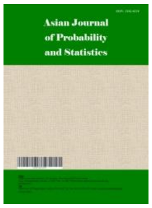

Asian Journal of Probability and Statistics

12(1): 68-78, 2021; Article no.AJPAS.65956
ISSN: 2582-0230

Road Traffic Offences in Nigeria: An Empirical Analysis using Buys-Ballot Approach

K. C. N. Dozie[1*] and C.C Ibebuogu[2]

[1]Department of Statistics, Imo State University Owerri, Imo State, Nigeria.
[2]Department of Computer Science, Imo State University Owerri, Imo State, Nigeria.

Authors' contributions

This paper was carried out in collaboration between both authors. Author KCND designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author CCI managed the analyses and literature searches of the study. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2021/v12i130280
Editor(s):
(1) 1. Dr. S. M. Aqil Burney, University of Karachi, Pakistan.
Reviewers:
(1) Fernando Gonzalez Tavares, Universidade de Ribeirão Preto, Brazil.
(2) Safat Bin Wali, University Tenaga Nasional (UNITEN), Malaysia.
Complete Peer review History: http://www.sdiarticle4.com/review-history/65956

Original Research Article

Received 29 December 2020
Accepted 09 March 2021
Published 14 April 2021

Abstract

Road traffic offences in time series analysis when trend-cycle component is quadratic is discussed in this study. The study is to investigate the variance stability, trend pattern, seasonal indices and suitable model for decomposition of study data. The study shows that, the series is seasonal with evidence of upward trend or downward trend. There is an upsurge of the series in the months of March, August and November and a drop in January, June and December. The periodic standard deviations are stable while the seasonal standard deviations differ, suggesting that the series requires transformation to make the seasonal indices additive.

Keywords: Decomposition model; buys-ballot table; successful transformation; seasonal mean; standard deviation; suitable model.

1 Introduction

Descriptive method of time series analysis is a set of observations taken at different time period especially equal time interval, this periods can be daily, weekly, monthly, quarterly etc. Generally, a time series may

**Corresponding author: E-mail: kcndozie@yahoo.com;*

usefully considered as a mixture of four components include, the trend, seasonal, cyclical and irregular components. Cyclical variation is regarded as long term oscillations. However, for short series, the cyclical component is jointly estimated into the trend [1] and the observed time series $\left(X_t, t=1,2,\ldots,n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \tag{3}$$

On when to apply each of these three models, Linde [2] stated that when the seasonal indices is independent of the absolute level of the time series, but it takes appropriately the same magnitude each period then is said to be additive model and shown in equation (1) may be employed. If the seasonal indices takes the same relative magnitude each period, then it is said to be multiplicative model obtained in equation (2).

Iwueze and Ohakwe [3] observed the Buys-Ballot procedure to the case in which the trend cycle component is quadratic. In their summary, they proposed that the estimation of the slope of the curve is as in Iwueze and Nwogu [4]. The difference in procedure lies in the calculation of *c* which is easily calculated from differences in the periodic means.

Dozie [5] discussed the expression for estimation of trend parameters and seasonal indices using periodic, seasonal and overall averages for the mixed model in time series. He observed that the estimate of trend parameters and seasonal indices for mixed model, when there is no trend and (b = 0).

2 Methodology

2.1 Buys-Ballot procedure for time series is employed in this study. For details of this procedure see Wei [6], Iwueze and Ohakwe [4], Dozie, et al. [7], Dozie and Ihekuna [8].

2.1 Quadratic trend cycle and seasonal components

The expression of the quadratic trend is given by

$$\bar{X}_i = a + bt + ct^2 \tag{4}$$

Iwueze and Nwogu [4] discussed estimation of the trend and seasonal indices for an additive model when trend-cycle component is quadratic as;

Table 1. Buys-Ballot tabular arrangement of time series data

Rows (i)	Columns j								
	1	**2**	⋯	***j***	...	***s***	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	⋯	X_j	⋯	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	⋯	X_{s+j}	⋯	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	⋯	X_{2s+j}	⋯	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	⋯	$X_{(i-1)s+j}$	⋯	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	⋯	$X_{(m-1)s+j}$	⋯	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	⋯	$T_{.j}$	⋯	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	⋯	$\bar{X}_{.j}$	⋯	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	⋯	$\hat{\sigma}_{.j}$	⋯	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

Where, m = number of periods, s = length of periodic interval and n = length of the series

$$\hat{a} = a^{|} + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \tag{5}$$

$$\hat{b} = \frac{b^{|}}{s} + \hat{c}(s-1) \tag{6}$$

$$\hat{c} = \frac{c^{|}}{s^2} \tag{7}$$

$$\hat{S}_j = \bar{X}_{.j} - d_j \tag{8}$$

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \hat{b} + \hat{c}(n-s)j + \hat{c}\, j^2 \tag{9}$$

2.2 Choice of model

2.2.1 Cochran's test for constant variance

To test the null hypothesis that the variances are equal, that is

$$H_0 = \sigma_1^2 = \sigma_2^2 = ... = \sigma_k^2$$

Against the alternative

$$H_1 \neq \sigma_1^2 \neq \sigma_2^2 \neq ... \neq \sigma_k^2 \ \left(Atleast\ one\ \text{var}iance\ is\ different\ from\ others\right)$$

The statistic is given as

$$C = \frac{\max\left(S_j^2\right)}{\sum_{j=1}^{k} S_j^2} \tag{10}$$

Where, $\max\left(S_j^2\right)$ is the maximum variance among all column variances $\sum_{j=1}^{k} S_j^2$ is the sum of the variances S_j has the range j = 1, 2, …, s, which are the variances of the j[th] sub-group.

Using the parameters of the Buys-Ballot table: $S_j^2 = \hat{\sigma}_j^2$, the statistic in (10) is then given as;

$$C = \frac{\max\left(\hat{\sigma}_j^2\right)}{\sum_{j=1}^{k} \hat{\sigma}_j^2} \tag{11}$$

Where, $\sigma_j^2 = (j = 1,\ 2,\ \ldots\ s)$ is the column variance of the Buys-Ballot table.

3 Real Life Example

Empirical example is given in this section to demonstrate the application of the Buys-Ballot method of time series decomposition. The series is listed in the Buys-Ballot table, with its row, column and overall means and standard deviations. The associated graphs are obtained in Figs. 3.1, 3.2 and 3.3. The graphs show that the series is seasonal with evidence of upward or downward trend.

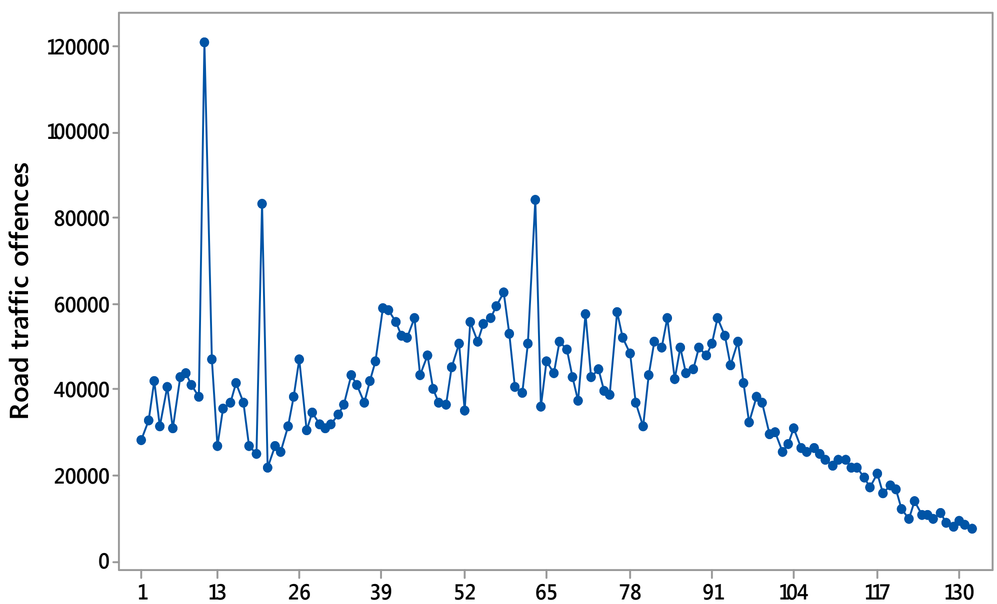

Fig. 3.1. Plot of road traffic offences between 2007-2017

There is an upsurge of the series in the months of March, August and November and a drop in January, June and December. There is a reasonably seasonal pattern over the period, suggesting that the series need transformation to make the seasonal effect additive.

The estimates of the parameters of quadratic trend cycle and seasonal components are obtained here, using (5), (6) and (7), we have

$$\bar{X}_i = 9.985 + 0.3642\, i - 0.0379\, i^2$$

$$\hat{c} = \frac{-0.0379}{144} = -0.0003$$

$$\hat{b} = \frac{0.3642}{12} - 0.0003(12-1) = 0.0271$$

$$\hat{a} = 9.985 + \left(\frac{12-1}{2}\right)(0.0271) - \left(\frac{(12-1)(24-1)}{6}\right)(-0.0003)$$

$$= 10.1467$$

$$\hat{S}_j = \bar{X}_{.j} - 10.2877 + 0.036\, j + 0.0003\, j^2$$

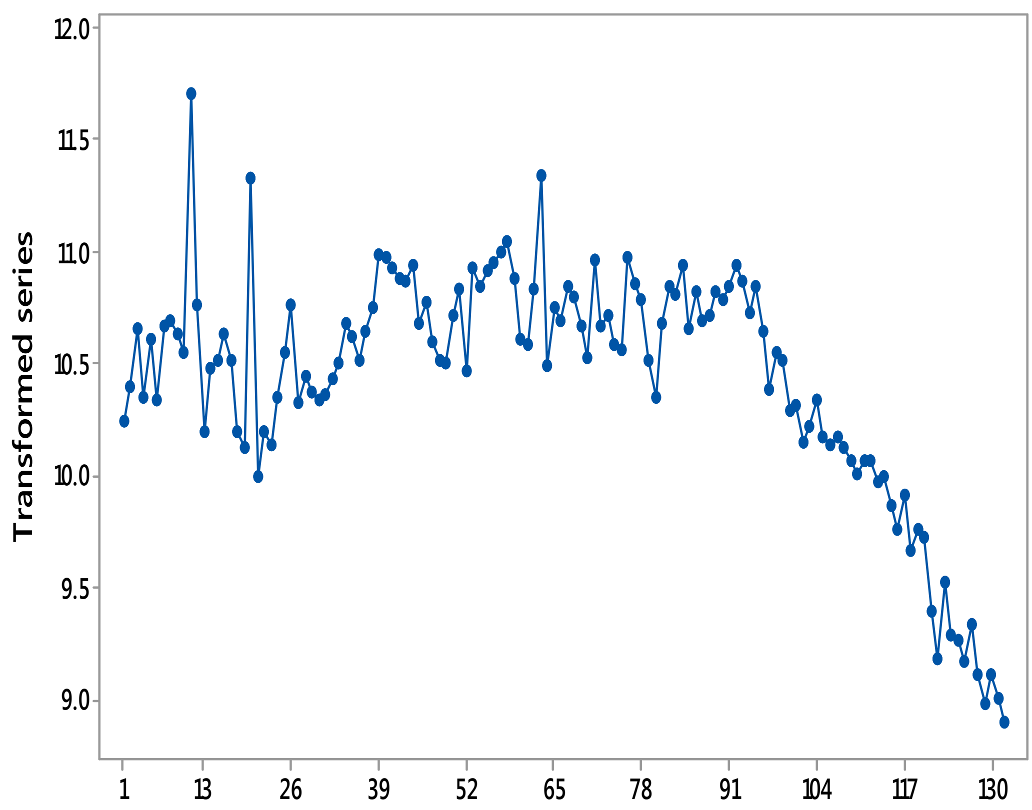

Fig. 3.2. Transformed series of road traffic offences between 2007-2017

Table 2. Estimates of seasonal effect

j	$\overline{X}_j$	S_j
1	10.353	-0.290
2	10.460	-0.146
3	10.544	-0.025
4	10.423	-0.108
5	10.483	-0.009
6	10.375	-0.078
7	10.410	-0.003
8	10.511	0.139
9	10.369	0.038
10	10.386	0.097
11	10.499	0.252
12	10.338	0.134
$\sum_{j=1}^{s} \hat{s}_j$		0.0000

Table 3. Estimates of quadratic trend parameters and seasonal indices

Parameter	**Quadratic trend and seasonal indices**
$\hat{a}$	10.147
$\hat{b}$	0.027
$\hat{c}$	-0.0003
$\hat{S}_1$	-0.2902
$\hat{S}_2$	-0.1463
$\hat{S}_3$	-0.0248
$\hat{S}_4$	-0.1077
$\hat{S}_5$	-0.009
$\hat{S}_6$	-0.0777
$\hat{S}_7$	-0.0028
$\hat{S}_8$	0.1387
$\hat{S}_9$	0.0378
$\hat{S}_{10}$	0.0965
$\hat{S}_{11}$	0.2518
$\hat{S}_{12}$	0.1337

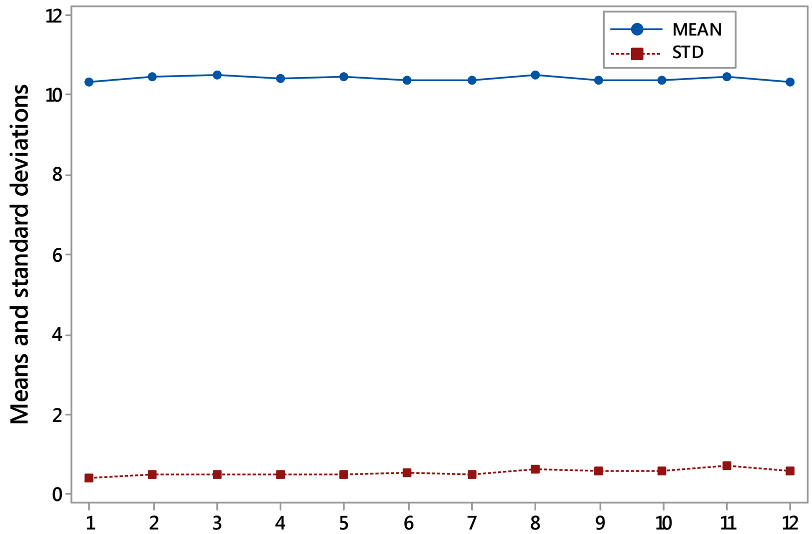

Fig. 3.3. Seasonal means and standard deviations

Table 4. Seasonal means and standard deviations

j	$\bar{X}_{j}$	$\sigma_{.j}$
1	10.353	0.382
2	10.460	0.487
3	10.544	0.477
4	10.423	0.468
5	10.483	0.501
6	10.375	0.508
7	10.410	0.500
8	10.511	0.629
9	10.369	0.576
10	10.386	0.573
11	10.499	0.712
12	10.338	0.580

3.1 Choice of model

Cochran's statistic in (11) is used to determine if the series accepts additive model. The null hypothesis that the series accepts additive model is rejected, if C is greater than the tabulated value $C_{tab}\ \{k, V, 1-\alpha\}$. level of significance, or do not reject null hypothesis otherwise.

From Appendix A and Table 5.

$$m=12,\ \max\left(\hat{\sigma}_j^2\right) = 896929395.00,\ \ \sum_{j=1}^{k}\hat{\sigma}_j^2 = 3458064838$$

hence,

$$C = \frac{896929395}{3458064838} = 0.2594$$

Decision rule: Reject H_0 if $C > C_{tab}$ {11,12:0.05} $\overline{X}_j$

Table 5. Seasonal means ($\overline{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$) of the actual series

j	$\overline{X}_j$	$\hat{\sigma}_j^2$
1	33152.73	97808446.00
2	37868.82	158792463.40
3	41744.45	349300360.10
4	36594.00	195739139.40
5	39142.73	214664393.80
6	35326.91	199927172.50
7	36601.64	222625161.90
8	42528.45	437170557.50
9	35918.09	233406043.70
10	36606.55	255221781.10
11	44553.73	896929395.00
12	34715.45	196479923.30
Total	454753.55	3458064838.00

Table 6. Seasonal means ($\overline{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$) of the transformed series

j	$\overline{X}_j$	$\hat{\sigma}_j^2$
1	10.35	0.15
2	10.46	0.24
3	10.54	0.23
4	10.42	0.22
5	10.48	0.25
6	10.37	0.26
7	10.41	0.25
8	10.51	0.40
9	10.37	0.33
10	10.39	0.33
11	10.50	0.51
12	10.34	0.34
Total	125.15	3.49

As shown in Table 5, the critical value (0.2353) is greater than the test statistic in (11), suggesting that the series does not accept additive model. Having verified that the actual series is not additive. We transformed the series to meet the constant variance and normality assumptions in the distribution. When the transformed series listed in Table 6 are subjected to test for constant variance, the test statistic (0.1461) is less than the tabulated (0.2353) at C_{tab} {k, V, 1 − α} level significant. Therefore, transformed series accepts additive model.

From Appendix B and Table 6.

m=12, $\max\left(\hat{\sigma}_j^2\right) = 0.51, \quad \sum_{j=1}^{k} \hat{\sigma}_j^2 = 3.49$

$$C = \frac{0.51}{3.49} = 0.1461$$

Decision rule: Reject H_0 if $C > C_{tab}$ {11,12:0.05}

4 Conclusion

We have discussed the road traffic offences in time series analysis. The emphasis is to investigate the variance stability, trend pattern, seasonal effect and suitable model for decomposition. The study indicates that, the series is seasonal with evidence of upward or downward trend. There is an upsurge of the series in the months of March, August and November and a drop in January, June and December. There is a reasonably seasonal pattern over the period, suggesting the additive model. Also, the suitable model that best describe the pattern in the transformed series shown in the summary table (Table 6) is additive.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Chatfield C. The analysis of time Series: An introduction. Chapman and Hall, CRC Press, Boca Raton; 2004.

[2] Linde P. Seasonal adjustment, Statistics Denmark; 2005. Available:www.dst.dk/median/konrover/13-forecasting-org/seasonal/001pdf

[3] Iwueze IS, Nwogu EC. Framework for choice of models and detection of seasonal effect in time series. Far East Journal of Theoretical Statistics. 2014;48(1):45– 66.

[4] Iwueze IS, Ohakwe J. Buys-Ballot estimates when stochastic trend is quadratic. Journal of the Nigerian Association of Mathematical Physics; 2004;8:311-318.

[5] Dozie KCN. Buys-Ballot estimates for mixed model in descriptive time series. International Journal of Theoretical and Mathematical Physics. 2020;10(1):22-27.

[6] Wei WWS. Time series analysis: Univariate and multivariate methods. Addison-wesley publishing Company Inc, Redwood; 1989.

[7] Dozie KCN, Ibebuogu CC, Mbachu HI, Raymond MC. Buys-Ballot modeling of church marriages in Owerri, Imo State, Nigeria. American Journal of Mathematics and Statistics. 2020;10(1):26-31.

[8] Dozie KCN, Ihekuna SO. Buys-Ballot estimates of quadratic trend component and seasonal indices and effect of incomplete data in time series. International Journal of Science and Healthcare Research; 2020;5(2):341-348.

Appendix A. Actual data on number of road traffic offences in Nigeria (2007-2017)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\overline{X}_{i.}$	$\sigma_{i.}^{2}$
2007	27905	32732	42104	31288	40529	30727	42742	43807	41146	38152	121049	47113	44941.2	610818258.7
2008	26613	35623	36655	41333	36827	26714	24782	83102	21793	26793	25334	31170	34728.3	268434131
2009	38268	47134	30474	34365	31714	30673	31584	34008	36479	43294	40956	36710	36304.9	28174340.8
2010	41683	46299	59036	58401	55681	52653	52107	56362	43392	47885	39863	36878	49186.7	57679005.33
2011	36223	45036	50404	35008	55690	50946	55095	56681	59273	62365	53103	40482	50025.5	80211908.3
2012	39294	50635	84203	35797	46285	43715	50989	49011	42607	37393	57381	42619	48327.4	166439233.7
2013	44724	39486	38661	58112	52004	48191	36777	31262	43449	51205	49543	56472	45823.8	67562322.7
2014	42376	49697	43837	44665	49674	48038	50812	56405	52633	45569	51234	41617	48046.4	20260646.3
2015	32164	38125	36687	29488	30114	25486	27264	30927	26211	25224	26201	24772	29388.6	19895997.2
2016	23421	22098	23519	23348	21468	21871	19224	17262	20173	15726	17288	16709	20175.6	8142224.6
2017	12009	9692	13609	10729	10584	9582	11242	8986	7943	9066	8139	7328	9909.1	3307375.2
$\overline{X}_{.j}$	33152.7	37868.8	41744.5	36594	39142.7	35326.9	36601	42528.5	35918	36606.6	44553.7	34715.5	37896.1	
$\sigma_{.j}^{2}$	9780446	158792463.4	349300360.1	195739139.4	214664393.8	199927172.5	222625161.9	437170557.5	233406043.7	255221781.1	896929395.0	196479923.3		3458064838

Appendix B. Transformed data on number of road traffic offences in Nigeria (2007-2017)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\overline{X}_{i.}$	$\sigma^2_{i.}$
2007	10.24	10.40	10.65	10.35	10.61	10.33	10.66	10.69	10.62	10.55	11.70	10.76	10.63	0.14
2008	10.19	10.48	10.51	10.63	10.51	10.19	10.12	11.33	9.99	10.20	10.14	10.35	10.39	0.13
2009	10.55	10.76	10.32	10.44	10.36	10.33	10.36	10.43	10.50	10.68	10.62	10.51	10.49	0.02
2010	10.64	10.74	10.99	10.98	10.93	10.87	10.86	10.94	10.68	10.78	10.59	10.52	10.79	0.03
2011	10.50	10.72	10.83	10.46	10.93	10.84	10.92	10.95	10.99	11.04	10.88	10.61	10.80	0.04
2012	10.58	10.83	11.34	10.49	10.74	10.69	10.84	10.80	10.66	10.53	10.96	10.66	10.76	0.05
2013	10.71	10.58	10.54	10.97	10.86	10.78	10.51	10.35	10.68	10.84	10.81	10.94	10.72	0.03
2014	10.65	10.81	10.69	10.71	10.81	10.78	10.84	10.94	10.87	10.73	10.84	10.64	10.78	0.01
2015	10.38	10.55	10.51	10.29	10.31	10.15	10.21	10.34	10.17	10.14	10.17	10.12	10.28	0.02
2016	10.06	10.00	10.07	10.06	9.97	9.99	9.86	9.76	9.91	9.66	9.76	9.72	9.90	0.03
2017	9.39	9.18	9.52	9.28	9.27	9.17	9.33	9.10	8.98	9.11	9.00	8.90	9.19	
$\overline{X}_{.j}$	10.35	10.46	10.54	10.42	10.48	10.37	10.41	10.51	10.37	10.39	10.50	10.34	10.43	
$\sigma^2_{.j}$	0.15	0.24	0.23	0.22	0.25	0.26	0.25	0.40	0.33	0.33	0.51	0.34		0.01

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle4.com/review-history/65956