



# Fixed Point Results on Interval-Valued Fuzzy Metric Space using notation of Pairwise Compatible Maps and Occasionally Weakly Compatible Maps with Application

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## Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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## Abstract

The purpose of the paper is to obtain common fixed point results on interval-valued fuzzy metric space for occasionally weakly compatible maps (OWC) using contractive conditions. With the concept of  $\tau_{norm}$ , interval numbers, and some important properties of interval-valued  $\tau_{norm}$ .

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## 1 Introduction

Zadeh [1] introduced the IVFS, which is a subset of fuzzy sets and is distinguished by its fuzzy membership function. Li [2] introduced three types of distance between two IVFS (or numbers) are defined real line  $\mathbb{R}$  and pointed out that each type of distance is a metric on the associated sets, and underlined that the metric space for any IVFS of numbers is complete. Shen, Li, and Wang [3] came up with the concept of IVFMS, which generalises fuzzy metric space based on the ideas of George and Veeramani [4]. Kramosil et al. [5] initially suggested fuzzy metrics, and instantly Kaleva et al.[6] and Abu Osman [7] used two distinct ways to create an independent, densely fuzzy metric space. The concept of weakly commuting maps on Probabilistic Metric spaces were introduced by Singh et al. [8]. Kumar and Chung developed some common fixed point theorems in metric space using  $\mathbb{R}$ -weakly commutative and reciprocal continuity of mappings. Mihet [9] used a contractual need satisfying an explicit relation to derive a widely used proof theorem. Fixed point results in Fuzzy Menger Space with Common Property and Fixed point results for P-1 Compatible in Fuzzy Menger Space were worked on by Ruchi Singh et al. [10] and [11]. It is clear from a literature review that no effort has been made to derive fixed point theorems with the requirement of occasionally weakly compatible (OWC) mapping on IVFMS. Sewani et al. [12] applied the fuzzy iterated contraction abstraction to create some new results in intuitionistic-fuzzy metric space. Here, we have derived several approximate points for OWC mappings satisfying explicit relationships using IVFMS.

## 2 Preliminaries

**Definition2.1** [3]: Assume that  $Q$  is a typical non empty set.

The mapping  $\mathfrak{R}_m : Q \rightarrow [I_{iv}]$  is referred to as an interval-valued fuzzy set on  $Q$ .  $IVF(Q)$  is used to identify all interval-valued fuzzy-set on  $Q$ .

If  $\mathfrak{R}_m \in IVF(Q)$ , let  $\mathfrak{R}_m(\varphi^*) = [\mathfrak{R}_m^-(\varphi^*), \mathfrak{R}_m^+(\varphi^*)]$ ,  $\mathfrak{R}_m^-(\varphi^*) \leq \mathfrak{R}_m^+(\varphi^*)$  for all  $\varphi^* \in Q$ , then the conventional fuzzy-set  $\mathfrak{R}_m^- : Q \rightarrow [I_{iv}]$  and  $\mathfrak{R}_m^+ : Q \rightarrow [I_{iv}]$  are referred to as the *Lower\**-fuzzy-set and *Upper\**-fuzzy-set respectively. In particular,  $\mathfrak{R}_m$  is referred to degenerate fuzzy-set if  $\mathfrak{R}_m^-(\varphi^*) = \mathfrak{R}_m^+(\varphi^*)$  for any  $\varphi^* \in Q$ .

**Definition2.2** [3]: A binary operation of the form is an interval-valued  $\tau_{norm}$  is  $*_{I_{iv}} : [I_{iv}]X[I_{iv}] \rightarrow [I_{iv}]$  on  $[I_{iv}]$ . All four of the following conditions are satisfied for  $\bar{\gamma}_{\bar{\alpha}}, \bar{\delta}_{\bar{\alpha}}, \bar{\eta}_{\bar{\alpha}} \in [I_{iv}]$ :

- (1) *Commutativity*:  $\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} \bar{\delta}_{\bar{\alpha}} = \bar{\delta}_{\bar{\alpha}} *_{I_{iv}} \bar{\gamma}_{\bar{\alpha}}$ ,
- (2) *Associativity*:  $\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} [\bar{\delta}_{\bar{\alpha}} *_{I_{iv}} \bar{\eta}_{\bar{\alpha}}] = [\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} \bar{\delta}_{\bar{\alpha}}] *_{I_{iv}} \bar{\eta}_{\bar{\alpha}}$ ,
- (3) *Monotonicity*:  $\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} \bar{\delta}_{\bar{\alpha}} \leq \bar{\delta}_{\bar{\alpha}} *_{I_{iv}} \bar{\eta}_{\bar{\alpha}}$  whenever  $\bar{\delta}_{\bar{\alpha}} \leq \bar{\eta}_{\bar{\alpha}}$ ,
- (4) *Boundary condition*;  $\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} \bar{1} = \bar{\gamma}_{\bar{\alpha}}$ ,  $\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} \bar{0} = [\bar{\gamma}_{\bar{\alpha}}, \bar{\gamma}_{\bar{\alpha}}^+] *_{I_{iv}} [0, 1] = [0, \bar{\gamma}_{\bar{\alpha}}^+]$

**Example 1:**(i)  $\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} \bar{\delta}_{\bar{\alpha}} = [\bar{\gamma}_{\bar{\alpha}} \cdot \bar{\delta}_{\bar{\alpha}}, \bar{\gamma}_{\bar{\alpha}}^+ \cdot \bar{\delta}_{\bar{\alpha}}^+]$ ;  
(ii)  $\bar{\gamma}_{\bar{\alpha}} *_{I_{iv}} \bar{\delta}_{\bar{\alpha}} = [\bar{\gamma}_{\bar{\alpha}} \wedge \bar{\delta}_{\bar{\alpha}}, \bar{\gamma}_{\bar{\alpha}}^+ \wedge \bar{\delta}_{\bar{\alpha}}^+]$

**Definition2.3** [3]: Let  $\{\alpha_{\bar{\alpha}\eta_a}\} = \{[\alpha_{\bar{\alpha}}^-, \alpha_{\bar{\alpha}}^+]\}$ ,  $\eta_a \in \mathbb{N}^+$  be a sequence of interval-numbers in  $[I_{iv}]$ ,  $\bar{\alpha}_{\bar{\alpha}} = [\alpha_{\bar{\alpha}}^-, \alpha_{\bar{\alpha}}^+] \in [I_{iv}]$ , if  $\lim_{\eta_a \rightarrow \infty} \alpha_{\bar{\alpha}\eta_a}^- = \bar{\alpha}_{\bar{\alpha}}^-$  and  $\lim_{\eta_a \rightarrow \infty} \alpha_{\bar{\alpha}\eta_a}^+ = \bar{\alpha}_{\bar{\alpha}}^+$ , then we say that the sequence  $\{\alpha_{\bar{\alpha}\eta_a}\}$  is convergent to  $\bar{\alpha}_{\bar{\alpha}}$ , and which is denoted by  $\lim_{\eta_a \rightarrow \infty} \alpha_{\bar{\alpha}\eta_a} = \bar{\alpha}_{\bar{\alpha}}$ .

**Definition2.4** [13]: An interval-valued  $\tau_{norm} *_{I_{iv}}$  is continuous if and only if it is continuous in its first component,

i.e., for each  $\bar{\beta}_{\bowtie} \in [I_{iv}]$ , if  $\lim_{\eta_a \rightarrow \infty} \bar{\alpha}_{\bowtie \eta_a} = \bar{\alpha}_{\bowtie}$   
 then  $\lim_{\eta_a \rightarrow \infty} (\bar{\alpha}_{\bowtie \eta_a} *_{I_{iv}} \bar{\beta}_{\bowtie}) = (\lim_{\eta_a \rightarrow \infty} \bar{\alpha}_{\bowtie \eta_a} *_{I_{iv}} \bar{\beta}_{\bowtie}) = \bar{\alpha}_{\bowtie} *_{I_{iv}} \bar{\beta}_{\bowtie}$ , where  $\{\bar{\alpha}_{\bowtie \eta_a}\} \subseteq [I_{iv}]$ ,  $\bar{\alpha}_{\bowtie} \in [I_{iv}]$ .

**Definition2.5** [4] :If Q is a temporary set,  $*_{I_{iv}}$  is a continuous interval-valued  $\tau_{norm}$  on  $[I_{iv}]$ , and is an IVF(Q) on  $Q^2 \times (0, \infty)$  meets the following conditions, the triple  $(Q, \underline{d}, *_I)$  is know as IVFMS on Q:  
 following requirements:

- (1)  $\underline{d}(\varphi^*, \varrho^*, \tau_{norm}) > \bar{0}$ ;
- (2)  $\underline{d}(\varphi^*, \varrho^*, \tau_{norm}) = \bar{1}$  if and only if  $\varphi^* = \varrho^*$ ;
- (3)  $\underline{d}(\varphi^*, \varrho^*, \tau_{norm}) = \underline{d}(\varrho^*, \varphi^*, \tau_{norm})$ ;
- (4)  $\underline{d}(\varphi^*, \varrho^*, \tau_{norm}) *_I \underline{d}(\varphi^*, r, s) \leq \underline{d}(\varphi^*, r, \tau_{norm} + s)$ ;
- (5)  $\underline{d}(\varphi^*, \varrho^*, \cdot) : (0, \infty) \rightarrow [I]$  is continuous;
- (6)  $\lim_{\eta \rightarrow \infty} \underline{d}(\varphi^*, \varrho^*, \tau_{norm}) = \bar{1}$ , where  $\varphi^*, \varrho^*, r \in Q$  and  $\tau_{norm}, s > \bar{0}$ .

**Definition2.6** [4] : Let  $(Q, \mathbb{k}, *_I)$  is an IVFMS,

- (1) For all  $\varphi^*, \varrho^* \in Q$ , if  $s > \tau_{norm} > 0$  then  $\mathbb{k}(\varphi^*, \varrho^*, \tau_{norm}) \leq \mathbb{k}(\varphi^*, \varrho^*, s)$ .
- (2) A sequence  $\{\varphi_{\eta_a}\}$  in Q is referred to as a cauchy sequence if for all  $\bar{\varepsilon} > \bar{0}$  and  $\tau_{norm} > 0$ , there is an exists a  $\eta_{a0} \in \mathbb{N}$  such that  $\mathbb{k}(\varphi_{\eta_a}, \varphi_{\ell_a}, \tau_{norm}) > \bar{1} - \bar{\varepsilon}$  for all  $\eta_a, \ell_a \geq \eta_{a0}$ .
- (3) An IVFMS in which every cauchy sequence is convergent, is said to be a complete IVFMS .

**Definition2.7** [14] : In interval-valued-metric-space, a function is continuous iff  $\varphi_{\eta_a} \rightarrow \varphi^*, \varrho_{\eta_a} \rightarrow \varrho^* \Rightarrow \lim_{\eta_a \rightarrow \infty} \mathbb{k}(\varphi_{\eta_a}, \varrho_{\eta_a}, \tau_{norm}) \rightarrow \mathbb{k}(\varphi^*, \varrho^*, \tau_{norm})$ .

**Definition2.8** [15] : Assume that Q is a non-empty set and that Y and Z are the self-maps of Q. A point  $\varphi^*$  in Q is said to be coincidence point of Y and Z iff  $Y\varphi^* = Z\varphi^*$ .

**Definition2.9** [16] : Two mapping Y and Z on IVFMS  $(Q, \mathbb{k}, *_I)$  are weakly compatible by the expression iff  $\mathbb{k}(YZ\varphi^*, ZY\varphi^*, \tau_{norm}) \geq \mathbb{k}(Y\varphi^*, Z\varphi^*, \tau_{norm})$  for  $\tau_{norm} \geq 0$  and  $\varphi^* \in Q$ .

**Definition2.10** [15] : Two mappings Y and Z on IVFMS  $(Q, \mathbb{k}, *_I)$  are compatible iff for sequence  $\{\varphi_{\eta_a}\}$  in Q,  $\mathbb{k}(YZ\{\varphi_{\eta_a}\}, ZY\{\varphi_{\eta_a}\}, t) \rightarrow \bar{1}$  whenever  $F\{\varphi_{\eta_a}\} \rightarrow v, G\{\varphi_{\eta_a}\} \rightarrow v$  for some  $v \in Q$ .

**Definition2.11** [15] : Two mappings Y and Z on IVFMS  $(Q, \mathbb{k}, *_I)$  are said to be OWC iff there exists a coincidence point  $\varphi^*$  of Y and Z in Q such that Y and Z commutes at the point.

**Definition2.12** [17] : Two mappings Y and Z on IVFMS  $(Q, \mathbb{k}, *_I)$  then they are said to satisfy E.A property if there exists a  $\{\varphi_{\eta_a}\}$  in Q such that  $\lim_{\eta_a \rightarrow \infty} F\{\varphi_{\eta_a}\} = \lim_{\eta_a \rightarrow \infty} G\{\varphi_{\eta_a}\} = v, v \in Q$ .

**Lemma2.13** [4] : IVFMS  $(Q, \mathbb{k}, *_I)$  is non-decreasing for all  $\varphi^*, \varrho^* \in Q$ .

**Proof:** Proof is followed by the definition of Interval-valued fuzzy metric space .

**Lemma2.14** : If for all  $\varphi^*, \varrho^* \in Q, \tau_{norm} > 0$  and for a number  $h \in (0, 1)$ ,  $\mathbb{k}(\varphi^*, \varrho^*, h\tau_{norm}) \geq \mathbb{k}(\varphi^*, \varrho^*, \tau_{norm})$  then  $\varphi^* = \varrho^*$ .

### 3 Results and Discussion

**Theorem3.1** : Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC mappings. If there exists  $h \in (0, 1)$  such that

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right], \dots (3.1)$$

for all  $\rho^*, \rho^* \in Q$  and  $\tau_{norm} > 0$ , then there exist unique points  $w_{up}^*, z_{up}^* \in Q$  such that  $ABw_{up}^* = CDw_{up}^* = w_{up}^*$  and  $EFz_{up}^* = GHz_{up}^* = z_{up}^*$ . Moreover, for  $z_{up}^* = w_{up}^*$ , A,B,C,D,E,F,G and H has unique common fixed point in Q.

**Proof :** Let us consider that  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC, therefore for  $\rho^*, \rho^* \in Q$  we have  $AB\rho^* = CD\rho^*$  and  $EF\rho^* = GH\rho^*$ .

Now we claim that  $AB\rho^* = EF\rho^*$ .

If not, then by inequality (3.1) we have

$$\begin{aligned} \Rightarrow \mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) &\geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, EF\rho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \\ &\geq \min [\mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, AB\rho^*, \tau_{norm})] \end{aligned}$$

Therefore  $AB\rho^* = EF\rho^*$ , i.e.  $AB\rho^* = CD\rho^* = EF\rho^* = GH\rho^*$ . Suppose that  $z_{up}^*$  is another point in Q such that  $ABz_{up}^* = CDz_{up}^*$  then by (3.1), we have  $ABz_{up}^* = CDz_{up}^* = EF\rho^* = GH\rho^*$ , so  $AB\rho^* = ABz_{up}^*$  and  $AB\rho^* = CD\rho^* = w_{up}^*$ .

Now by Lemma 2.13,  $w_{up}^*$  is the common fixed point of AB and CD, and it is unique.

In the same way, there is a unique point  $z_{up}^* \in Q$  such that  $z_{up}^* = EFz_{up}^* = GHz_{up}^*$ .

Assuming that  $w_{up}^* \neq z_{up}^*$ , we have

$$\begin{aligned} \mathbb{k}(w_{up}^*, z_{up}^*, h\tau_{norm}) &= \mathbb{k}(ABw_{up}^*, EFz_{up}^*, h\tau_{norm}) \\ &\geq \min \left[ \begin{array}{l} \mathbb{k}(CDw_{up}^*, GHz_{up}^*, \tau_{norm}), \mathbb{k}(CDw_{up}^*, ABw_{up}^*, \tau_{norm}), \mathbb{k}(EFz_{up}^*, GHz_{up}^*, \tau_{norm}), \\ \mathbb{k}(ABw_{up}^*, GHz_{up}^*, \tau_{norm}), \mathbb{k}(EFz_{up}^*, CDw_{up}^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[ \begin{array}{l} (\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, w_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm})), \\ \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm}) \end{array} \right] \\ &\geq \min [(\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm}))] \end{aligned}$$

$$\geq \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm})$$

by Lemma 2.13, we have  $w_{up}^* = z_{up}^*$  and  $z_{up}^*$  is the common fixed point A,B,C,D,E,F,G and H. The uniqueness holds from (3.1).

**Theorem 3.2 :** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q.

Let  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \varphi \left[ \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \right] \dots (3.2)$$

for all  $\rho^*, \rho^* \in Q$  and  $\varphi \in *_I$  for all  $0 < \tau_{norm} < 1$ , then A,B,C,D,E,F,G and H has unique common fixed point in Q.

**Proof:** As proved in the theorem 3.1.

**Theorem 3.3 :** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \varphi \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \dots (3.3)$$

for all  $\rho^*, \rho^* \in Q$  and  $\varphi : [I_{iv}]^5 \rightarrow [I_{iv}]$  so that

$\varphi(\tau_{norm}, \bar{1}, \bar{1}, \tau_{norm}, \tau_{norm}) > \tau_{norm}$  for all  $0 < \tau_{norm} < 1$ , then A,B,C,D,E,F,G and H has unique common fixed point in Q.

**Proof:** Let us consider that  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC, therefore for  $\rho^*, \rho^* \in Q$ , we have  $AB\rho^* = CD\rho^*$  and  $EF\rho^* = GH\rho^*$ . Now we claim that  $AB\rho^* = EF\rho^*$ .

If not, then by inequality (3.3). We have,

$$\begin{aligned} \mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) &\geq \varphi \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \\ &= \varphi \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, AB\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, EF\rho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \\ &\geq [\mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, AB\rho^*, \tau_{norm})] \end{aligned}$$

$$\geq \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm})$$

Therefore  $AB\rho^* = EF\rho^*$ , i.e.  $AB\rho^* = CD\rho^* = EF\rho^* = GH\rho^*$ . Suppose that  $z_{up}^*$  is another point in Q such that  $ABz_{up}^* = CDz_{up}^*$  then by (3.3). We have  $ABz_{up}^* = CDz_{up}^* = EF\rho^* = GH\rho^*$ , so  $AB\rho^* = ABz_{up}^*$  and  $AB\rho^* = CD\rho^* = w_{up}^*$  is point of coincidence of AB and CD.

Now by Lemma 2.13,  $w_{up}^*$  is the common fixed point of AB and CD, and it is unique.

In the same way, there is a unique point  $z_{up}^* \in Q$  such that  $z_{up}^* = EFz_{up}^* = GHz_{up}^*$ .

Hence  $z_{up}^*$  is the common fixed point A,B,C,D,E,F,G and H.

**Example:** Let  $I = [0, 1]$  with the usual metric  $\mathbb{k}(\rho^*, \rho^*, \tau_{norm}) = |\tau_{norm}\rho^* - \tau_{norm}\rho^*|$  and set up the following maps:

$AB\rho^* = \frac{1}{2}$  for  $\rho$  in  $[0,1]$ ,  $CD\rho^* = \frac{3}{4}$  for  $\rho$  in  $[0,1]$ ,  $EF\rho^* = 1$  for  $\rho$  in  $[0,1]$ ,  $GH\rho^* = \frac{1}{4}$  for  $\rho$  in  $[0,1]$ ,  $\{AB, CD\}$  and  $\{EF, GH\}$  are pair of OWC mapps. Putting  $\varphi(\S_1, \S_2, \S_3, \S_4, \S_5) = -\S_1 + \frac{1}{2} \min(\S_2, \S_3) + \frac{1}{3}(\S_4 + \S_5)$ . we get

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \varphi \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right]$$

$$\Rightarrow |1 - \frac{1}{2}| = \frac{1}{2} \geq \varphi \left\{ \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \frac{1}{4} \right\}$$

$$\Rightarrow \frac{1}{2} \geq -\frac{1}{2} + \frac{1}{2} \max \left( \frac{1}{4}, \frac{3}{4} \right) + \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} \right) \geq -\frac{1}{2} + \frac{1}{2} \left( \frac{3}{4} \right) + \frac{1}{3} \left( \frac{1}{2} \right) \geq \frac{1}{24}$$

The eight maps accept 1 as the unique fixed point between them, and hence, all of the theorem’s requirements are satisfied.

**Theorem3.4:** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}) \end{array} \right] \dots(3.4)$$

for all  $\rho^*, \rho^* \in Q$  and  $\tau_{norm} > 0$ , then A,B,C,D,E,F,G and H has unique common fixed point in Q.

**Proof:** Let us consider that  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC, therefore for  $\rho^*, \rho^* \in Q$  we have  $AB\rho^* = CD\rho^*$  and  $EF\rho^* = GH\rho^*$ .

Now we claim that  $AB\rho^* = EF\rho^*$ .

If not, then by inequality (3.4) we have

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}) \end{array} \right]$$

$$\geq \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}) \end{array} \right]$$

$$\geq \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm})$$

Therefore  $AB\rho^* = EF\rho^*$ , i.e.  $AB\rho^* = CD\rho^* = EF\rho^* = GH\rho^*$ . Suppose that  $z_{up}^*$  is another point in Q such that  $ABz_{up}^* = CDz_{up}^*$  then by (3.4). We have  $ABz_{up}^* = CDz_{up}^* = EF\rho^* = GH\rho^*$ , so  $AB\rho^* = ABz_{up}^*$  and  $AB\rho^* = CD\rho^* = w_{up}^*$ .

Now by Lemma 2.13,  $w_{up}^*$  is the common fixed point of AB and CD, and it is unique. In the same way, there is a unique point  $z_{up}^* \in X$  such that  $z_{up}^* = EFz_{up}^* = GHz_{up}^*$ . Hence  $w_{up}^*$  is the common fixed point of A,B,C,E,F,G and H. The uniqueness Of  $w_{up}^*$  holds from (3.4).

**Corollary3.5 :** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let  $\{AB, CD\}$  and  $\{EF, GH\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, 2\tau_{norm}) \end{array} \right] \dots(3.5)$$

for all  $\rho^*, \rho^* \in Q$  and  $\tau_{norm} > 0$ , then A,B,C,D,E,F,G and H has unique common fixed point in Q.

**Proof:** We have

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, 2\tau_{norm}) \end{array} \right]$$

$$\begin{aligned} &\geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}) \end{array} \right] \\ &\geq \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}) \end{array} \right] \end{aligned}$$

From Theorem 3.4, unique common fixed point exists for A,B,C,D,E,F,G and H.

**Theorem3.6 :** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let  $\{R, AB\}$  and  $\{V, CD\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \dots (3.6)$$

for all  $\varrho^*, \rho^* \in Q$  and  $\tau_{norm} > 0$ , then there exist unique points  $w_{up}^*, z_{up}^* \in Q$  such that  $ABw_{up}^* = Rw_{up}^* = w_{up}^*$  and  $CDz_{up}^* = Vz_{up}^* = z_{up}^*$ . Moreover, for  $z_{up}^* = w_{up}^*$ , A,B,C,D,R and V has unique common fixed point in Q.

**Proof:** Let us consider that  $\{R, AB\}$  and  $\{V, CD\}$  are pairs of OWC, therefore for  $\rho^*, \varrho^* \in Q$ , we have  $AB\rho^* = R\rho^*$  and  $CD\varrho^* = V\varrho^*$ .

Now we claim that  $R\rho^* = V\varrho^*$ .

if not, then by inequality (3.6) we have

$$\begin{aligned} \mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) &\geq \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[ \begin{array}{l} \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, R\rho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, V\varrho^*, \tau_{norm}), \\ \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, R\rho^*, \tau_{norm}) \end{array} \right] \\ &\geq [\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm})\mathbb{k}(V\varrho^*, R\rho^*, \tau_{norm})] \end{aligned}$$

$$\geq \min [\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm})]$$

Therefore  $AB\rho^* = R\rho^*$ , i.e.  $AB\rho^* = R\rho^* = CD\varrho^* = V\varrho^*$ . Suppose that  $z_{up}^*$  is another point in Q such that  $ABz_{up}^* = Rz_{up}^*$  then by (3.6). We have  $ABz_{up}^* = Rz_{up}^* = CD\varrho^* = V\varrho^*$ , so  $AB\rho^* = ABz_{up}^*$  and  $AB\rho^* = R\rho^* = w_{up}^*$ . Now by Lemma 2.13,  $w_{up}^*$  is the common fixed point of AB and R, and it is unique.

In the same way, there is a unique point  $z_{up}^* \in Q$  such that  $z_{up}^* = CDz_{up}^* = Vz_{up}^*$ .

Assuming that  $w_{up}^* \neq z_{up}^*$ , we have

$$\begin{aligned} \mathbb{k}(w_{up}^*, z_{up}^*, h\tau_{norm}) &= \mathbb{k}(Rw_{up}^*, Vz_{up}^*, h\tau_{norm}) \\ &\geq \min \left[ \begin{array}{l} \mathbb{k}(ABw_{up}^*, CDz_{up}^*, \tau_{norm}), \mathbb{k}(ABw_{up}^*, Rw_{up}^*, \tau_{norm}), \mathbb{k}(Vz_{up}^*, CDz_{up}^*, \tau_{norm}), \\ \mathbb{k}(Rw_{up}^*, CDz_{up}^*, \tau_{norm}), \mathbb{k}(Vz_{up}^*, ABw_{up}^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[ \begin{array}{l} (\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, w_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \\ \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm}) \end{array} \right] \end{aligned}$$

$$\begin{aligned} &\geq \min [\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm})] \\ &\geq \mathbb{k}(w_{up}^*, z_{up}^*, h\tau_{norm}) \end{aligned}$$

by Lemma 2.13, we have  $w_{up}^* = z_{up}^*$  and  $z_{up}^*$  is the common fixed point A,B,C,D,R and V. The uniqueness holds from (3.6).

**Theorem3.7 :** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let  $\{R, AB\}$  and  $\{V, CD\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \varphi \left[ \min \left[ \begin{aligned} &\mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ &\mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ &\mathbb{k}(V\varrho^*, AB\rho^*, \tau_{norm}) \end{aligned} \right] \right] \dots(3.7)$$

for all  $\varphi^*, \varrho^* \in Q$  and  $\varphi \in *_I$  for all  $0 < \tau_{norm} < 1$ , then A,B,C,D,R and V has unique common fixed point in Q.

**Proof:** As proved in the theorem 3.6.

**Theorem3.8 :** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let  $\{R, AB\}$  and  $\{V, CD\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \min \left[ \begin{aligned} &\mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ &\mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \end{aligned} \right] \dots(3.8)$$

for all  $\varphi^*, \varrho^* \in Q$  and  $\tau_{norm} > 0$ , then A,B,C,D,R and V has unique common fixed point in Q.

**Proof:** Let us consider that  $\{R, AB\}$  and  $\{V, CD\}$  are pairs of OWC, therefore for  $\rho^*, \varrho^* \in Q$  we have  $AB\rho^* = R\rho^*$  and  $CD\varrho^* = V\varrho^*$ .

Now we claim that  $R\rho^* = V\varrho^*$ .

if not, then by inequality (3.8) we have

$$\begin{aligned} &\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \min \left[ \begin{aligned} &\mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ &\mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}) \end{aligned} \right] \\ &= \min \left[ \begin{aligned} &\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, R\rho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, V\varrho^*, \tau_{norm}), \\ &\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}) \end{aligned} \right] \\ &\geq \left[ \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}) \right] \\ &\geq \left[ \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}) \right] \end{aligned}$$

Therefore  $AB\rho^* = R\rho^*$ , i.e.  $AB\rho^* = R\rho^* = CD\varrho^* = V\varrho^*$ . Suppose that  $z_{up}^*$  is another point in Q such that  $ABz_{up}^* = Rz_{up}^*$  then by (3.8). We have  $ABz_{up}^* = Rz_{up}^* = CD\varrho^* = V\varrho^*$ , so  $AB\rho^* = ABz_{up}^*$  and  $AB\rho^* = R\rho^* = w_{up}^*$ .

Now by Lemma 2.13,  $w_{up}^*$  is the common fixed point of AB and R, and it is unique.

In the same way, there is a unique point  $z_{up}^* \in Q$  such that  $z_{up}^* = CDz_{up}^* = Vz_{up}^*$ .

Hence we get  $w_{up}^*$  is a common fixed point of A,B,C,D,R and V.

**Corollary3.9 :** Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let  $\{AB, R\}$  and  $\{CD, V\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, AB\rho^*, 2\tau_{norm}) \end{array} \right]$$

for all  $\varphi^*, \varrho^* \in Q$  and  $\tau_{norm} > 0$ , then A,B,C,D,R and V has unique common fixed point in Q.

**Proof** We have

$$\begin{aligned} \mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) &\geq \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, AB\rho^*, 2\tau_{norm}) \end{array} \right] \\ &\geq \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}) \end{array} \right] \\ &\geq \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \end{array} \right] \end{aligned}$$

From theorem 3.8, unique common fixed point exists for A,B,C,D,R and V.

**Theorem3.10** : Let  $(Q, \mathbb{k}, *_I)$  be a complete IVFMS and let A,B,C,D be self mappings of Q. Let  $\{AB\}$  and  $\{CD\}$  are pairs of OWC mappings. If there exists  $h \in (0,1)$  such that

$$\mathbb{k}(CD\rho^*, CD\varrho^*, h\tau_{norm}) \geq [\alpha \mathbb{k}(AB\rho^*, AB\varrho^*, \tau_{norm})] + \beta \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, AB\varrho^*, \tau_{norm}), \\ \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(CD\varrho^*, AB\varrho^*, \tau_{norm}) \end{array} \right]$$

where  $\alpha, \beta > 0, \alpha + \beta > 1$ . Then A,B,C and D has unique common fixed point in Q.

**Proof:** Let AB and CD are OWC, therefore for point  $\rho^* \in Q$  such that  $AB\rho^* = CD\rho^*$ . Suppose that there exist  $\varrho^* \in Q$  for which  $AB\varrho^* = CD\varrho^*$ .

On contrary let us assume  $AB\rho^* = CD\varrho^*$ . We have,

$$\begin{aligned} \Rightarrow \mathbb{k}(CD\rho^*, CD\varrho^*, h\tau_{norm}) &\geq [\alpha \mathbb{k}(AB\rho^*, AB\varrho^*, \tau_{norm})] + \beta \min \left[ \begin{array}{l} \mathbb{k}(AB\rho^*, AB\varrho^*, \tau_{norm}), \\ \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(CD\varrho^*, AB\varrho^*, \tau_{norm}) \end{array} \right] \\ &= [\alpha \mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] + \beta \min \left[ \begin{array}{l} \mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(CD\varrho^*, CD\varrho^*, \tau_{norm}) \end{array} \right] \\ &\geq [\alpha \mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] + \beta \min [\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm}), \bar{1}, \bar{1}] \\ &\geq [\alpha \mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] + \beta [\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] \\ &\geq (\alpha + \beta) [\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] \end{aligned}$$

Which is a contradiction, since  $(\alpha + \beta) > 1$ .

Therefore  $CD\rho^* = CD\rho^*$  and hence  $AB\rho^* = AB\rho^*$  and  $AB\rho^*$  is unique.

Therefore from lemma 2.13 A,B ,C and D have a unique fixed point [18] - [22].

## 4 Application

An essential component of analytic and engineering mathematics research is the examination of the existence, nonexistence, and uniqueness of solutions to differential and integral equations. An important tool created in this field is the use of the fixed-point theorem. Consider the integral equation

$$\rho(\check{z}, \tau) = \check{g}_n(\rho(\check{z}), \tau) + \int_{\alpha}^{\beta} \check{y}(\check{z}, \check{s}, \tau) \check{h}_n(\check{s}, \check{u}(\check{s}), \tau) ds + \int_{\alpha}^{\beta} \check{x}(\check{z}, \check{s}, \tau) \check{j}_n(\check{s}, \check{u}(\check{s}), \tau) ds \text{ for all } \check{z} \in [\alpha, \beta] \text{ where}$$

1.  $\check{g}_n : [\alpha, \beta] \rightarrow [0, 1]$  are continuous.
2.  $\check{y}(\check{z}, \check{s}, \tau), \check{x}(\check{z}, \check{s}, \tau) : [\alpha, \beta] \times [\alpha, \beta] \rightarrow [0, 1]$  are continuous.
3.  $\check{h}_n, \check{j}_n : [\alpha, \beta] \times [0, 1] \rightarrow [0, 1]$  are continuous.

Let  $\mathbb{E} = \mathbb{C}[0, 1]$  be the set of continuous function on  $[\alpha, \beta]$

$\mathbb{k}(\rho, \varrho, \tau) = |\tau\rho|_1 + |\tau\varrho|_1 = \max(\rho(\check{z}), \tau) + \max(\varrho(\check{z}), \tau)$  for all  $\rho, \varrho \in \mathbb{E}$ . It is evident that  $\mathbb{k}(\rho, \varrho, \tau)$  is a dislocated fuzzy metric space.

## 5 Conclusions

We have shown that there is a unique common fixed point for eight self-mappings, six self-mappings, and four self-mappings in a complete IVFMS using the concepts of contractive conditions and OWC. Our results are useful for theoretical mathematics and computer science.

## Competing Interests

Authors have declared that no competing interests exist.

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