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Cyclical Consumption, Real Interest Rate Deviations and Output Gaps in a Large Emerging Economy: Expected and Unexpected Responses under Different Regimes

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Abstract

This article proposed and tested the hypothesis that both expected and unanticipated responses of cyclical consumption to changes in real interest rate (and income) are marked by non-linearities and asymmetries, depending on the regime concerning the underlying real consumption time trend. The reference model is in line with the micro-founded optimizing behavior of consumers, and different inference methods were employed to test our hypothesis for the case of a large emerging economy, like the Brazilian one, over the monthly period from February 2003 to July 21, thereby covering a post COVID-19 sample. Under different model specifications and alternative methods for extracting latent cycles, the evidence indicated that in a regime of stagnation/decline in real consumption, the latter becomes more sensitive to both real interest rate deviations and output gaps, compared to what is observed under a growth regime.

Keywords

Cyclical Consumption, Consumption Trend, Real Interest Rate, Output Gap, Risk Aversion, Brazil

1. Introduction

Understanding how private consumption behaves is crucial for both the formulation of economic policies and the decision-making regarding production and investments by firms. This stems from the high weight of household consumption into the GDP of modern economies. Based on U.S. time series data from the *Bureau of Economic Analysis*, for instance, private consumption constituted roughly 68% of the GDP in June 2023, while it averaged around 63% from 1947 to 2023.

Empirical research on consumption behavior tests for micro-founded relationships in the face of the dynamics of relevant explanatory variables, such as real interest rates and disposable income. A large literature has been developed based on a canonical consumption model, inspired by the contributions of Friedman (1957), Hall (1978), Campbell and Mankiw (1989), among others. However, there is still limited available evidence regarding possible non-linear effects of such factors on consumption, especially under different structural regimes regarding the underlying *consumption trend*. This study emphasizes these aspects based on recent stylized facts in an emerging economy of significant importance.

Specifically, the Brazilian economy (where private consumption accounts for over 60% of the GDP, according to the *Brazilian Institute of Geography and Statistics*) has experienced several stress scenarios in recent decades, some of which had external origins (such as the Subprime crisis and the COVID-19 pandemic), while others were of domestic origin, such as the recession of 2015/2016 and the truck drivers' strike event in 2018.

The goal of this study was to provide evidence that contributes to closing gaps regarding four motivating questions: 1) How does the consumption cycle in an emerging economy react to anticipated changes in the real interest rate and disposable income? 2) Depending on the prevailing consumption trend regime (growth or stagnation/decline), can we expect significant changes in cyclical consumption responses to short-term deviations in real interest rates and GDP? 3) How do *unexpected changes* in cyclical consumption behave in the face of *structural innovations* in (or shocks to) real interest rates and output? 4) Is there any substantial alteration in such short-term relationships under those different regimes?

Empirical studies have been conducted to test the applicability of the canonical model in explaining private consumption behavior, based on the well-known Euler equation. Flavin (1981), Hall (1988), and Carroll (2001) are examples of evidence that estimating log-linear approximations for that equation led to results contrary to the optimization hypothesis, even suggesting, as in Carroll (op. cit.), to abandon empirical estimations for the Euler equation. However, more recently, Gomes & Issler (2017) used aggregate data from the United States and found evidence of consumption optimization from a wide range of specifications for the Euler equation. According to the authors, optimization in consumption (i.e., the role of real interest rates in consumption changes) is the rule, not the exception.

Micro-founded models have been also employed to test hypotheses regarding consumption determination in Brazil. Some noteworthy works include Reis et al. (1998), who, drawing from Campbell & Mankiw (1989), for instance, provided evidence suggesting that consumption in Brazil could be impacted by economic uncertainty while confirming the credit constraint hypothesis for the majority of the country's population, thereby widening the dependence on current disposable income. In turn, Gomes (2013) tested for substitution and complementarity relationships between private and public consumption decisions. The results rejected both relationships, but the study provided evidence of significant sensitivity of consumption to income.

However, more recently, Moreira (2023) found evidence of consumption optimization in Brazil, at least partially. Despite disposable income maintaining its explanatory power, the inverse relationship between real interest rates and consumption was confirmed from Feb/2003 to Feb/2016. The author also inferred that after the Subprime crisis there was an increase in risk aversion, thus reducing the consumption optimization degree.

Methodologically, this study followed the approach developed by Moreira (2023), which involves expanding time series observations by using monthly frequency. By using quarterly or annual data, previous studies were limited by a small number of observations. For example, Reis et al. (1998) used only 76 quarterly observations, while Gomes (2013) used 33 annual observations.

By using data at a monthly frequency, the present study obtained 222 observations for its estimations. Furthermore, in this the research hypotheses were tested by using variables in deviations from their trend levels, which more consistently characterizes an economic cycle phenomenon.

A stylized fact that motivated the division into two regimes (one of growth and the other of stagnation/contraction) was the Brazilian economic crisis starting in 2014 (specifically in October 2014). Either based on consumption itself or economic activity, it was possible to observe a significant regime change from that year onwards. Prior to this, and since 2003, Brazil experienced consistent real growth in consumption, despite cyclical fluctuations. With the deterioration of the macroeconomic context in the country, especially regarding the fiscal budget path, the last quarter of 2014 marked the beginning of a regime of stagnation in real consumption.

The study then developed and tested the hypothesis that in the face of prolonged real consumption stagnation and/or contraction, the latter becomes more sensitive to deviations in real interest rates and output, as well as more responsive to structural shocks to these latter drivers. In other words, both anticipated and unexpected changes in real interest rate deviations and output gaps have a stronger impact on cyclical consumption when it is under a structural stagnation regime.

This means that under such a regime, occasional reductions (even if temporary) in income (as well as occasional increases in real interest rates) can have a greater impact on real consumption, making it more difficult to resume a path of permanent growth. The other side of this phenomenon is the increased effectiveness of monetary policy and fiscal instruments (with impacts on disposable income). Additionally, if it is considered that in a stagnation regime there may be an underlying effort by the Central Bank to achieve disinflation, real income (and real interest rates) finds more difficulty in assuming an upward (and downward, respectively) trend; thus, a situation of inertia or lock-in at stagnant levels is created. So, our hypothesis suggests non-linearity in consumption short-term responses, conditional on the current structural regime[1]. Such a proposition and its corresponding estimation can be regarded as a novelty brought by the present work.

From a long-term perspective, such a hypothesis suggests that it is more costly and/or difficult to initiate a new regime of consistent consumption growth (as observed between 2003 and 2014 in Brazil) than to preserve it once it has started. This is because, under a growth regime, consumption becomes less sensitive to real interest rate and income, most likely reflecting a more expressive impact of long-term growth drivers, such as positive changes in *total factor productivity* and gains in terms of broad economic efficiency. Both aspects are associated with the process of technological advancement and/or structural and institutional improvements in the overall business environment.

This article has been structured as follows: in the second section, theoretical aspects regarding the determination of household consumption are addressed. Section 3 presents the data, the methodological strategy, and some preliminary empirical considerations. In turn, the main research results are analyzed in Section 4, while a robustness checking is implemented in Section 5. Following that, we have final remarks and references.

2. Theoretical Aspects: Consumption, Output, and Real Interest Rate in an Uncertain Context

Consumption models take into account the *permanent income hypothesis*, random walk, and the role of the interest rate as important counterpoints to Keynesian consumption theory, which posits that the dynamics of current disposable income are the primary precursor to the households consumption. However, under an optimizing context, consumers respond to changes in the real interest rate, implementing an intertemporal substitution of consumption. Therefore, the result is that an increase in the real interest rate is accompanied by higher long-term consumption growth, given the incentives to save and, thus, reducing current consumption.

In turn, the permanent income hypothesis and the random walk theory, developed by Friedman (1957) and Hall (1978), respectively, predict consumption smoothing in the face of variations in current disposable income. This is because the postulated maximization depends on an expected present value for the future income stream (over the life horizon), thereby mitigating effects of transitory income change on household decisions.

On the one hand, in the absence of uncertainty, the consumer has a reliable estimate of how much he/she needs in terms of consumption to maximize their utility over the life horizon. So a reduction in current consumption is accompanied by a decrease in the expected marginal utility for future consumption, and

[1]In this sense, linear models would not be able to handle this complexity and asymmetry.

such a decline in marginal utility in consumption is linear. On the other hand, when uncertainty is introduced into the model, the consumer is uncertain about the exact volume of consumption needed to meet future scenarios and thus maximize expected utility. In this case, the expected marginal utility of future consumption decreases more slowly as current consumption decreases, which represents an additional incentive for savings, known as the precautionary motive. More formally, the optimizing attitude of household consumption can be expressed by[2],

$$u'(C_t) = E_t\left[u'(C_{t+1})\right] \tag{1}$$

where $u'(C_t)$ represents the marginal utility of current consumption, with E_t being the expectation operator for future events. This conventional form assumes that the marginal utility is linear as a consequence of a quadratic utility function. This leads to the condition that,

$$E_t\left[u'(C_{t+1})\right] = u'\left[E_t(C_{t+1})\right] \tag{2}$$

Hence, the Euler equation can be also expressed as,

$$C_t = E_t\left[(C_{t+1})\right] \tag{3}$$

However, in the context of uncertainty regarding both the level and variance of future consumption (and income), there may be an additional and unanticipated reduction in consumption. This leads to the case of a positive third derivative of the utility function, or $u'''(\cdot) > 0$. Under this functional form, marginal utility becomes convex and nonlinear. According to Romer (2018), as a consequence we have,

$$E_t\left[u'(C_{t+1})\right] > u'\left[E_t(C_{t+1})\right] \tag{4}$$

As $C_t = E_t\left[(C_{t+1})\right]$, so replacing it into (4),

$$E_t\left[u'(C_{t+1})\right] > u'(C_t) \tag{5}$$

In this way, a marginal reduction in current consumption becomes a generator of expected utility. As Leland (1968) suggests, this postulate underlies the concept of precautionary savings. In general terms, a positive third derivative in the context of uncertainty means that an increase in the latter results in a rise in expected marginal utility for a given level of expected intertemporal consumption. Therefore, a rise in uncertainty increases the incentive to save.

Alternatively, consider a reduction in current consumption, dC_t. This allows for an increase in future consumption, denoted as $dC_{t+1} = (1+r)dC_t$, with r as the real interest rate. In this process, there is a decrease in the marginal utility of future consumption while an increase in the marginal utility of current consumption. The cost of current utility can be measured as $dC_t u'(C_t)$, while the benefit of future utility is given by $(1+r)dC_t E_t\left[u'(C_{t+1})\right]$. In a condition of

[2]The following analysis is consistent with the main results from a canonical *constant-relative-risk-aversion* (CRRA) utility function model.

equilibrium or optimization, costs and benefits are equal, such that,

$dC_t u'(C_t) = dC_{t+1} E_t\left[u'(C_{t+1})\right]$, or,

$$\frac{E_t\left[u'(C_{t+1})\right]}{u'(C_t)} = \frac{dC_t}{dC_{t+1}} \tag{6}$$

In the absence of both uncertainty and a precautionary motive, and with a positive real interest rate, typically $E_t\left[u'(C_{t+1})\right] < u'(C_t)$. However, as mentioned, under uncertainty, the consumer doesn't know the quantity of intertemporal consumption that will allow to a maximization of expected utility over the life horizon. The positive third derivative and a convex marginal utility explain how an increase in future consumption can be accompanied by a very small decrease in its marginal utility. This, once again, leads to $E_t\left[u'(C_{t+1})\right] > u'(C_t)$, which can be explained by a level of risk aversion σ influencing the intertemporal path. Let $dC_{t+1} = \frac{1+r}{1+\sigma} dC_t$ be. Hence,

$$\frac{E_t\left[u'(C_{t+1})\right]}{u'(C_t)} = \frac{dC_t}{\frac{1+r}{1+\sigma} dC_t} \tag{7}$$

Given that $1+\sigma > 1+r$, we have $E_t\left[u'(C_{t+1})\right] > u'(C_t)$. Therefore, *coeteris paribus*, an increase in σ is accompanied by a stronger incentive for precautionary savings, which is micro-founded in the optimizing process of the household with an increase in the ratio $\frac{E_t\left[u'(C_{t+1})\right]}{u'(C_t)}$. This implies that the optimization by the representative consumer, in the face of an initial reduction in current consumption, is accompanied by a more significant stimulus for the decrease in C_t as σ grows, when compared to an analysis that disregards the uncertainty component.

The other side of this process is precisely the low elasticity of intertemporal substitution (EIS), under a context of high risk aversion. It is known that consumption optimizing models presuppose a process of EIS that negatively depends on the degree of risk aversion. Under $1+\sigma > 1+r$, the decision to save implies $dC_{t+1} < dC_t$ (7). This maintains a stimulus for more saving, which results from its scarcity adjusted for high risk.

In general, we have the following form, as in the classic *Ramsey-Cass-Koopmans* (RCK) model,

$$\frac{\dot{c}(t)}{c(t)} = \frac{r - \rho - (n+g)}{\sigma} \tag{8}$$

where $\frac{\dot{c}(t)}{c(t)}$ is the intertemporal consumption growth rate, ρ is the household's subjective discount rate, *n* is the population growth, and *g* is productivity growth. It is important to emphasize that the higher the households' risk aversion *σ*, *coeteris paribus*, the smaller the effect of real interest rates on consumption dynamics. In other words, there is a reduced effectiveness of real interest rates in stimulating intertemporal consumption reallocation under contexts of higher risk aversion degrees.

Following Campbell and Mankiw (1989), we can establish that a fraction λ of households decide on their consumption based on a rule of thumb, giving more weight to the variation in current income $(y_t - y_{t-1})$, while the rest $(1-\lambda)$ optimize based on estimated permanent income as proposed by Friedman (1957), resulting in the random walk recommended by Hall (with noise ϵ_t representing other factors beyond changes in current disposable income),

$$c_t - c_{t-1} = \lambda(y_t - y_{t-1}) + (1-\lambda)\epsilon_t \tag{9}$$

The idea is that if $\lambda = 0$, then consumption changes could not be predicted from changes in current disposable income, resulting in a random walk in consumption. Conversely, in the context of $\lambda > 0$, part of consumption decisions would be explained by reactions to changes in current disposable income, providing a counterpoint to the permanent income theory. However, this version does not consider a possible component of inertia in consumption changes. Introducing the hypothesis of persistence in these fluctuations, i.e., an inertial component $\phi > 0$, leads to the autoregressive form,

$$c_t - c_{t-1} = \phi(c_{t-1} - c_{t-2}) + (1-\phi)\left[\lambda(y_t - y_{t-1})\right] + (1-\phi)(1-\lambda)\epsilon_t \tag{10}$$

In both the Campbell and Mankiw (1989) version and the inertial version, the uncertainty component is represented in the residual term ϵ_t, which also includes the effects of real interest rates on consumption changes. Expanding on the latter,

$$c_t - c_{t-1} = \phi(c_{t-1} - c_{t-2}) + (1-\phi)\left[\lambda(y_t - y_{t-1}) + \beta(r_t - r_{t-1})\right] + (1-\phi)(1-\lambda)\epsilon_t \tag{11}$$

Given β as a measure of the response of consumption changes to real interest rate variations. Since the present study aimed to focus on the dynamics of consumption, output, and real interest rate cycles, it is convenient to transform the previous equation into a version with variables in cyclical form (i.e., in terms of deviations or gaps around the time trend). Furthermore, it is assumed that the data-generating process exhibits some degree of time lag in the effects of *yc* and *rc* on cyclical consumption. The empirical convenience of this assumption also serves to reduce the likelihood of endogeneity issues in the regressions. Thus,

$$cc_t = \phi cc_{t-1} + (1-\phi)\left[\lambda yc_{t-1} + \beta rc_{t-1}\right] + (1-\phi)(1-\lambda)\epsilon_t \tag{12}$$

where *cc* is the *cyclical consumption*, *yc* is the *output gap*, and *rc* stands for the *real interest rate deviation*. A time lag is also assumed in the effects of output and real interest rate cycles, both at $t-1$, on cyclical consumption at t. This was the empirical baseline version used in the subsequent regressions.

3. Data, Preliminary Analysis, and Methodological Strategy

3.1. Data

The household consumption series was represented by the *real retail sales index*

(CONS)—provided by the *Monthly Trade Survey* (source: *Brazilian Institute of Geography and Statistics*)—as a proxy, with data spanning from Feb/2003 to Jul/2021, totaling 222 observations. This is because the directly obtained household consumption series, from the National Accounts, is not available on a monthly basis but only on a quarterly and annual basis. However, adopting this low frequency would lead to the problem identified by Moreira (2023), in which the empirical literature on consumption in Brazil generally had a low number of observations precisely because it relied on the National Accounts System. In an effort to conduct the research using higher-frequency data and thus significantly increasing the number of observations, we can use the *real retail sales series*, which is primarily linked to household consumption. The index is seasonally adjusted and deflated by the *Broad Consumer Prices Index* (basis, average 2014 = 100).

A significant regime shift is notable starting from 2014. Until this year, consumption exhibited sustained growth with minor fluctuations, including those resulting from the Subprime crisis. However, starting from 2014, the national economy entered into a regime of stagnation in real consumption levels.

Besides, the real interest rate series (R) was calculated using a standard Fisher equation, by taking the difference between the annual Selic rate (i.e. the short-term interest rate) and inflation expectations for the following 12 months. Both the annual Selic rate and expectations were obtained from the *Time Series Manager System* of the Central Bank of Brazil. There was a clear trend of decreasing short-term real interest rates (interrupted in 2013 and resumed in 2017), with cyclical fluctuations, responding to monetary policy decisions as well as to fluctuations in inflation expectations.

In turn, the behavior of the Brazilian real GDP or output (Y) was represented by the *Central Bank's Economic Activity Index*, the IBC-Br, due to its versatility in providing monthly frequency data and covering all major sectors of the economy. The index is in real terms and seasonally adjusted (basis, average 2002 = 100). A structural break in 2014 can also be observed in its pattern, albeit with noticeably higher volatility compared to what occurred with consumption. **Figure 1** shows the time behavior of CONS, R and Y.

These series were then used for the extraction of their respective latent cyclical components. The *Hodrick-Prescott (HP) filter* was so adopted, which allows for the extraction of moving averages or smoothed trends, as well as the resulting cyclical component.

Figure 2 illustrates the behavior of consumption (*cc*), income (*yc*), and real interest rate (*rc*) cycles. These cyclical components are fluctuating stationarily around zero, which normalizes the time series' trend value. Based on the output gap (*yc*), it is possible to observe that since 2014, there was a more volatile behavior compared to the previous period. This was the period during which Brazil experienced a severe recession in 2015/2016, the truck drivers' strike crisis in 2018, and the onset of the COVID-19 in 2020. Although there was the Subprime crisis (2008/2009) in the earlier period, the subsequent one was still more

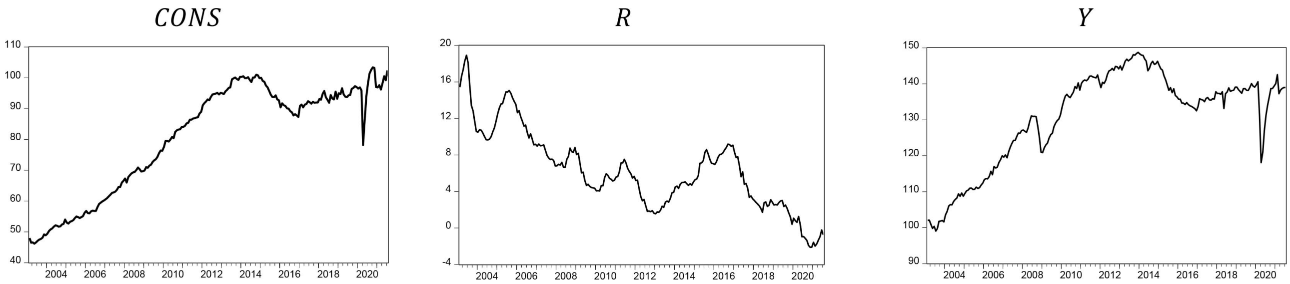

Figure 1. Consumption, real interest rate and output: Feb/2003-Jul/2021. Source: Own elaboration.

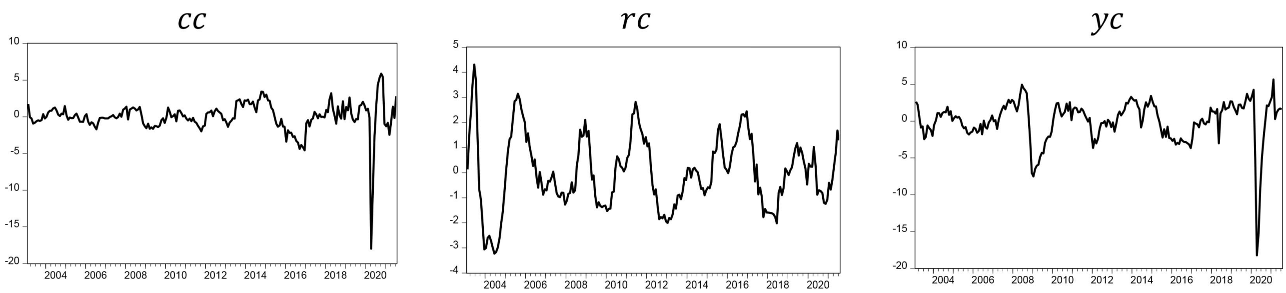

Figure 2. Consumption, real interest rate and output cyclical components. Source: Own elaboration.

unstable. While the standard deviation of the series from Feb/2003 to Oct/2014 was 2.23, for the period between Nov/2014 to Jul/2021, this statistics was 3.64.

In fact, the same happened with cyclical consumption. In this case, the difference is even greater. The standard deviation for cc in the first period was 1.03, while in the second subperiod, it was 3.12. Because of this, we will adopt these two subsamples for comparison purposes, as there were indications that in the more recent subsample there was an increase in economic uncertainty.

It is also important to note that the standard deviation measure for these two cycle variables confirms the stylized fact of consumption smoothing in relation to output, as in both subsamples, the standard deviation of cc was lower than that of yc. As for the real interest rate cycle (rc), the inertial characteristic and the reversals between hawkish and dovish cycles around the trend are evident. Over the entire sample, the trend was declining, with the lowest historical real level (−2.1% per annum) observed after the COVID-19 shock (specifically in November and December 2020).

It should be noted how the cyclical variables relate to each other, specifically in light of the theoretical prediction. Table 1 presents the correlation matrix, along with their respective t-statistics and probabilities. The aforementioned positive correlation (between cc and yc) and negative correlation (between cc and rc) are confirmed. Of course, these statistics are only initial clues, which will be further examined in subsequent analyses.

3.2. A descriptive Characterization of Periods with Significant Contractions

The cyclical approach gains details when distinct events are evaluated, albeit with similar effects mainly on cc. The first moment of crisis and pronounced

Table 1. Ordinary correlation matrix, t-stat and prob.: Feb/2003-Jul/2021.

	cc	*yc*	*rc*
cc	1.000		

	-----		
yc	0.699	1.000	
	[14.517]	-----	
	(0.000)	-----	
rc	−0.294	−0.200	1.000
	[−4.571]	[−3.037]	-----
	(0.000)	(0.002)	-----

Notes: [ ] for t-stat and ( ) for prob. Source: own elaboration.

effects on the cyclical consumption to be analyzed was the Subprime crisis. The peak of consumption prior to that event was identified in September 2008, with a respective positive gap at 1.35 in relation to the trend. The bottom after the shock was observed between March and May 2009, with a negative gap at −1.64 over 8 months, representing a cumulative drop of 221%[3] from a cyclical perspective. Figure 3 illustrates the behavior of *cc* during the periods of expressive contraction, with the area filled in black color for these time intervals.

The period between 2014 and 2016 (also highlighted with a dashed line in Graph 1) was the second moment with pronounced effects on the cyclical consumption. The peak of *cc* prior to the shock was identified in October 2014, at a level of 3.4. Afterward, consumption continued to decline, although gradually, reaching a bottom of −4.6 in December 2016, from which there was a recovery. Over the 26-month period, the cycle reversed, with a cumulative decline of 235%.

In sequence, we had the "*truck drivers' strike*", which lasted for 10 days in May 2018 and had significant effects on the short-term production capacity in Brazil, due to shortages of inputs and goods. The peak before the start of that event occurred in April 2018, with a positive consumption gap of about 3.2. The bottom after the shock occurred in July of the same year, with a negative gap of −0.9. This represented a contraction of 128% in the cyclical component over just 3 months, meaning a monthly decline of 42.6%, higher than the monthly decline observed during the Subprime crisis (27.6% per month) and also during the 2014-2016 crisis (9% per month).

Most recently, we experienced the COVID-19 pandemic. This was likely the event with the most significant variation in the cyclical consumption over the country's history. The peak prior to the initial effects occurred in February 2020,

[3]This is because the accumulated change can be calculated by [(−1.64 − 1.35)/1.35] = −2.21, or −221%. The same calculation holds for other similar considerations into this section.

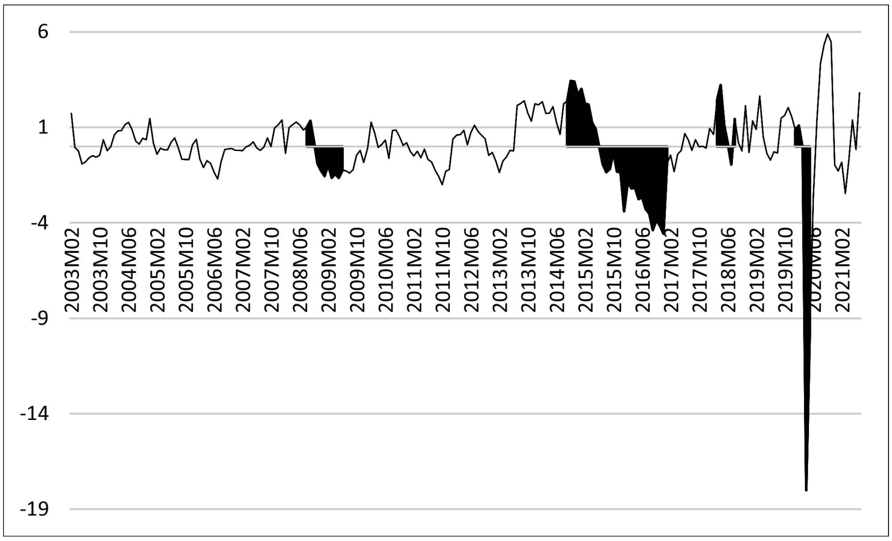

Figure 3. Periods of expressive cyclical consumption contraction. Source: Own elaboration.

with *cc* marking 1.09. In April of the same year, the cyclical consumption reached its support, with a negative level of −17.9. In this way, the observed retraction rate in the cyclical component was an astonishing 1.742%, or 871% per month. The "V" or "square root" recovery was a characteristic of this period, amid waves of lockdowns and business closures.

3.3. Methodological Strategy

Two stages of inference and identification were performed. In the first one, the predicted effects of real interest rate and output cycles on the cyclical consumption were inferred. The response of the cyclical consumption to predicted (or anticipated) changes in real interest rate and income cycles was analyzed based on models inspired by Equations (11) and (12). However, initially, regressions were tested without the presence of the first-order autoregressive component, so $\phi = 0$. Thus, in the baseline version,

$$cc_t = \lambda yc_{t-1} + \beta rc_{t-1} + \epsilon_t \tag{13}$$

In turn, in order to verify the inertial pattern of consumption deviations and allow for a better fit to the data, the first-order autoregressive component (AR (1)) was included, yielding the inertial dynamic version (12),

$$cc_t = \phi cc_{t-1} + (1-\phi)\left[\lambda yc_{t-1} + \beta rc_{t-1}\right] + (1-\phi)(1-\lambda)\epsilon_t$$

For this specification, regressions were estimated using both OLS and GMM approaches in order to improve the robustness of the estimated coefficients in the face of the common problem of endogeneity of regressors in macroeconomics. In fact, the Generalized Method of Moments is considered robust in dealing with common issues in time series, such as heteroscedasticity, serial correlation of residuals, and especially endogeneity (Hansen, 1982), by means of the adoption of instrumental variables to control the latter.

The following instruments were then used: $cc(t-2)$, $yc(t-2)$, $rc(t-2)$, $cc(t-3)$, $yc(t-3)$, $rc(t-3)$, $cc(t-4)$, $yc(t-4)$, $rc(t-4)$. Thus, it was recognized that there is a *general equilibrium* process among the three macro-level variables[4]. Illustratively, the lagged cycle of real interest rates (e.g., $rc(t-2)$, $rc(t-3)$ and $rc(t-4)$) can affect the subsequent levels of both the output gap and cyclical consumption; alternatively, lagged values of the cyclical consumption can also have statistically significant impacts on output gaps (an issue already recognized in Campbell & Mankiw, 1989). From the set of instrumental variables, the exogeneity condition was satisfied as suggested in Johnston (1984). Furthermore, the J-test was implemented to test the null hypothesis of model over identification, ensuring the validity of the adopted instruments (Hansen, 1982; Cragg, 1983).

In the second stage, the aim was to infer unanticipated effects on cyclical consumption from structural innovations in real interest rate deviations and output gaps. For this purpose, estimation was carried out through *Structural Vector Autoregressive models* (SVAR). A SVAR model can be considered a method to address the well-known identification problem in macroeconomics (Sims, 1980). By imposing recursive restrictions on unanticipated changes in endogenous variables, a SVAR focuses on the unexpected responses of a specific variable to structural shocks to itself and/or to the other variables of the model. A reduced form for the SVAR can be,

$$X_t = \Gamma^{-1}B(L)X_t + \Gamma^{-1}e_t \tag{14}$$

where X_t is a vector of endogenous variables (e.g., those in Equation (12)), Γ^{-1} is an inverse matrix of the contemporaneous coefficients (Γ), $B(L)$ denotes the lagged coefficients matrix, and e_t represents the matrix of variance-covariance innovations. By defining $B^* = \Gamma^{-1}B$ and $u_t = \Gamma^{-1}e_t$, we can express it as follows,

$$X_t = B^*(L)X_t + u_t \tag{15}$$

The identification of restrictions in the SVAR assumes that the structural innovations are orthogonal. This means that the covariance between two shocks to the variances is restricted to zero. Furthermore, the SVAR imposes a normalization on those variances, so that each of them becomes unitary. By doing this, we can interpret the impulse response functions as a response of the economy to unitary innovations, where $\Sigma_e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$, for the case of an illustrative bivariate model.

In turn, the identification, or causality ordering, can be achieved by imposing an exclusion restriction on Γ. Just to illustrate, consider a small system composed of two variables x_{1t} and x_{2t}. If we assume that an unitary structural in-

[4]Such a general equilibrium is consistent with the canonical New-Keynesian cycle model (Clarida et. al., 1999), composed by its well-known three equations: a dynamic IS curve, a New-keynesian Phillips Curve and a Taylor rule.

novation in x_{1t} does not result in unexpected changes in x_{2t} ($\delta_{1,2} = 0$), a matrix Γ would be,

$$\Gamma = \begin{pmatrix} 1 & \delta_{2,1} \\ 0 & 1 \end{pmatrix} \tag{16}$$

In other words, an exogeneity condition was imposed (i.e., x_2 is contemporaneously exogenous, and x_1 is endogenous), so that an innovation $e_{2t} > 0$ causes unexpected changes in x_{2t} ($u_{2t} > 0$) and also in x_{1t} (indicated by the contemporaneous parameter $\delta_{2,1}$ to be estimated). It is important to note that this type of identification should be grounded in theory and/or previous evidence.

Regarding our underlying model, and in order to identify the exclusion restrictions in the Γ matrix, we assumed that real interest rates are autonomously determined by the Central Bank of Brazil, under the inflation targeting regime. Therefore, being the most exogenous variable in the model, $\delta_{x_i,rc} = 0$, with x_i representing the other variables (*yc* and *cc*). This indicates that unitary structural shocks to both *yc* and *cc* do not create contemporaneous unexpected changes in *rc*. In turn, it was considered that structural shocks to *rc* impact both output gaps and cyclical consumption. In this case, $\delta_{rc,yc}$ and $\delta_{rc,cc}$ are contemporaneous relationships different from zero. Furthermore, we assumed that structural shocks to output gaps have contemporaneous unexpected effects on cyclical consumption, so that $\delta_{yc,cc}$ is not restricted to zero. The matrix is thus represented as follows,

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 \\ \delta_{rc,yc} & 1 & 0 \\ \delta_{rc,cc} & \delta_{yc,cc} & 1 \end{pmatrix} \tag{17}$$

The adopted set of structural restrictions imposes a causality ordering starting from the most exogenous variable (*rc*) to the most endogenous one (*cc*), with the output gap as the intermediate variable. In other words, the ordering follows the direction *rc*-*yc*-*cc*. Such an ordering of contemporaneous recursive restrictions, or structural decomposition, was then used to estimate impulse-response functions (IRFs).

4. Results: Anticipated and Unanticipated Responses of Cyclical Consumption

4.1. Predicted Responses and Regime Shift

The estimates for the entire sample and for the first and second subsamples are presented in **Table 2**. At this initial stage, only Ordinary Least Squares (OLS) regressions were estimated, while controlling for autocorrelation and heteroskedasticity in the residuals using the Newey-West (1987) procedure. For all three samples, parameters estimates confirm the underlying theory, with statistical significance. Cyclical consumption responds positively to output gaps, while responding inversely to real interest rate deviations. However, there is a regime

Table 2. Expected responses of cyclical consumption (OLS/HAC).

Sample	Regressor	Coef.	St-dev.	t-stat
Feb/2003-Jul/2021	$yc(-1)$***	0.333	0.055	6.025
	$rc(-1)$***	−0.284	0.091	−3.101
R_I	$yc(-1)$***	0.238	0.072	3.267
	$rc(-1)$**	−0.158	0.073	−2.169
R_II	$yc(-1)$***	0.362	0.114	3.155
	$rc(-1)$**	−0.776	0.378	−2.049

Notes: *** for significance at 1%; ** for 5%. Coefficients covariance estimated by HAC (Newey-West). Source: Own elaboration.

shift concerning the investigated subsamples. In the first regime (Feb/2003-Oct/2014: *R_I*), during which stable consumption growth was observed in Brazil, there was lower sensitivity of cyclical consumption to both explanatory variables. Either a potential reduction in output gaps, or a rise in the real interest rate deviation, had a smaller expected contraction effect on cyclical consumption, during that "golden phase".

In the second regime (Nov/2014-Jul/2021: *R_II*), under which a phase of stagnation in real consumption levels was observed, there was an increase in the sensitivity of cyclical consumption to changes in output gaps (λ from 0.23 in *R_I* to 0.36 in *R_II*) and real interest rate deviations (β from −0.15 to −0.77, respectively). This evidence confirms the hypothesis that in *R_II* any deterioration regarding output gaps (i.e. decreasing) and real interest rate deviations (i.e. increasing) was followed by stronger effects on short-term consumption levels, compared to what would be observed under *R_I*. This structural change in λ and β under *R_II*, compared to *R_I*, can be interpreted as a feedback regime of real consumption stagnation, making it more difficult to return to a regime of sustained growth.

4.2. Predicted Responses in the Presence of Cyclical Consumption Inertia

Table 3 presents the estimates for the three time samples using both OLS and GMM methods while controlling for cyclical consumption inertia (AR(1)). Overall, when accounting for cyclical consumption inertia, estimates corroborate the evidence that in *R_II* there was a rise in the sensitivity of cyclical consumption to anticipated changes in real interest rate deviations in Brazil, compared to *R_I*. This holds true for both OLS and GMM methods. For instance, under *R_II*, the response of *cc* to *yc* was at −0.385 by GMM regression, and −0.415 based on OLS estimation; on the other hand, under *R_I*, these same estimates were −0.060 and −0.149, respectively, thereby indicating lower sensitivity.

On the other hand, the output gap (*yc*) loses its explanatory power in the AR(1) specification, either with absence of statistical significance or with an inversion of the expected signal. It is essential to highlight that none of the GMM

Table 3. Expected responses of inertial cyclical consumption OLS/GMM. Dependent variable: cc.

Samples	Method	Regressor	Coef.	St-dev.	t-stat
Feb/2003-Jul/2021	OLS	$cc(-1)$***	0.761	0.071	10.625
		$yc(-1)$*	0.039	0.020	1.889
		$rc(-1)$**	−0.068	0.029	−2.358
	GMM [0.284]	$cc(-1)$***	0.701	0.118	5.913
		$yc(-1)$	−0.134	0.161	−0.829
		$rc(-1)$***	−0.186	0.063	−2.934
R_I	OLS	$cc(-1)$***	0.559	0.076	7.273
		$yc(-1)$	0.063	0.043	1.441
		$rc(-1)$***	−0.149	0.045	−3.260
	GMM [0.662]	$cc(-1)$***	0.815	0.105	7.728
		$yc(-1)$	0.018	0.033	0.542
		$rc(-1)$**	−0.060	0.029	−2.042
R_II	OLS	$cc(-1)$**	0.475	0.193	2.457
		$yc(-1)$	0.081	0.160	0.506
		$rc(-1)$*	−0.415	0.247	−1.674
	GMM [0.839]	$cc(-1)$***	0.876	0.168	5.205
		$yc(-1)$**	−0.363	0.143	−2.527
		$rc(-1)$***	−0.385	0.132	−2.901

Notes: [ ] refers to J-test prob.; *** for significance at 1%; ** for 5%; * for 10%. HAC for OLS. St-deviation and covariance estimated by HAC in the GMM. Source: Own elaboration.

regressions rejected the null hypothesis of model over identification based on the J-test, thereby supporting our instruments list.

It should be noted that in the inertial specification the parameters λ and β need to be computed *a posteriori* from the coefficients yielded by the regression, particularly from ϕ. Table 4 then presents the structural parameters for each sample through the GMM regression (reported in Table 3). Again, the structural parameters indicate distinct regimes of cyclical consumption sensitivity to deviations in income and real interest rates. Specifically, β presents significant change between the two regimes: from −0.3 in *R_I* to −3.1 in *R_II*. This corroborates the evidence reported by both OLS and GMM methods (as shown in Table 2 and Table 3). However, when adjusting those estimates for the degree of cyclical consumption inertia (ϕ), the change in λ and β between both regimes becomes even more pronounced.

Nevertheless, the structural parameter λ, which measures the correlation between cyclical consumption and income deviations, exhibits opposite signs between both regimes, with a negative sign in *R_II* (contrary to the theory) and a

Table 4. Computed structural parameters based on (12) and GMM estimates.

Sample	ϕ	λ	β
Feb/2003-Jul/2021	0.701***	−0.448	−0.622***
R_I	0.815***	0.097	−0.324**
R_II	0.876***	−2.927**	−3.104***

Notes: *** for significance at 1%; ** for 5%; * for 10%. Source: Own elaboration.

lack of statistical significance in the overall sample. This suggests that, when controlling for the inertial component, output gaps loses explanatory power. This result implies more weight to the permanent income hypothesis, the consumer optimizing behavior, and/or Hall's (1978) random-walk theory. Specifically for Brazil, these findings partially align with those obtained in Moreira (2023).

4.3. Unanticipated Responses: Structural Shocks

The first step in estimating the SVAR was to identify the optimal lag of the simultaneous equations structure, i.e., the lag order of the coefficient matrix B(L) (Equations (14) and (15)). Initially, the Akaike Information Criterion (AIC) suggested the use of 4 lags in the structural equations, but both the Schwarz and Hannan-Quinn criteria pointed to only 2 lags. Subsequently, the LM test for residual autocorrelation was applied to this preliminary structure with 2 lags. Since the test rejected the null hypothesis of no autocorrelation, additional lags were added to the $B(L)$ matrix. Following this procedure, it was found that using 4 lags was necessary to not reject the null hypothesis of the LM test. Furthermore, a test of inverse roots of the characteristic polynomial was applied to this structure and confirmed the hypothesis of parameter stability, as all 12 inverse roots (= 3 endogenous variables × 4 lags) remained within the unitary circle.

Figure 4 presents impulse-response functions (IRFs) for the overall sample, focusing on the cyclical consumption responses to positive structural shocks to cc, rc, and yc at a 12-month horizon after the occurrence of the structural innovation. A structural shock to cyclical consumption is accompanied by an unexpected positive level in cc itself, with statistical significance (as indicated by the confidence intervals, i.e., both dashed lines in the positive quadrant), until approximately the 4th month after the structural innovation. This corroborates the estimate of cyclical consumption structural inertia for the total sample (ϕ = 0.701) through the GMM findings (**Table 4**).

Regarding shocks to the real interest rate deviation, an unanticipated decrease (with statistical significance) in cyclical consumption is observed from the 1st to the 2nd month, again in the 4th month, and also around the 7th month after the initial shock. This evidence is consistent with the structural parameter obtained from GMM estimates ($\beta = -0.622$).

Furthermore, the IRFs also showed unanticipated effects on the cyclical consumption as a response to structural innovations in the output gap (yc). These

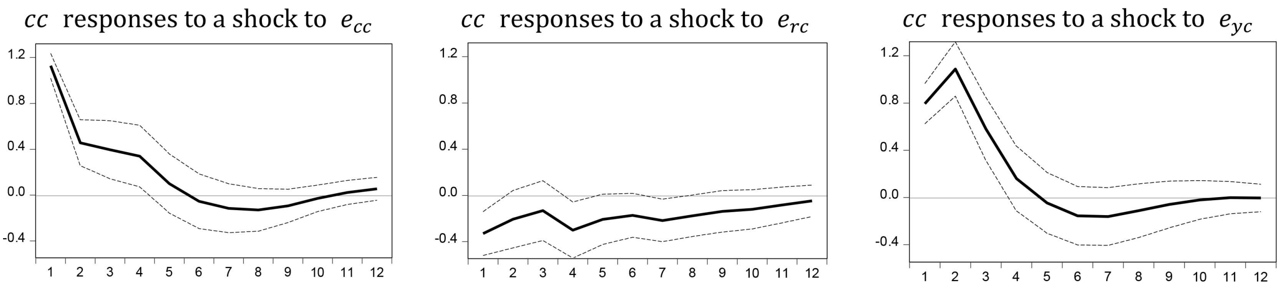

Figure 4. Unexpected responses of cyclical consumption to structural innovations. Source: Own elaboration.

shocks bring a positive surprise to cyclical consumption that persists for approximately 4 months after the structural event. Thus, even though the estimated structural parameter (λ) by GMM did not present statistical significance, the inference from the estimated SVAR points to the significance of positive (and unanticipated) effects on cyclical consumption.

4.4. Unanticipated Responses: Effect of Regime Change

The SVAR methodology was also applied to the two regimes (R_I and R_II) experienced in Brazil. **Figure 5** presents the unanticipated response of cc to structural shocks to rc and yc under both regimes. The solid line is used for R_I, while the dashed line is used for R_II.

It can be observed that a structural shock to both real interest rate deviations and output gaps is accompanied by more intense unanticipated effects on cyclical consumption under R_II compared to those observed under R_I. Furthermore, the unanticipated effects on cc from structural innovations are consistent with the direction of the estimates obtained by GMM, thus reinforcing the evidence that a regime of consumption stagnation/decline is reflected in structural changes of the cyclical consumption sensitivity (amplifying it) to unexpected variations in real interest rates and output, compared to what is verified under a regime of real consumption growth.

5. Robustness Checking: An Alternative Approach to Extract Latent Cycle Components

As aforementioned, the HP filter was used to extract the cyclical components cc, rc, and yc. However, Hamilton (2018) raised concerns about the accuracy of the HP filter in measuring the underlying cycles of macroeconomic variables. There could be biases in the estimation of cycles, especially at the end of the sample, as well as the alleged issue of *ad hoc* choice for the smoothing parameter. Despite subsequent works (Jonsson, 2020; Phillips & Shi, 2021, among others) criticizing Hamilton's (2018) arguments and even yielding results in favor of using the HP filter (such as in Dritsaki & Dritsaki, 2022), this study preferred to perform a robustness test based on the estimation of cyclical components using the method proposed in Hamilton (2018).

Essentially, Hamilton (2018) proposed a method for extracting the cyclical component of a variable Z based on its forecast over $t+h$ periods ahead, using

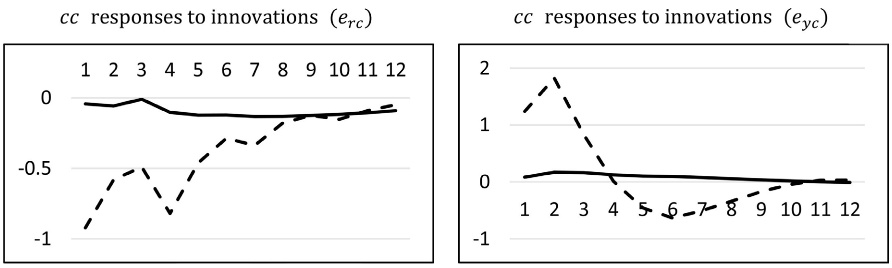

Figure 5. Unanticipated responses to innovations in distinct regimes (*R_I*: *solid line*; *R_II*: *dashed line*). Source: Own elaboration.

the four most recent values of the own time series *Z*. In the case of monthly frequency with $h = 0$, the specification would be as follows:

$$Z_t = \alpha_0 + \alpha_1 Z_{t-1} + \alpha_2 Z_{t-2} + \alpha_3 Z_{t-3} + \alpha_4 Z_{t-4} + \varepsilon_t \quad (18)$$

Thus, the cyclical component would be captured by the residual of the regression ε_t, as follows:

$$Z_t - \hat{Z}_t = \varepsilon_t \quad (19)$$

In this way, this Hamilton-based procedure was applied to extract the cyclical components of consumption *cc_H*, real interest rates *rc_H*, and output *yc_H*. The time behavior of these three components is depicted in **Figure 6**.

After extracting the cyclical components, the SVAR was applied with the same contemporaneous restrictions as in the previous section. To identify the optimal lag of the model, the same criteria as before were used. Based on the Akaike Information, Schwarz and Hannan-Quinn criteria, as well as the LM test for serial autocorrelation and the test for inverse roots of the characteristic polynomial, it was found that only 01 lag should be adopted. In our view, this results from the fact that the variables measured by the Hamilton filter become more volatile than when measured by the HP filter, as also observed in Dritsaki & Dritsaki (2022). **Figure 7** brings the IRFs for the robustness test.

Similar to the evidence previously presented, the regime of stagnation/decline in real consumption (*R_II*) brings greater sensitivity of unexpected cyclical consumption to structural shocks to both real interest rate deviations and output gaps, when compared to responses under a growth regime (*R_I*). The shortening of unforeseen effects from such shocks results from the aforementioned increase in volatility (and reduction in persistence) in the cyclical components extracted by the Hamilton filter.

6. Final Remarks and Policy Implications

This article proposed and tested the hypothesis that the responses of cyclical consumption to changes in conventionally accepted determining factors—real interest rates and output—are non-linear under different structural regimes. In particular, it was proposed that when an economy is under a regime of stagnation/decline in real consumption, it becomes more sensitive to changes in those factors, compared to what is observed under a regime of stable consumption

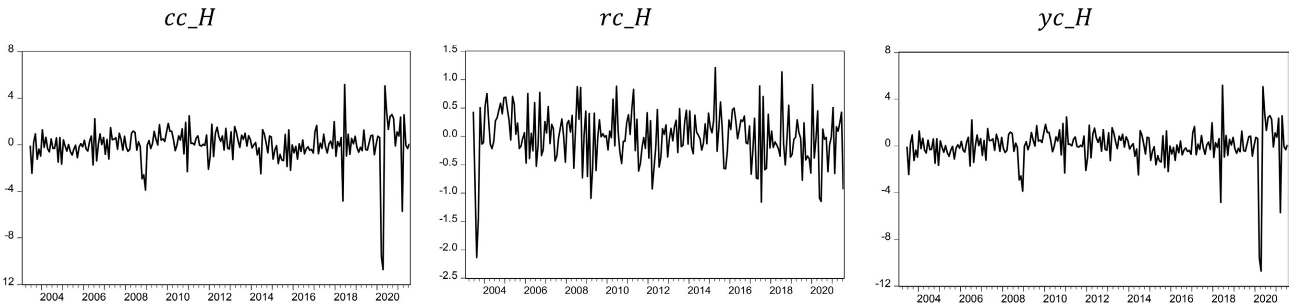

Figure 6. Cyclical components with the Hamilton's filter Source: Own elaboration.

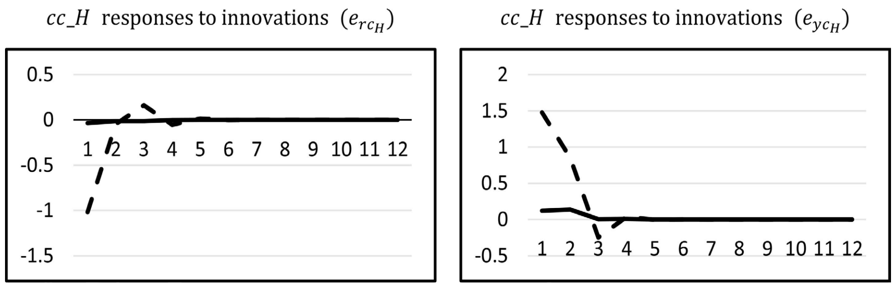

Figure 7. Innovations under distinct regimes: Hamilton's filter. (*R_I*: *solid line*; *R_II*: *dashed line*) Source: Own elaboration.

growth. This implies that there would be non-linearity in these responses across different regimes. In other words, a change in the underlying consumption trend would impact the parameters of short-term relationships. There is therefore a complexity that linear models would not be suitable to deal with.

Based on stylized facts in Brazil, according to which there was a change in the dynamics of real consumption from October 2014 onwards (with stagnation since then), this study applied different methods of inference and over identification, such as OLS, GMM, and SVAR, relying on a canonical model of consumption that assumes optimizing behavior.

Regarding the overall sample (Feb/2003-Jul/2021), the evidence obtained aligns with previous studies that found an important role for income in determining consumption. However, when controlling for the inertial component, the explanatory power of the output gap was eliminated through OLS and GMM methods. This suggests that, assuming a degree of persistence for cyclical consumption, the key variable for explaining its predicted changes becomes the real interest rate deviations, implying an optimizing behavior (partially in line with Moreira, 2023). Nonetheless, through SVAR, it was observed that there are *unanticipated* responses of cyclical consumption to *innovations* to output gaps, as well as to real interest rate deviations.

Furthermore, we did not find evidence to reject its central hypothesis. Both the GMM and SVAR estimations identified structural changes in the predicted and unanticipated responses of cyclical consumption to variations and shocks (respectively) in real interest rates and output, depending on the considered regime. The hypothesis that under a regime of stagnation/decline in consumption trend (*R_II*) there was stronger sensitivity of cyclical consumption to (anticipated and unanticipated) changes in its determining factors was confirmed. In contrast, a growth regime (*R_I*) resulted in lower sensitivity over the same relationships. It is important to note that the estimates were also corroborated even when adopting the inference method proposed by Hamilton (2018) for extracting cycle components.

In terms of micro-foundation for such non-linearity, it is suggested that a likely explanation lies in the households' lower precautionary savings motive under *R_II*, in line with the discussion in the theoretical scope. As pointed out in Aydin (2022), precautionary behavior stems from the desire to build up a buffer by depressing spending and delevering when households deal with atypical constraints. Thus, a change in disposable income under *R_II* would be associated with a greater change in consumption, given the reduced propensity to save preventively in a stagnation regime. In turn, stagnation/decline regimes may be associated with a scenario of reduced risk aversion among households. Although the perception of risk or uncertainty is higher, individuals need to take more risks to generate income. According to our reference model, a reduction in σ implies a higher elasticity of intertemporal substitution, so explaining the greater sensitivity of cyclical consumption to changes in real interest rates compared to what is observed in a growth regime. The underlying idea is that risk appetite (i.e., the opposite of aversion) arises out of necessity and scarcity, more easily found in declining regimes. On the other hand, growth and prosperity regimes would lead households to gradually adopt a more conservative behavior, with an increase in risk aversion and precautionary savings.

The current research dealt with limitations which are idiosyncratic to the empirical case of study, the Brazilian economy. There exist some constraints on data availability (such as on real GDP indicators) that prevent larger sample horizons. However, as aforementioned, this challenge was addressed with higher frequency data (monthly instead to annual or quarterly), so that we could enlarge the number of observations in comparison with previously published works in the related literature for Brazil.

Lastly, but not least, our evidence has implications from a policymaking perspective, specifically for monetary policy reaction functions. For instance, when central banks are pursuing disinflation goals, interest rate decision-makers must take into account the regime they are in, as this impacts the effectiveness of anticipated and unanticipated changes in real interest rates. Particularly, under a stagnation/decline real consumption trend, additional caution should be assumed in raising real interest rates, given the higher impacts on cyclical consumption levels and, consequently, in social losses.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Aydin, D. (2022). Consumption Response to Credit Expansions: Evidence from Experimental Assignment of 45,307 Credit Lines. *American Economic Review, 112,* 1-40. https://doi.org/10.1257/aer.20191178

Campbell, J. Y., & Mankiw G. N. (1989). Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence. In O. J. Blanchard, & S. Fischer, (Eds.), *NBER Macroeconomics Annual* (pp. 185-214). MIT Press. https://doi.org/10.1086/654107

Carroll, C. D. (2001). Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second-Order Approximation). *Topics in Macroeconomics, 1,* 153460131003. https://doi.org/10.2202/1534-6013.1003

Clarida, R., Gali, J., & Gertler, M. (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature, 37,* 1661-1707. https://doi.org/10.1257/jel.37.4.1661

Cragg, J. G. (1983). More Efficient Estimation in the Presence of Heteroscedasticity of Unknown form. *Econometrica, 51,* 751-763. https://doi.org/10.2307/1912156

Dritsaki, M., & Dritsaki, C. (2022). Comparison of HP Filter and the Hamilton's Regression. *Mathematics, 10,* Article 1237. https://doi.org/10.3390/math10081237

Flavin, M. A. (1981). The Adjustment of Consumption to Changing Expectations about Future Income. *Journal of Political Economy, 89,* 974-1009. https://doi.org/10.1086/261016

Friedman, M. (1957). *A Theory of the Consumption Function*. Princeton University Press. https://doi.org/10.1515/9780691188485

Gomes, F. (2013). Gasto do Governo e Consumo Privado: Substitutos ou Complementares? *Revista Brasileira de Economia, 67,* 219-234. https://doi.org/10.1590/S0034-71402013000200004

Gomes, F., & Issler, J. (2017). Testing Consumption Optimality Using Aggregate Data. *Macroeconomic Dynamics, 21,* 1119-1140. https://doi.org/10.1017/S1365100515000085

Hall, R. E. (1978). Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence. *Journal of Political Economy, 86,* 971-987. https://doi.org/10.1086/260724

Hall, R. E. (1988). Intertemporal Substitution in Consumption. *Journal of Political Economy, 96,* 339-357. https://doi.org/10.1086/261539

Hamilton, J. D. (2018). Why You Should Never Use the Hodrick-Prescott Filter. *The Review of Economics and Statistics, 100,* 831-843. https://doi.org/10.1162/rest_a_00706

Hansen, L. P. (1982). Large Sample Properties of Generalized Method of Moments Estimators. *Econometrica, 50,* 1029-1054. https://doi.org/10.2307/1912775

Johnston, J. (1984). *Econometric Methods.* McGraw-Hill Book.

Jonsson, K. (2020). Real-Time US GDP Gap Properties Using Hamilton's Regression-Based Filter. *Empirical Economics, 59,* 307-314. https://doi.org/10.1007/s00181-019-01631-6

Leland, H. E. (1968). Saving and Uncertainty: The Precautionary Demand for Saving. *The Quarterly Journal of Economics, 82,* 465-473. https://doi.org/10.2307/1879518

Moreira, R. R. (2023). Consumption Per Effective Labour in Brazil: Testing for the Optimising Behaviour. *International Journal of Computational Economics and Econometrics, 13,* 23-34. https://doi.org/10.1504/IJCEE.2023.127291

Newey, W. K., & West, K. D. (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica, 55,* 703-708. https://doi.org/10.2307/1913610

Phillips, P. C. B., & Shi, Z. (2021). Boosting: Why You Can Use the HP Filter. *International Economic Review, 62,* 521-570. https://doi.org/10.1111/iere.12495

Reis, E., Issler, J. V., Blanco, F., & Carvalho, L. (1998). Renda permanente e poupança precaucional: Evidências empíricas para o Brasil no passado recente. *Pesquisa e Planejamento Econômico, 28,* 233-272.

Romer, D. (2018). *Advanced Macroeconomics*. McGraw Hill.

Sims, C. A. (1980). Macroeconomicsand Reality. *Econometrica, 48,* 1-48. https://doi.org/10.2307/1912017