqrt{B^2 + 2k + 4A(1-C)},\ a_1 = 0,\quad w = \pm\sqrt{B^2 + 2k + 4A(1-C)}$

On behalf of the set 1, we have the solutions

$$u(\xi) = B \pm \sqrt{B^2 + 2k + 4A(1-C)} + (2C-2)(G'/G) \tag{15}$$

According to the cases in the method we have

Case-1: When $B \neq 0,\ \Delta_1 = B^2 + 4A(1-C) \geq 0,$ then

$$u(\xi) = \pm\sqrt{2k + \Delta_1} - \sqrt{\Delta_1}\left(\frac{C_1 \exp\left(\frac{\sqrt{\Delta_1}}{2}\xi\right) + C_2 \exp\left(-\frac{\sqrt{\Delta_1}}{2}\xi\right)}{C_1 \exp\left(\frac{\sqrt{\Delta_1}}{2}\xi\right) - C_2 \exp\left(-\frac{\sqrt{\Delta_1}}{2}\xi\right)}\right) \tag{16}$$

where $\xi = x \mp \sqrt{2k + \Delta_1}\,t$ and C_1, C_2 are arbitrary constants. The Figs. 1 and 2 indicates the nature of the solution.

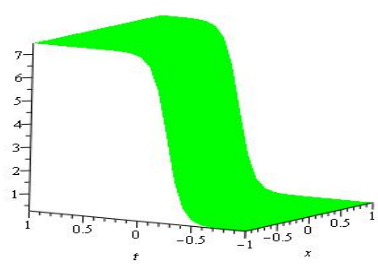

Fig. 1. 3D profile of kink soliton solution of Eq. (16) for $C_1 = A = B = k = 1$, $C = -2, C_2 = -1$

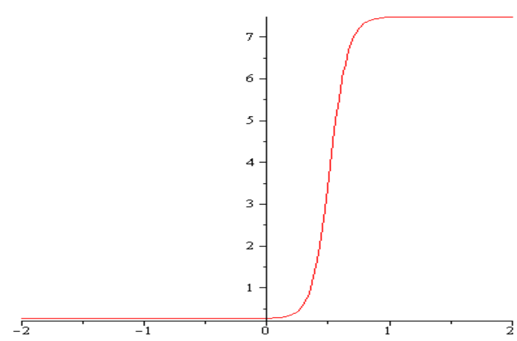

Fig. 2. 2D profile of kink soliton solution of Eq. (16) for $C_1 = A = B = k = 1$, $C = -2, C_2 = -1$ with $x = 2$

Case-2: When $B \neq 0,\ \Delta_1 = B^2 + 4A(1-C) < 0,$ then

$$u(\xi) = \pm\sqrt{2k + \Delta_1} - \sqrt{-\Delta_1}\left(\frac{iC_1 \cos\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right) - C_2 \sin\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right)}{iC_1 \sin\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right) + C_2 \cos\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right)}\right) \tag{17}$$

where $\xi = x \mp \sqrt{2k+\Delta_1}\,t$ and C_1, C_2 are arbitrary constants. The Figs. 3 and 4 indicates the nature of the solution.

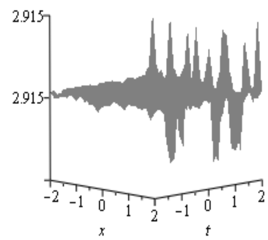

Fig. 3. 3D profile of singular kink soliton solution of Eq. (17) for $C_1 = A = B = 1$**,** $C = 2, C_2 = -1,\ k = 0.8$

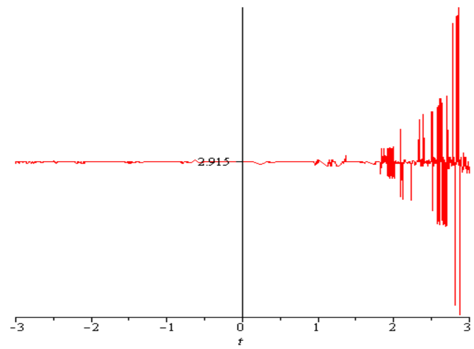

Fig. 4. 2D profile of singular kink soliton solution of Eq. (17) $C_1 = A = B = 1$**,** $C = 2, C_2 = -1,\ k = 0.8$ **with** $x = 2$

Case-3: When $B = 0,\ \Delta_2 = A(C-1) \geq 0,$ then

$$u(\xi) = \pm\sqrt{2k+4\Delta_2} - 2\sqrt{\Delta_2}\left(\frac{C_1\cos\left(\sqrt{\Delta_2}\,\xi\right) + C_2\sin\left(\sqrt{\Delta_2}\,\xi\right)}{C_1\sin\left(\sqrt{\Delta_2}\,\xi\right) - C_2\cos\left(\sqrt{\Delta_2}\,\xi\right)}\right) \tag{18}$$

where $\xi = x \mp \sqrt{2k+4\Delta_2}\,t$ and C_1, C_2 are arbitrary constants. The Figs. 5 and 6 indicates the nature of the solution.

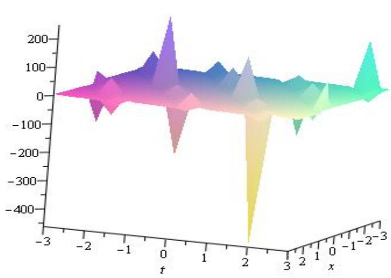

Fig. 5. 3D profile of periodic solution of Eq. (18) for $C_1 = A = k = 1, B = 0,,$ $C = 2, C_2 = -1$

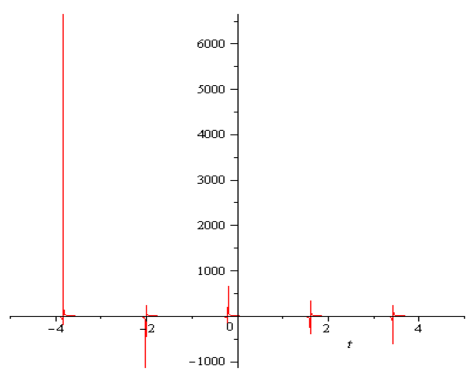

Fig. 6. 2D profile of periodic solution of Eq. (18) for $C_1 = A = k = 1, B = 0,,$ $C = 2, C_2 = -1$ **with** $x = 2$

Case-4: When $B = 0,\ \Delta_2 = A(C-1) < 0,$ then

$$u(\xi) = \pm\sqrt{2k+4\Delta_2} - 2\sqrt{-\Delta_2}\,(C-1)\left(\frac{iC_1\cosh\left(\sqrt{-\Delta_2}\,\xi\right) - C_2\sinh\left(\sqrt{-\Delta_2}\,\xi\right)}{iC_1\sinh\left(\sqrt{-\Delta_2}\,\xi\right) - C_2\cosh\left(\sqrt{-\Delta_2}\,\xi\right)}\right) \tag{19}$$

where $\xi = x \mp \sqrt{2k+4\Delta_2}\,t$ and C_1, C_2 are arbitrary constants. The Figs. 7 and 8 indicate the nature of the solution.

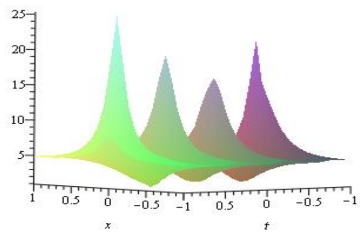

Fig. 7. 3D profile of soliton solution of Eq. (19) for $A = k = 1, B = 0,,$ $C = -2, C_1 = 2, C_2 = 4$

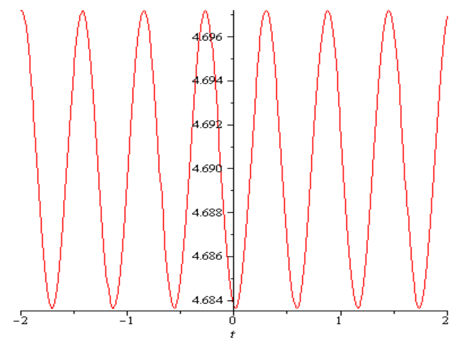

Fig. 8. 2D profile of soliton solution of Eq. (19) for $A = k = 1, B = 0,,$ $C = -2, C_1 = 2, C_2 = 4$ **with** $x = 2$

In favor of the set 2, we have the solutions

$$u(\xi) = -B \pm \sqrt{B^2 + 2k + 4A(1-C)} - 2A(G'/G)^{-1} \tag{20}$$

According to the cases in the method we have

Case-1: When $B \neq 0,\ \Delta_1 = B^2 + 4A(1-C) \geq 0,$ then

$$u(\xi) = -B \pm \sqrt{2k+\Delta_1} - 2A\left(\frac{B}{2(1-C)} + \frac{\sqrt{\Delta_1}}{2(1-C)}\frac{C_1\exp\left(\frac{\sqrt{\Delta_1}}{2}\xi\right) + C_2\exp\left(-\frac{\sqrt{\Delta_1}}{2}\xi\right)}{C_1\exp\left(\frac{\sqrt{\Delta_1}}{2}\xi\right) - C_2\exp\left(-\frac{\sqrt{\Delta_1}}{2}\xi\right)}\right)^{-1} \tag{21}$$

where $\xi = x \mp \sqrt{2k+\Delta_1}\,t$ and C_1, C_2 are arbitrary constants. The Fig. 9 indicate the nature of the solution.

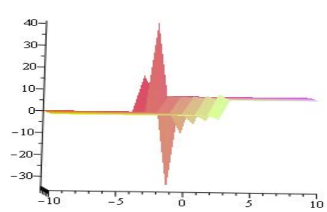

Fig. 9. 3D profile of singular kink soliton solution of Eq. (21) for $A = B = k = 1,,$ $C = -2, C_1 = 1, C_2 = -1$

Case- 2: When $B \neq 0,\ \Delta_1 = B^2 + 4A(1-C) < 0,$ then

$$u(\xi) = -B \pm \sqrt{2k+\Delta_1} - 2A\left(\frac{B}{2(1-C)} + \frac{\sqrt{-\Delta_1}}{2(1-C)}\frac{iC_1\cos\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right) - C_2\sin\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right)}{iC_1\sin\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Delta_1}}{2}\xi\right)}\right)^{-1} \quad (22)$$

Where $\xi = x \mp \sqrt{2k+\Delta_1}\,t$ and C_1, C_2 are arbitrary constants. The figure of the solution (22) is similar to the figure of the solution (17). So we omit this.

Case-3: When $B = 0,\ \Delta_2 = A(C-1) \geq 0,$ then

$$u(\xi) = \pm\sqrt{2k+4\Delta_2} - 2A\left(\frac{\sqrt{\Delta_2}}{1-C}\frac{C_1\cos\left(\sqrt{\Delta_2}\xi\right) + C_2\sin\left(\sqrt{\Delta_2}\xi\right)}{C_1\sin\left(\sqrt{\Delta_2}\xi\right) - C_2\cos\left(\sqrt{\Delta_2}\xi\right)}\right)^{-1} \quad (23)$$

Where $\xi = x \mp \sqrt{2k+4\Delta_2}\,t$ and C_1, C_2 are arbitrary constants. The figure of the solution (23) is similar to the figure of the solution (18). So we omit this.

Case-4: When $B = 0,\ \Delta_2 = A(C-1) < 0,$ then

$$u(\xi) = \pm\sqrt{2k+4\Delta_2} - 2A\left(\frac{\sqrt{-\Delta_2}}{1-C}\frac{iC_1\cosh\left(\sqrt{-\Delta_2}\xi\right) - C_2\sinh\left(\sqrt{-\Delta_2}\xi\right)}{iC_1\sinh\left(\sqrt{-\Delta_2}\xi\right) - C_2\cosh\left(\sqrt{-\Delta_2}\xi\right)}\right)^{-1} \quad (24)$$

Where $\xi = x \mp \sqrt{2k+4\Delta_2}\,t$ and C_1, C_2 are arbitrary constants. The figure of the solution (24) is similar to the figure of the solution (19). So we omit this.

Remark: All of the solutions available in this latter have been checked with Maple by putting them back into the original equations.

4. COMPARISON

Many researchers solved the Burgers equation for obtaining analytical and exact solutions by using different methods. Kheiri et al. [29] studied this equation by applying basic (G'/ G) - expansion method to construct traveling wave solutions and only solutions Eq. (13), Eq. (14) and Eq. (15) are obtained. If we setting $C = 0, A = -\mu, B = -\lambda$ then their auxiliary equation reduced to our auxiliary equation and our set -1 being similar to their obtained solutions set. It is worth declaring that two of our obtained solutions are in good agreement with already published results which is presented in the following Table 1 obtained by Kheiri et al. [29]. Hence their solutions Eq. (13), Eq. (14) are achieved in case-1 of our set-1. Moreover, in this article, profuse traveling wave solutions of the Burgers equation are constructed by using the method. The others solutions Eq. (16), Eq. (17), Eq.(21), Eq. (22), Eq. (23) and Eq. (24) are completely new achieved by improved (G'/G) expansion method.

Table 1. Comparison our solutions with Kheiri et al. [29] solutions

When $A(1-C)\le 0, A=-\mu$, $C=0$ replace C_2 by $-C_2$ in our Eq. (18) $u(\xi)=\pm\sqrt{2k-4\mu}-2\sqrt{\mu}\left(\frac{C_1\cos(\sqrt{\mu}\xi)+C_2\sin(\sqrt{\mu}\xi)}{C_1\sin(\sqrt{\mu}\xi)-C_2\cos(\sqrt{\mu}\xi)}\right)$, $\xi=x\mp\sqrt{2k-4\mu}t$	When $\lambda=0, D=k$, replace C_1 by C_2 replace C_2 by C_1 in Eq. (14) of Ref. [29] $u(\xi)=\pm\sqrt{-4\mu+2D}-2\sqrt{\mu}\left(\frac{C_1\cos(\sqrt{\mu}\xi)-C_2\sin(\sqrt{\mu}\xi)}{C_1\sin(\sqrt{\mu}\xi)+C_2\cos(\sqrt{\mu}\xi)}\right)$, $\xi=x\mp\sqrt{-4\mu+2k}t$
When $A(1-C)\le 0, A=-\mu$ and $C=0$ in our solution Eq. (19) $u(\xi)=\pm\sqrt{2k-4\mu}+2\sqrt{-\mu}\left(\frac{iC_1\cosh(\sqrt{-\mu}\xi)-C_2\sinh(\sqrt{-\mu}\xi)}{iC_1\sinh(\sqrt{-\mu}\xi)-C_2\cosh(\sqrt{-\mu}\xi)}\right)$	When $\lambda=0, D=k$, replace C_1 by iC_1 and C_2 by $-C_2$ in Eq. (13) of Ref. [29] $u(\xi)=\pm\sqrt{2k-4\mu}-2\sqrt{-\mu}\left(\frac{C_1\sinh(\sqrt{-\mu}\xi)+C_2\cosh(\sqrt{-\mu}\xi)}{C_1\cosh(\sqrt{-\mu}\xi)+C_2\sinh(\sqrt{-\mu}\xi)}\right)$

5. CONCLUSION

In this paper we successfully used the new approach of improved (G'/G)-expansion in the Burgers equation and got some new traveling wave solutions, including the hyperbolic functions, trigonometric functions, exp-functions and rational function solutions. Kheiri et al. [29] obtained only three solutions but we obtained eight solutions. Beside this, both two and three-dimensional plots of the obtained wave solutions are also provided to visualize the dynamics of the equation. Moreover, the method appears to be easier, faster and can be handle by computer easily to solve NLEEs and we used Maple-13 to solve the equation. This will have a good wisdom to promote the extensive application of the equations.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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