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Thermodynamical Modeling Stability of Financial Network Based on Their Structure on Fractal and Rule Driven Spin Lattice

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Abstract

The stability and order of globalised financial network is crucial for economic health. Prediction of instabilities, or defaults of such macroscopic system based on many microscopic states of individual behaviour of assets, institutions, clients, and traders is a challenging problem. Is it possible to estimate a failure of financial institution on a large scale, or domain? Many approaches in financial mathematics were used for this and usually without quantitative success. They are struggling with complexity and NP difficulty given by complex relations in financial network. Such a network can contain some recurrent patterns and can be described like fractal or other patterns in structure. In this article, we are modelling the financial network as a thermodynamical rule-driven board, fractal spin lattice to describe equilibrial behaviour and analyse phase transitions and other thermodynamis quantities describing macroscopic behaviour of such structure system with recurrent patterns. Assets and liabilities of financial institutions and companies are represented with spin binary variables. Spin interactions are interpreted as mutual trading among assets and liabilities. The thermal coefficient introduces assets and liabilities fluctuations. Below phase transition temperature are large-scale clusters of assets formed and are dominant. Assets cluster size can be an order parameter. After phase transition, when fluctuations of prices increase, actors on the market are disturbed with increasing fluctuation and are not correlated on larger network distances into the communication, which suppresses long-range correlations. Crash of such a system can be represented with change of lattice order parameter, when system temperature jumps up over critical value and long range domain structure of assets is broken, which usually means that financial institutions fail to meet interest or payment obligations.

Keywords

Numerical Model of Financial Network, Financial Crash Prediction

1. Introduction

Global financial networks delivered well-known economic benefits. As discussed in the literature, parameters as the size, and composition of the financial positions, can matter for financial network stability as well as for shock transmission. Organizations with larger assets exposition would be under greater pressure to adjust while facing a shock. The financial and ownership links provide an important transition mechanism for the financial market and a way how to affect the value of financial assets and liabilities in global funding and liquidity [1].

Nowadays financial globalisation and market integration, using many financial products and derivates, complex owners and investment schemas require relevant risk management and monitoring solutions which adequately avoid banks and financial institutions' failures. Despite a lot of work in predicting bank failure, many banks the world over have failed or are fragile as a result of a number of financial quakes and are sensitive in context of prices of their collaterals are changed. Many such approaches [2] [3] use machine learning techniques, regressions, classificators, and deriving new indicators, but is obvious, that they are not very successful regarding repeating financial crisis. Common indicators of bank capital adequacy are the core capital ratio and the risk-weighted capital ratio, prescribed by the Basel II (and III) Capital Accords. Capital ratio, impaired loans to equity, rate of loan growth, return on average assets, net interest margin, net loans to total assets, loans to deposits ratio, and impaired loans to gross loans are used in many methodologies to predict bank healthiness classification, but ability to predict bank failure is severely lacking.

This suggestion needs to redesign bank health evaluation using a new set of approaches and solutions that can be continuously used to monitor the health of complex financial networks as well as individual banks. An algorithm to describe all combinations of asset-liability modelling requirements can be an NP, nondeterministic (super) polynomial time problem and needs new models and technics how to handle it [4].

Financial relations between entities can be described with recurrent or other rule terms between assets and owners. Not each asset or owner has a relation to another and then some pattern and fractal geometry can occur. Due financial network, like many other spontaneous networks generated in nature, is based on some recurrence principles or other rules and can contain design patterns in its structure. Like fractals or tiling.

In this article, we model the financial network as a thermodynamical rule-driven spin lattice to describe equilibrial behaviour and analyse phase transitions and other thermodynamis quantities describing macroscopic behaviour of such a system.

Recurrent or hierarchy chararacter of financial relation can be reflected on ownership of other companies with assets and liabilities. To express this, equity value V_i ownership of all assets and cross-holdings of other institutions $V_i = A_{ik} p_k + C_{ij} V_j$ are used often. There is summed via two times repeated index

(Einstein summation convention) and $A_{ik}p_k$ is asset matrix element multipled by k-element of asset's price vector, $C_{ij}V_j$ is ownership, cross-holdings matrix element multiplied by j-element of equity vector.

For our investigations is numeric value knowledge of equity not necessary, we investigating qualitative changes of system based on fractal geometry or different pattern geometry of bindings between institutions and cross-holding.

This means, organisation defined with spin vertex in centre of basic lattice is unit owning or is owned, related (by shares, liabilities or other fiancial product) with other organization represented by spin on edges of unit. Recursive joining process of the vertex can generate a fractal structure of interacting actors in lattice. For example for simple Fibonnaci sequence of ownership

$$\{F_n = F_{n-1} + F_{n-2}\}_n, F_1 = F_2 = 1$$

or connected nodes one can find related Sierpinski triangle structure where number of triangles at each stage is equal to the Fibonacci number for that stage. This can be simple ilustrated on Pascal triangle where all odd numbers can be represented with number one and all even numbers with number zero or vice versa, or use black/white color. To calculate Hausdorff dimension D is used the scale factor (S = 2) and the number of self-similar objects (N = 3) in each node of previous triangle $D = (\ln N)/(\ln S) = (\ln 3)/(\ln 2) = 1.5849$. Or have 3 × 3 unit with 9 vertexes and not connected 4 corner's nodes with scale factor 3 we get Hausdorff dimension uses the scale factor (S = 3) and the number of self-similar objects (N = 5) to calculate the dimension D. Because origin basic unit have 3 × 3 nodes and then scale factor 3 and 5 nodes connected after the first iteration Hausdorff dimension $D = (\ln N)/(\ln S) = (\ln 5)/(\ln 3) = 1.4649$. This fractal lattice is sort of Vicsek fractal.

Some of fractals or other interesting patterns can be generated using board matrix. So we introduce board matrix representation of the Ising bond vertex model generated with cellural automata, discrete models defined by a board and transition rule [5].

Board is a lattice, array, where cells can take values 0, 1 and representing interaction coupling constant. For cell = 0 no interaction, J = 0, no bonds between nodes. Transition rule is a method that describes how the values of each cell on the board changes from one time step to the next. As we are describing in article, some structural patterns in lattice geometry have an impact on phase transition and other theromodynamics quantities. Structure of financial network is thus important for stability of financial market also.

We can generalize this and create a recurrent matrix generator refers to a matrix generation process where each element of the matrix depends on some previously generated elements. This could be seen as a form of recursion, but it doesn't necessarily produce a fractal pattern. One common example of a recurrent matrix generator is generating a matrix where each element is the sum of its adjacent neighbors (e.g., the elements to the left, right, above, and below it). This process is often used in cellular automata also and by image processing algorithms.

Ising model exhibits a phase transition when the lattice dimension is larger than one as was demonstrated by analysis of renormalization flow for ϕ^4 Ising model. Despite this, Ising model on Sierpinski gasket lattice with Hausdorff dimension = 1585 does not show a phase transition at any finite temperature [6].

But as we show in article when Sierpinski triangle fractal is projected in inverse sense, that mean nodes of fractal has J = 0 and any other J = 1 or any other non zero value, we can find a phase transition at finite temperature. Depends on fractal type, but for some, when fractal lattice has sufficient size, we can see critical behaviour of this Ising model also. As is well known, critical indices are different from standard square lattice Insing model.

2. Lattice Models of Financial Relations

Binary spin variables are used in standard asset-liability optimization portfolio modelling with well-defined Quadratic Unconstrained Binary Optimization (QUBO) objective function. QUBO is an optimization framework working with binary decision variables 0, 1, and quadratic objective functions. It is used in computer science and optimization. QUBO problems aim to find the binary variable assignment that minimizes or maximizes a quadratic objective function subject to certain constraints. There can be used classical optimization techniques, such as integer linear programming solvers and simulated annealing, or with quantum annealer, is possible to map QUBO problem to Ising model on the quantum hardware.

In Ising model spin takes values from the set −1, +1, I (sing). QUBO binary variable takes the values from the set 0, 1, B (oolean). This can be easy mapped one to other with relation between elements of both sets $B = (1 + I)/2$

In this model, assets and liabilities of financial institutions and companies are represented whit spin binary variable $\sigma = \{1, -1\}$. Spin interactions J_{ij} are interpreted as a mutual trading among assets and liabilities. The thermal coeficient introduce a assets and liabilities prices fluctuations. Below phase transition temperature T_c are large scale clusters of assets formed and are dominant as huge volume of derivates on balance sheet, bonds etc. Assets cluster size is order parameter.

After phase transition, when fluctuations of prices increase, actors on the market are disturbed with increasing fluctuation and are not correlated on larger network distances into the communication, which suppresses long-range correlations. Crash of such a system can be represented with the change of lattice order parameter, when system temperature jumps up over critical value and the long-range domain structure of assets is broken, which usually means that financial institutions fail to meet interest or payment obligations.

Investigation and classification of model's phase transition is done by numerical calculation of the partition function and its derivates. Summing is over each spins nodes, H is Hamiltionan, k - Boltzmann constant, T - temperature.

$$Z = \sum_{\sigma} e^{-\frac{H}{kT}}$$

Any observable thermodynamics function *A* is given

$$\langle \hat{A} \rangle = Z^{-1} \sum_{\sigma} \hat{A} e^{-\frac{H}{kT}} \quad (1)$$

Free energy *F*, or other function (internal energy *U*, specific heat *C*) used for phase transition classification can be such calculated.

$$F = -k_B T \ln(Z)$$

$$U = -t^2 \frac{\partial (F/T)}{\partial T}$$

$$C = \frac{\partial U}{\partial T}$$

Because our lattice models can have a different structure, including some different patterns (fractals, cellural automat board) to have generic and reliable results, we need a reliable numerical method to calculate change of order parameter and we choose Monte Carlo simulation. To imitate a financial network of financial derivates we do the following.

Define a lattice graph with vertices. Each vertex represents an entity, which can be a bank, hedge fund, investor, or derivatives exchange. Each neighbours' relations represent a financial relationship or contract between two entities. The spin indicates the direction of ownership or contract, such as a long or short position in a specific financial instrument, product.

First, let's consider a financial network, where each participant is connected to all its neighbors in the network and thus through intermediaries, *i.e.* neighboring points can interact with each other. In such a case, we have a situation like in **Figure 1**.

Picture shows assets-liabilities fluctuation dependence of the complete order parameter $\langle O \rangle$ on the two-dimensional square lattice of the thermodynamic version financial network. Non-zero order parameter $\langle O \rangle$ represents a stability of financial network. The phase transitions in the classical spin systems are induced by the thermal fluctuations by varying the temperature T representing prices fluctuation in Equation (1). The second-order phase transition is resulted at the critical temperature $T_c \approx 2.5$ which indicates prices fluctuation level 67%.

Structure of investment can have random recurrent pattern, when investor with some random approach investing parts in some portfolio and this matrix of ownership is replicated on higher scale of network.

To generate such approach we use so called rule 30 in Wolfram code [5] because from simple well defined rules (on row cellural automat left cell XOR (central cell OR right cell)) produce complex random patterns. This rule and following complex pattern is observed in nature and is used as random generator in software systems or cryptography. On **Figure 2** is displayed such pattern in ownership structure in financial market, which represent a random investments relations with some recurrent structure of ownership. Some nodes in lattice are still connected to their neighbors, but as random investment is spreading from some init condition, some parts of financial relationship have different structure.

The second-order phase transition is pointed at the critical temperature $T_c \approx 2.25$ which indicates prices fluctuation level 64%. Some instabilities occured on $T \approx 0.5$, fluctuation level 13.5%. There is some stabilization point also in $T \approx 1.6 \approx 53\%$.

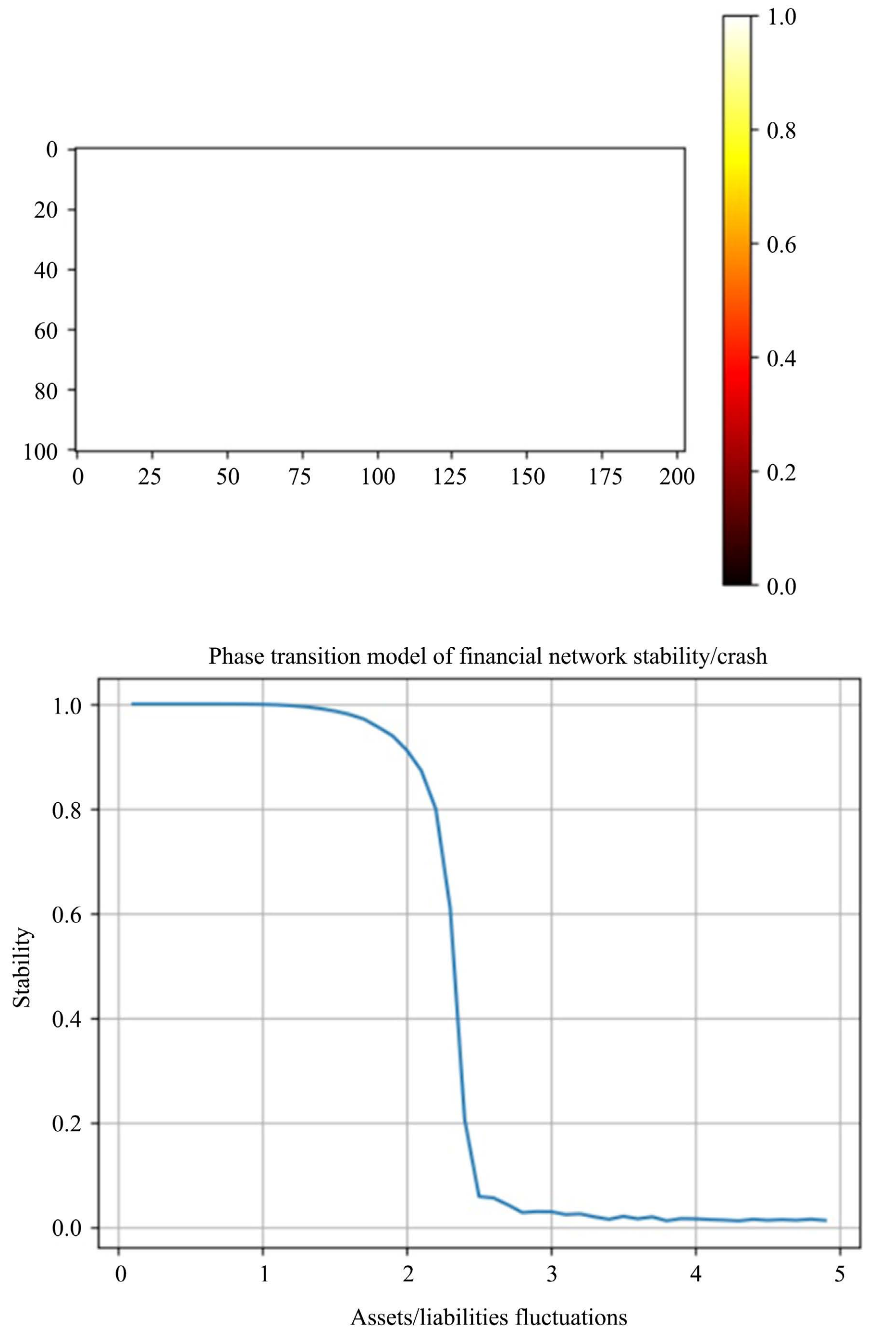

Figure 1. Ising matrix lattice: map for same values of spin interaction constant and phase transition plot.

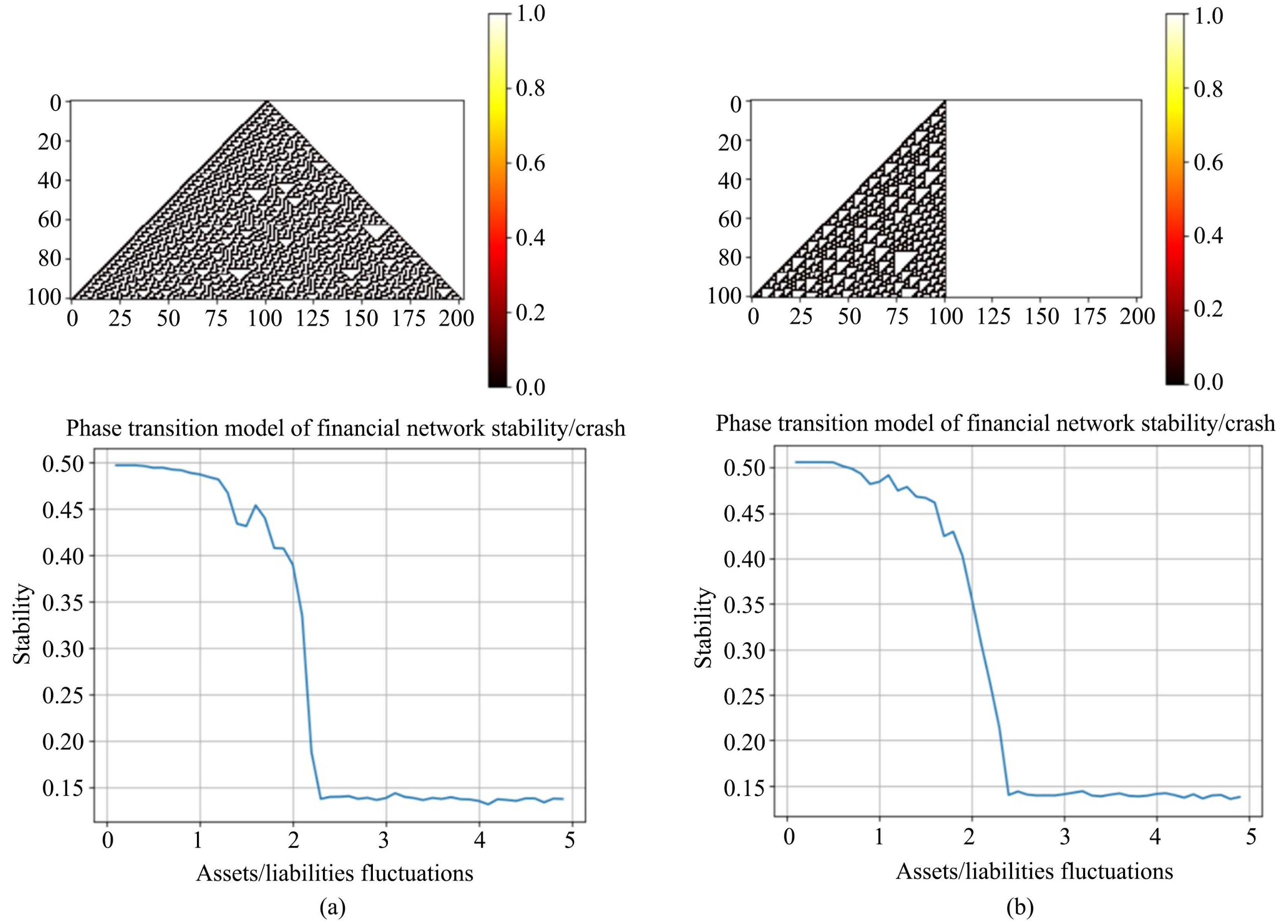

Figure 2. Ising matrix lattice: map for different values of spin interaction constant and phase transition plot. (a) rule 30; (b) rule 100.

An interesting picture can occur, rule 110, when a finite number of localized patterns are embedded within an infinitely repeating background pattern. Time evolution of such structure brings so-called moving spaceship pattern well known from Conway's Game of Life [7] and creates a cyclic tag system of relations patterns.

Fibonacci series in form of golden ratio is frequently used by traders in their trading activities, because has a strong psychological importance in human behaviour. Traders often take profits or cover losses in certain points described by golden ratio. As was mentioned in introduction, Fibonacii series can be transformed to Sierpinski fractal and such structure has implication to behaviour of network order parameter as we can see in **Figure 2**.

When we have more recurrent structure like Sierpinski fractal, no such randomized as in previous case, critical temperature is little bit higher $T_c \approx 2.4$, what is fluctuation on 66% what is not such difference comparing structure of rule 30, but we can observe more resilience to change order parameter till $T \approx 0.7 \approx 24\%$ of fluctuation and moved stabilization point at $T \approx 2.15 \approx 63\%$ of fluctuation value from basic state.

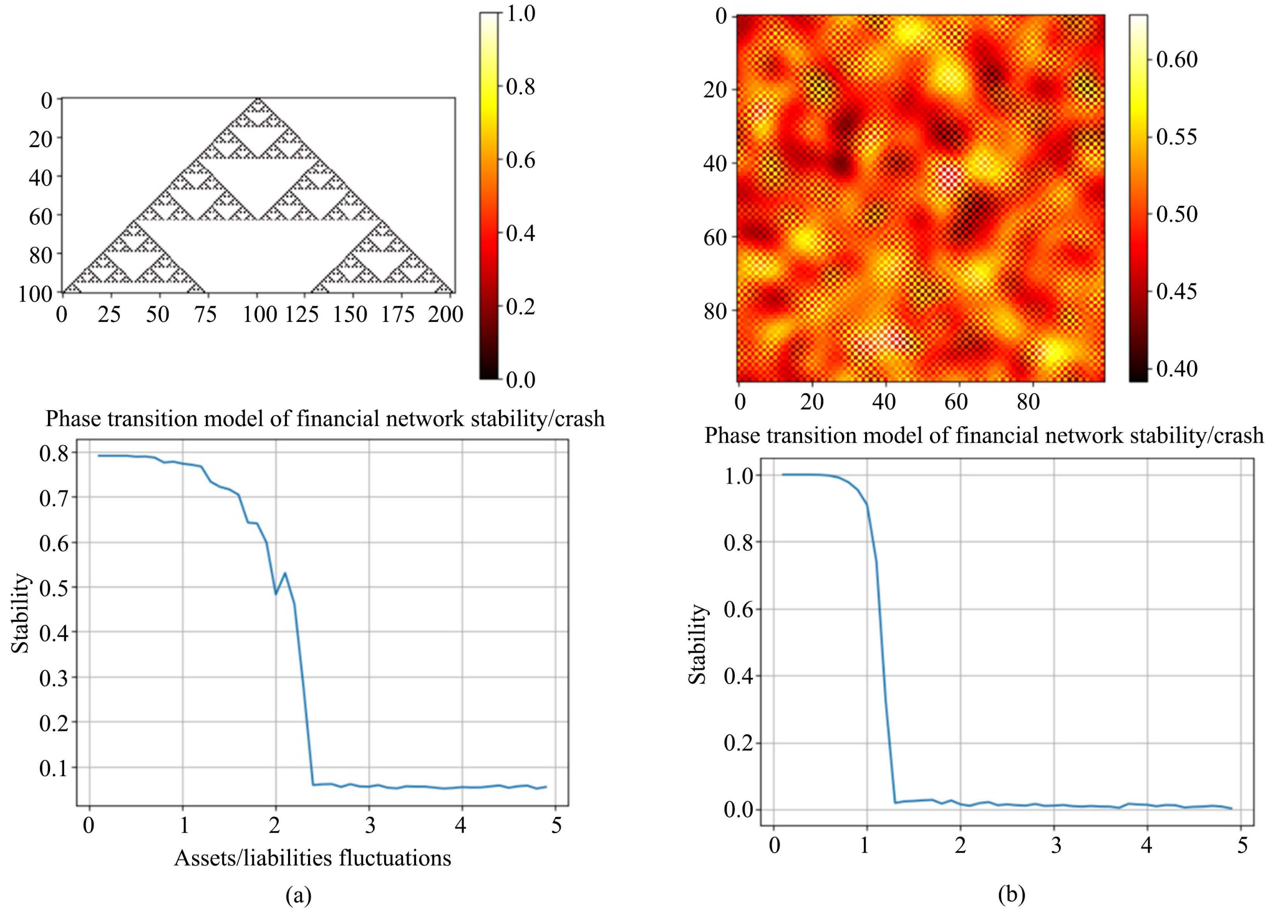

Figure 3. Ising matrix lattice: map for different values of spin interaction constant and phase transition plot. (a) rule 90; (b) Recurrent matrix.

Securitization is a process where a certain group of assets is combined and transferred or sold to a third party, a company designated as a Special Purpose Vehicle (SPV) or Special Purpose Entities (SPEs). The named company will sell this group of assets as valuable paper on the public market. Thanks to this step, the original institution will reduce its capital requirements.

This can be described with following recursion code for spin lattice with size

$$newmatrix[i, j] = \big(matrix[(i-1)\%size, j] + matrix[(i-1)\%size, j] + matrix[i, (j-1)\%size] + matrix[i, (j+1)\%size]\big)/4$$

where % mean modulo operation. How looks such spin lattice with different spin coupling values in displayed on Figure 3. Critical temperature $T_c \approx 1.25$, fluctuation on 45% where network change order parameter. We see, that such dilution of capital requirements decrease resilience of financial network and system is more fragile, what is obvious from simple logical consideration also.

3. Conclusions and Comments

In the framework of the statistical mechanics, we investigate a two state Ising model of lattice representing financial network and their structure based on ownership or assets relations. We see, that relation in financial network a crucial for their stability and have important role in resilience against price fluctuations.

Many financial institutions have highly-levered balance sheets, and large exposures to interest rate, commodity, and currency risks and need effective ways to manage these exposures. They use many financial instruments like derivatives to provide an efficient tool for off-balance sheet risk management to hedge the residual risk from commercial operations. We know that standard formulas do not always reflect the prices of derivatives contracts in the real world, because derivative transactions are structured in different manners as cache. Derivatives remain on a bank's balance sheet for the duration of the contract.

This creates complex ownership relations topology and stability of such system can be investigated with metology mentioned in this article.

In future this can be extended and combined in multi-state Potts lattice model $\theta_{i,j} = 2\pi\sigma_{i,j}$, to include other features of financial market as well as use quantum Monte Carlo numerical methods to evaluate phase transition of order parameter for large spin lattice.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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