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Parameters and States Estimates of COVID-19 Model Using Lagrange Polynomial, Least Square Approximation and Kenya Quarantine Data

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Authors' contributions

This work was carried out in collaboration among all authors. CGN estimation of the parameters, simulation and discussion. GGM Designed the model,literature review and model analysis. MKJ designed the model and the model analysis. All authors read and approved the final manuscript.

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ABSTRACT

Aims/ Objectives: To develop a compartment based mathematical model, fit daily quarantine data from Ministry of Health of Kenya, estimate individuals in latency and infected in general community and predict dynamics of quarantine for the next 90 days.

Study Design: Cross-sectional study.

Place and Duration of Study: 13^{th}March 2020 to 30^{th} June 2020.

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Methodology: The population based model was developed using status and characteristic of COVID-19 infection. Quarantine data up to 30/6/2020 was fitted using integrating and differentiating theory of odes and numerical differentiation polynomials. Parameter and state estimates was approximated using least square. Simulations were carried out using ode Matlab solver. Daily community estimates of individuals in latency and infected were obtained together with daily estimate of rate of enlisting individual to quarantine center and their proportions were summarized.
Results: The results indicated that maximum infection rate was equal 0.892999 recorded on 28/6/2020, average infection rate was 0.019958 and minimum 0.00012 on 26/6/2020.
Conclusion: Predictions based on parameters and state averages indicated that the number of individuals in quarantine are expected to rise exponentially up to about 26,855 individuals by 130th day and remain constant up to 190th day.

Keywords: COVID-19; reproduction number; quarantine; Lagrange polynomial; least square approximation; infection rate.

2010 Mathematics Subject Classification: 53C25, 83C05, 57N16.

1 INTRODUCTION

Novel Coronavirus (2019-nCov) -infected Pneumonia (NCIP), or simply COVID-19 was first identified in Wuhan China, a city with eleven million people on 29th December 2019 [1]. Covid-19 belong to a family of viruses called coronavirus that affect the respiratory system. According to [2], the coronavirus disease was named COVID-19 by World Health Organization (WHO) with collaboration with International classification of Diseases(ICD). It spread mostly from person to person. Most people with the COVID-19 will experience mild to moderate respiratory illness and recover without requiring treatment. Older people or those with other medical conditions like diabetes, cardiovascular disease, chronic respiratory disease and cancer are prone to serious illness.

A study done by Prof Nan-Shan Zhong's team found the clinical symptom's of the virus included fever (88.7%), cough (69.8%), fatigue (38.1%), sputum production (33.4%), shortness of breath (18.6%), sore throat (13.9%) and headache (13.6%) [3].

Bentout et al (2020), investigated how Covid 19 epidermic would evolve with or without interventions. They used Covid 19 data collected upto 31^{st} March 2020. They used the SIER model. They also used the least square method and the best fit curves that minimizes the sum of square residues to estimate the parameters and basic reproduction number R_0 of the model. They discussed the effects of the interventions using numerical simulation. They found out that $R_0 = 4.1$ which implied that epidermic in Algeria would occur in a strong way if intervention are not implemented. The interventions had a positive effect on the time delays of the epidermic peak.[4]

In Kenya the virus was detected on 13th March 2020, from a Kenyan who had traveled back to the country on 5th march 2020 from United States. The virus has spread exponentially since then, and by the time of writing this paper, Kenya had over 10,000 people infected with the virus.

Different techniques can be used to work out numerical integration which include: interpolation, undetermined coefficient and the finite difference operator method. Lagrange polynomial($P_n x$), is used when dealing with interpolation [5, 6, 7]. The Lagrange polynomials offer a suitable alternative to solving the simultaneous equations that result from requiring the polynomials to pass through the data values. This is a mostly suitable way to interpolate among tabulated values with polynomials. The core advantage of the Lagrange polynomial is that the data may be unequally spaced. Another advantage of the Lagrange formula is that it does not depend on the order in which the nodes are arranged. In the Newton formula, the divided differences do have such a dependence [8].

Uniform and uniform mess points $h(x)$, is fitted into polynomial of the form $P_n x = \sum_{k=0}^{n} L_k(x) f_k$, where $P_n(x)$ denotes an n-degree polynomial in x.

The commonly used method for approximating a linear and non linear function (f) is the absolute mean and least square approximation. This method estimate the parameter by minimizing the squared error from the difference between the function (f) and the model [6, 9].

Roddam (2001) in their book describes how one can learn about mathematical modeling of infectious diseases. They considered epidermic modeling using simple ideas of an epidermic in a closed population. They also covered simple concepts such as basic reproduction number, incorporating age structure, spatial spread of diseases and issues on contact or infection transmission [10].

Mathematical models projects how infectious diseases progress and predicts the possible outcome of an epidemic for the purpose of advising public health. These models use assumptions or collected data together with the knowledge of mathematics to find parameters to be used in the infectious diseases models [11].

Mathematical modeling of infectious diseases follow seven stages which are in fig. 1. [12].

Even if all stages in fig. 1. are important, it should be noted that disease control models should have a realistic validation. These validations comes from comparison of model solutions and predictions with certain data [13, 14].

Several researchers have modeled Covid 19 pandemic, considering different data fitting tools. Babacar M. N., Lena T., and Diaraf S., (2020) predicted the confirmed cases of Covid-19 by machine learning, deterministic and stochastic SIR models. From the parameters estimated and fitted in the SIR model, they concluded that pandemic in most countries would end within few weeks and the hit of anti-pandemic would be in mid-May [15].

The main objective of this paper is to propose a Covid 19 model in Kenya,estimate the parameters and use the parameters to find the numerical simulations.

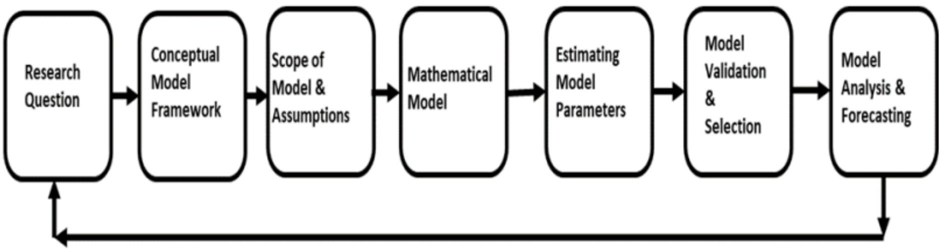


Fig. 1. Seven stages of modeling

2 MODEL ANALYSIS

2.1 Introduction

Our model involves person to person infection and has four human classes namely Susceptible (S), Latent (N), Infected (I) and Quarantine (Q). All people in the country are susceptible. The N class represent the individuals exposure to the virus and this can take 2-14 days. Infectious class involves individuals who have symptoms of the coronavirus. Q are the individuals who are quarantine because they have history of travel to countries with the virus or had contact with infected individuals or have tested positive for coronavirus.

Table 1. gives the parameters of our model and their descriptions.

Table 1. Parameters and their descriptions

Parameter	Description
π_1, π_2, π_3	Net immigration to S, N, I
β	Effective contact rate
η	Modification parameter
ω	Rate of individuals developing Covid-19 symptoms
γ	Rate of individuals joining quarantine
h	Fraction of people to be quarantined
δ_1, δ_2	Death rate due to Covid-19 infection of I and Q
Θ	Recovery rate of the individuals in the quarantined

We made the following assumptions;

- Individuals in quarantine are not infectious, since they are isolated and are under supervision.
- Any COVID-19 case in which Kenya Ministry of health is aware of is assumed to be in quarantine.
- Individuals in latency state (asymptomatic) in the community are assumed to be more than those in infected state (symptomatic).

Our model is formulated using first order differential equations given in equation (1)

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi_1 S + \theta Q - \lambda S,\\
\frac{dN}{dt} &= \pi_2 N + \lambda S - (h\gamma + \omega)N,\\
\frac{dI}{dt} &= \pi_3 I + \omega N - ((1-h)\gamma + \delta_1)I\\
\frac{dQ}{dt} &= h\gamma N + (1-h)\gamma I - (\theta + \delta_2)Q,
\end{aligned}\right\} \tag{1}$$

Where λ is the rate of the individuals joining the latent class N and it is given by

$$\lambda = \frac{\beta(N + \eta I)}{P}$$

.

2.2 Re-scaling the Model

The total population is given by P where

$$P = S(t) + N(t) + I(t) + Q(t). \tag{2}$$

Dividing (2) by P we obtain;

$$1 = \frac{S}{P} + \frac{N}{P} + \frac{I}{P} + \frac{Q}{P} \tag{3}$$

From equation (3) we can let

$$s = \frac{S}{P}, n = \frac{N}{P}, i = \frac{I}{P}, q = \frac{Q}{P}, t = t \tag{4}$$

Substituting equation (4) into equation (3) and making s the subject we obtain $s = 1 - n - i - q$. We now substitute s into equation (1) to get the reduced equation (5)

$$\left.\begin{aligned}\frac{dn}{dt} &= \pi_2 n + \lambda(1-n-i-q) - (h\gamma+\omega)n,\\ \frac{di}{dt} &= \pi_3 i + \omega n - ((1-h)\gamma + \delta_1)i\\ \frac{dq}{dt} &= h\gamma n + (1-h)\gamma i - (\theta+\delta_2)q,\end{aligned}\right\} \tag{5}$$

Where $\lambda = \beta(n + \eta i)$.

2.3 Boundedness of the Model

Lemma 1. *The feasible region* Ω *defined by the set*

$$\Omega = \left\{(S, N, I, Q) \in R^4_+ : 0 \leq N(t) \leq \frac{\tau}{\mu}\right\}.$$

with initial data $S > 0, N > 0, I > 0, Q > 0$*, is bounded for* $t \geq 0$*.*

Proof. Taking the sum of the the derivatives we obtain

$$\frac{dP}{dt} = \pi_1 S + \pi_2 N + \pi_3 I - \delta_1 I - \delta_2 Q. \tag{6}$$

If we let τ to represent the cumulative net migration times total population and μ to represent death rate due to COVID-19 as a proportion of total population we obtain;

$$\frac{dP}{dt} = \tau - \mu P.$$

Integrating and taking the limit of P as $t \to \infty$ we obtain;

$$\lim_{t\to\infty} P(t) \leq \frac{\tau}{\mu}. \tag{7}$$

Equation (7) implies that $P(t)$ is bounded for all $t \geq 0$ and $P(t) \leq \frac{\tau}{\mu}$. □

2.4 Positivity of the Model

We prove that the variables S, N, I, Q remain positive for all time $t \geq 0$.

Theorem 1. *If all variables of the model equation* (1) *are non-negative and the initial conditions satisfy*

$$\{(S(0), N(0), I(0), Q(0)) \geq 0\} \in \Omega,$$

then the solutions set $\{S(t), N(t), I(t), Q(t)\}$ *of the model system* (1) *is positive for all* $t \geq 0$*.*

Proof. Starting with the first equation in model system (1) we have

$$\frac{dn}{dt} = -h\gamma n + \lambda - \lambda n - \lambda i - \lambda q - \omega n + \pi_2 n \tag{8}$$

The parameters π_2 and λ are positive hence

$$\frac{dn}{dt} \geq -h\gamma n + \lambda n + \lambda i + \lambda q + \omega n \tag{9}$$

Dividing all through with n and integrating with respect to t we obtain

$$ln\ n \geq -\int_0^t (h\gamma + \frac{\lambda i}{n} + \lambda + \frac{\lambda q}{n} + \omega) + c_1 \tag{10}$$

Substituting the initial conditions $n(0)$ we get

$$n(t) \geq n(0)e^{-\int_0^t (h\gamma + \frac{\lambda(s)i(s)}{n(s)} + \lambda(s) + \frac{q\lambda(s)}{n(s)} + \omega)ds} \tag{11}$$

□

2.5 Disease Free Equilibrium

The DFE point is the point where there is not disease in the population. It is the point where $n = i = q = 0$. From equation (1) DFE represented by $E^0 = \{S^0, N^0, I^0, Q^0\} = \{\pi_1, 0, 0, 0\}$. From equation (5), $E^0 = \{0, 0, 0\}$.

Hence we find the reproduction number of the model. We use the second generation matrix as in [16, 17] to find the **reproduction number** R_0. The reproduction number represent the number of secondary infection one individual transmit in the population. The reproduction number is found by identifying the largest eigenvalue from the matrix FV^{-1} where matrix F represent new infection and matrix V represent transfer of new infections. We let $\psi_1 = h\gamma,\ \psi_2 = (1-h)\gamma$ and $\psi_3 = \theta + \delta_2$ so that equation (5) becomes:

$$\left.\begin{aligned} \frac{dn}{dt} &= \pi_2 n + \lambda(1-n-i-q) - \psi_1 n - \omega n, \\ \frac{di}{dt} &= \pi_3 i + \omega n - \psi_2 i - \delta_1 i \\ \frac{dq}{dt} &= \psi_1 n + \psi_2 i - \psi_3 q, \end{aligned}\right\} \tag{12}$$

Matrices F and V are given by:

$$F = \begin{pmatrix} \beta & \beta\eta \\ 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \omega - \pi_2 + \psi_1 & 0 \\ -\omega & -\pi_3 + \delta_1 + \psi_2 \end{pmatrix}$$

$$F.V^{-1} = \begin{pmatrix} \frac{\beta}{\omega-\pi_2+\psi_1} + \frac{\eta\omega\beta}{(\omega-\pi_2+\psi_1)(-\pi_3+\delta_1+\psi_2)} & \frac{\beta\eta}{-\pi_3+\delta_1+\psi_2} \\ 0 & 0 \end{pmatrix}$$

The largest eigenvalue of FV^{-1} is given by

$$R_0 = \frac{\beta\eta\omega}{(\delta_1 + \phi_2 - \pi_3)(\omega + \phi_1 - \pi_2)} + \frac{\beta}{\omega + \phi_1 - \pi_2} \tag{13}$$

2.6 Endemic Equilibrium Point

This is the point E^* when the disease persists in the community. To find the EEP we equate the second and the third equation of equation (5) to zero then solving in terms of n to obtain:

$$i^* = -\frac{n^*\omega}{\pi_3 - \delta_1 - \psi_2}$$

$$q^* = \frac{n^*\left(\psi_1 - \frac{\psi_2\omega}{-\delta_1-\psi_2+\pi_3}\right)}{\psi_3}$$

Substituting i^* and q^* into the first equation of (5), with $\lambda^* = \beta(n^* + \eta i^*)$, we obtain:

$$A_2 n^{*2} + A_1 n^* + A_0 = 0 \tag{14}$$

Where

$$A_2 = -\frac{\beta\left(\psi_3\left(\delta_1 + \omega - \pi_3\right) + \psi_1\left(\delta_1 + \psi_2 - \pi_3\right) + \psi_2\left(\psi_3 + \omega\right)\right)\left(\delta_1 + \eta\omega + \psi_2 - \pi_3\right)}{\psi_3\left(\delta_1 + \psi_2 - \pi_3\right)^2}$$

$$A_1 = -\frac{\beta\eta\omega}{-\delta_1 - \psi_2 + \pi_3} + \beta - \psi_1 - \omega + \pi_2$$

$$A_0 = 0$$

Writing A_2 and A_1 in term of R_0 and letting $\Omega_1 = -\pi_2 + \omega + \psi_1$ and $\Omega_2 = -\pi_3 + \delta_1 + \psi_2$ we get:

$$A_2 = -\frac{\Omega_1 R_0}{\psi_2 \Omega_2}((\psi_2 + \psi_3)\omega + (\psi_1 + \psi_3)\Omega_2)$$

$$A_1 = R_0 - 1$$

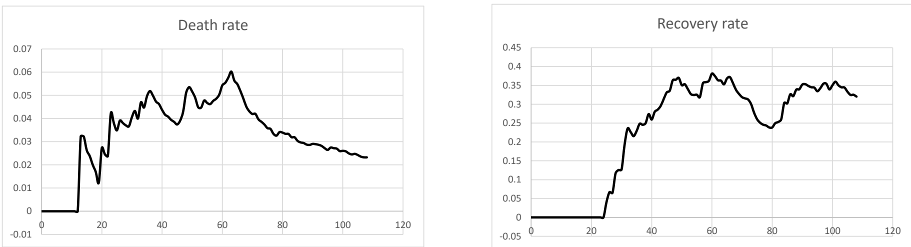
Clearly $A_2 < 0$ when $R_0 > 0$ and $n^* = -\frac{A_1}{A_2} < 0$ when $R_0 > 1$ from equation (14), hence there is no endemic point for this model when $R_0 < 1$. We have only one nonzero endemic point which is positive when $A_1 > 0$ and $R_0 > 1$.

Lemma 2. *A unique endemic equilibrium point E^* exist and is positive when $R_0 > 1$*

3 PARAMETERIZATION

3.1 Parameter Estimation

The total population of Kenya was $47,564,300$ according to census of 2019 [18]. Therefore $P = 47,564,300$. Government of Kenya keep records information of individuals it is aware of their whereabouts and status. It is assumed that such individuals cannot transmit COVID-19 because they are in quarantine. The recovery rate of individuals, using data from [19], in quarantine is obtained by: $\Theta = \frac{ToatalRecovery}{TotalInfected}$ which is plotted as below in fig. 2b. The disease induced death rate of individuals in quarantine, is obtained by: $\delta_2 = \frac{ToatalDeath}{TotalInfected}$, which is plotted in fig. 2a.



(a) Death rate due to Covid-19 (b) Recovery rate of Covid-19

Fig. 2. Recovery and death rate

Mean values of Θ and δ_2 obtained in Table 3. were substituted in discrete quarantine equations as below,

$$\frac{dln\frac{q(t)}{q(0)}}{dt} = h\gamma\frac{n(t)}{q(t)} + (1-h)\gamma\frac{i(t)}{q(t)} - (0.189353 + 0.033786) \quad (15)$$

Parameters (γ, h) and State variable estimation $n(t)$ and $i(t)$ are estimated from differential equation using Lagrange polynomials and least squares method. Note that

$$q(t) = \frac{Total\,infectives}{Population\,of\,Kenya(P)}$$

Initial time at 13/3/2020 was considered as $t = 0$ and 30/6/2020 considered as $t = 108$. Using Matlab software and the data from [19], we obtained the following polynomial and its approximate numerical differentiation.

$$ln\frac{q(t)}{q(0)} \approx -3.8 \times 10^{-7}t^4 + 9.7 \times 10^{-5}t^3 - 0.0088t^2 + 0.37t - 0.14 \quad (16)$$

This equation (16) is of the 5^{th} degree and the norm of residue is 1.3188 We get the first derivative of equation (16) to obtain;

$$dln\frac{q(t)}{q(0)} \approx -3.8 \times 4 \times 10^{-7}t^3 + 3 \times 9.7 \times 10^{-5}t^2 + 0.0088t + 0.37 \quad (17)$$

Equation (15) was fitted using least square approximation in excel and with the following conditions;

- $0 \leq h \geq 1$; On assumption that the government of Kenya has equal chance of putting individuals in latent state and infectious state in quarantine center.
- $n \geq i$; On assumption that individuals latent state are likely to be more than individuals in infectious state. Note that n and i are simple fractions which lie from 0 and 1.
- All state variables and parameters are non-negative.

Fig. 3. was obtained after fitting $q(t)$ in the model to the observed data with time using least square approximation method. The total sum of least square was $2.54656x10^{-9}$

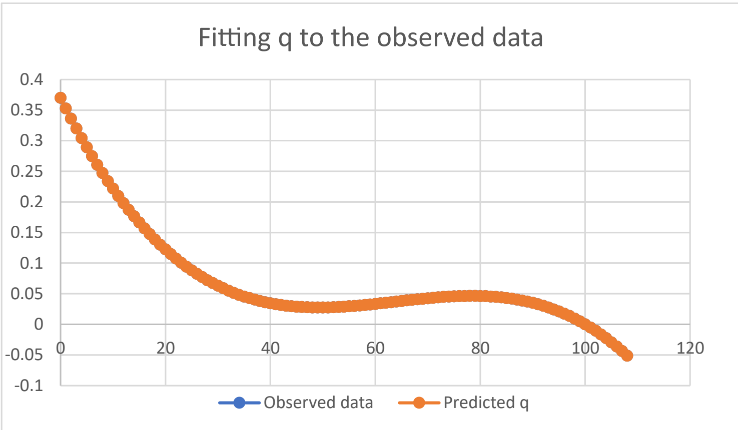
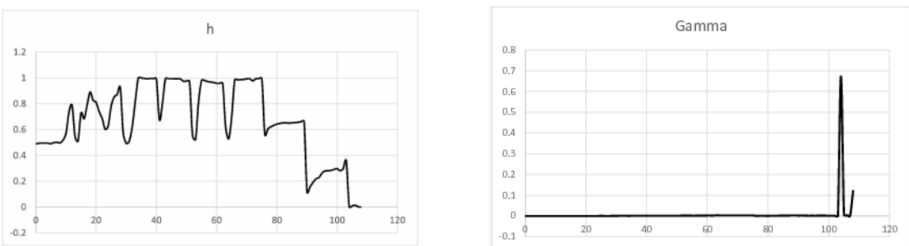


Fig. 3. Fitting q in the model to the observed data

Fig. 4c and Fig. 4d are plotted using h values and γ values from Table 2. We used excel software to plot these graphs which represent the fraction of people to be quarantine (Fig. 4c) and the rate of individuals joining quarantine (Fig. 4d).



(c) h values

(d) Gamma values

Fig. 4. h and gamma values

Table 2a. Estimated state variables and parameters

Date	h	Gamma	n	i	Observed data	Predicted q	Least Square
3/13/2020	0.48875	0.00006	0.00013	0.00012	0.37000	0.37000	8.8377E-14
3/14/2020	0.49412	0.00007	0.00011	0.00010	0.35269	0.35269	1.5958E-13
3/15/2020	0.49483	0.00007	0.00011	0.00010	0.33595	0.33595	2.0991E-14
3/16/2020	0.49487	0.00012	0.00017	0.00016	0.31978	0.31978	8.0312E-14
3/17/2020	0.49456	0.00012	0.00017	0.00016	0.30416	0.30416	5.0819E-15
3/18/2020	0.48991	0.00018	0.00025	0.00023	0.28909	0.28909	2.7603E-15
3/19/2020	0.49933	0.00017	0.00025	0.00022	0.27455	0.27455	1.2232E-13
3/20/2020	0.50156	0.00017	0.00024	0.00022	0.26054	0.26054	3.0129E-13
3/21/2020	0.49680	0.00016	0.00024	0.00022	0.24705	0.24705	7.5442E-14
3/22/2020	0.51701	0.00023	0.00034	0.00031	0.23406	0.23406	2.2529E-14
3/23/2020	0.56870	0.00023	0.00034	0.00031	0.22158	0.22158	1.3837E-13
3/24/2020	0.73686	0.00028	0.00040	0.00038	0.20959	0.20959	1.2857E-13
3/25/2020	0.79161	0.00029	0.00041	0.00038	0.19808	0.19808	9.5356E-15
3/26/2020	0.54558	0.00038	0.00038	0.00038	0.18704	0.18704	3.3802E-14
3/27/2020	0.50971	0.00034	0.00042	0.00039	0.17647	0.17647	7.736E-14
3/28/2020	0.73080	0.00035	0.00044	0.00043	0.16635	0.16634	2.7506E-15
3/29/2020	0.68156	0.00034	0.00046	0.00046	0.15667	0.15667	2.9288E-20
3/30/2020	0.78779	0.00036	0.00049	0.00049	0.14743	0.14743	2.2671E-14
3/31/2020	0.89191	0.00038	0.00051	0.00050	0.13862	0.13862	1.5037E-13
4/01/2020	0.82607	0.00041	0.00062	0.00044	0.13023	0.13023	5.4659E-14
4/02/2020	0.81156	0.00049	0.00078	0.00041	0.12224	0.12224	3.0573E-13
4/03/2020	0.73838	0.00050	0.00082	0.00039	0.11465	0.11465	1.456E-13
4/04/2020	0.68769	0.00050	0.00084	0.00039	0.10746	0.10746	1.824E-14
4/05/2020	0.60047	0.00057	0.00099	0.00037	0.10065	0.10065	6.7265E-15
4/06/2020	0.62729	0.00057	0.00098	0.00042	0.09420	0.09420	2.4529E-13
4/07/2020	0.78403	0.00065	0.00104	0.00042	0.08813	0.08812	2.772E-14
4/08/2020	0.85254	0.00070	0.00112	0.00042	0.08240	0.08240	1.5781E-13
4/09/2020	0.87191	0.00069	0.00110	0.00038	0.07702	0.07702	1.2396E-15
4/10/2020	0.93016	0.00075	0.00125	0.00039	0.07198	0.07198	4.1349E-14
4/11/2020	0.57137	0.00106	0.00095	0.00076	0.06726	0.06726	6.554E-13
4/12/2020	0.49238	0.00107	0.00100	0.00079	0.06286	0.06286	1.8452E-14
4/13/2020	0.50994	0.00126	0.00115	0.00088	0.05877	0.05877	6.3713E-14
4/14/2020	0.62185	0.00135	0.00128	0.00095	0.05498	0.05498	1.5861E-13
4/15/2020	0.82097	0.00130	0.00129	0.00093	0.05147	0.05147	1.1464E-13
4/16/2020	0.99993	0.00124	0.00129	0.00090	0.04825	0.04825	9.3668E-14
4/17/2020	0.99929	0.00126	0.00141	0.00088	0.04531	0.04530	9.5985E-14
4/18/2020	0.99608	0.00129	0.00151	0.00085	0.04262	0.04262	9.9179E-14
4/19/2020	0.99287	0.00129	0.00154	0.00082	0.04019	0.04019	8.2823E-14
4/20/2020	0.99633	0.00129	0.00162	0.00081	0.03800	0.03800	7.0229E-14
4/21/2020	0.99489	0.00133	0.00171	0.00079	0.03605	0.03605	5.8198E-14

Table 2b. Estimated state variables and parameters (continued.....)

Date	h	γ	n	i	Observed data	Predicted q	Least Square
4/22/2020	0.99406	0.00130	0.00175	0.00076	0.03432	0.03432	6.384E-14
4/23/2020	0.67440	0.00172	0.00160	0.00114	0.03281	0.03281	1.1922E-13
4/24/2020	0.80036	0.00170	0.00163	0.00111	0.03151	0.03151	5.4683E-14
4/25/2020	0.99463	0.00166	0.00165	0.00107	0.03041	0.03041	1.3066E-13
4/26/2020	0.99362	0.00168	0.00174	0.00106	0.02950	0.02950	1.0493E-12
4/27/2020	0.99460	0.00171	0.00183	0.00104	0.02877	0.02876	2.0918E-15
4/28/2020	0.99276	0.00173	0.00189	0.00104	0.02821	0.02821	8.9699E-15
4/29/2020	0.99311	0.00178	0.00204	0.00102	0.02781	0.02781	1.6154E-12
4/30/2020	0.99219	0.00180	0.00213	0.00099	0.02756	0.02756	1.7643E-13
5/01/2020	0.97389	0.00182	0.00218	0.00102	0.02746	0.02746	7.9083E-14
5/02/2020	0.97562	0.00180	0.00236	0.00094	0.02750	0.02750	1.1262E-12
5/03/2020	0.97363	0.00183	0.00246	0.00088	0.02766	0.02766	3.4777E-13
5/04/2020	0.55024	0.00258	0.00205	0.00150	0.02794	0.02794	1.2258E-13
5/05/2020	0.52157	0.00254	0.00220	0.00162	0.02833	0.02833	5.8815E-13
5/06/2020	0.81488	0.00242	0.00225	0.00150	0.02881	0.02881	3.5327E-12
5/07/2020	0.98053	0.00234	0.00226	0.00145	0.02939	0.02939	1.4355E-13
5/08/2020	0.98008	0.00231	0.00235	0.00139	0.03004	0.03004	9.5781E-14
5/09/2020	0.97166	0.00242	0.00257	0.00137	0.03077	0.03076	3.8244E-12
5/10/2020	0.96763	0.00243	0.00270	0.00134	0.03155	0.03155	1.2763E-12
5/11/2020	0.96496	0.00246	0.00277	0.00133	0.03239	0.03239	5.1377E-14
5/12/2020	0.95785	0.00254	0.00293	0.00131	0.03328	0.03328	3.8602E-13
5/13/2020	0.95808	0.00253	0.00300	0.00130	0.03420	0.03420	9.6388E-12
5/14/2020	0.95918	0.00252	0.00304	0.00121	0.03515	0.03514	1.9962E-12
5/16/2020	0.61501	0.00337	0.00269	0.00187	0.03611	0.03611	1.5664E-13
5/17/2020	0.52631	0.00336	0.00284	0.00207	0.03708	0.03708	3.5501E-14
5/18/2020	0.70072	0.00332	0.00295	0.00198	0.03805	0.03804	3.8511E-13
5/19/2020	0.98449	0.00311	0.00303	0.00172	0.03900	0.03900	1.1654E-12
5/20/2020	0.98502	0.00300	0.00322	0.00166	0.03994	0.03994	1.267E-12
5/21/2020	0.98673	0.00291	0.00342	0.00156	0.04085	0.04085	5.0359E-15
5/22/2020	0.98793	0.00287	0.00353	0.00150	0.04172	0.04172	3.2047E-13
5/23/2020	0.99259	0.00331	0.00306	0.00202	0.04254	0.04254	8.2125E-14
5/24/2020	0.99265	0.00326	0.00315	0.00193	0.04331	0.04331	5.3047E-14
5/25/2020	0.97754	0.00321	0.00337	0.00184	0.04401	0.04401	3.9437E-14
5/26/2020	0.99377	0.00311	0.00351	0.00184	0.04463	0.04463	1.2789E-17

Table 2c. Estimated state variables and parameters (continued.....)

Date	h	γ	n	i	Observed data	Predicted q	Least Square
5/27/2020	0.99462	0.00299	0.00373	0.00166	0.04518	0.04518	3.2351E-14
5/28/2020	0.99671	0.00297	0.00392	0.00161	0.04563	0.04563	2.8704E-13
5/29/2020	0.55547	0.00113	0.01192	0.00938	0.04597	0.04597	6.9983E-14
5/30/2020	0.60495	0.00112	0.01298	0.00940	0.04621	0.04621	1.0832E-13
5/31/2020	0.62198	0.00113	0.01316	0.00943	0.04632	0.04632	1.5776E-13
6/01/2020	0.63308	0.00114	0.01325	0.00947	0.04631	0.04631	4.129E-13
6/02/2020	0.64160	0.00119	0.01311	0.00947	0.04616	0.04616	1.0187E-12
6/03/2020	0.64822	0.00132	0.01273	0.00945	0.04586	0.04586	7.5668E-13
6/04/2020	0.65131	0.00146	0.01210	0.00947	0.04540	0.04540	8.1045E-14
6/05/2020	0.65175	0.00167	0.01101	0.00954	0.04478	0.04478	8.3397E-14
6/06/2020	0.64992	0.00212	0.00988	0.00963	0.04399	0.04399	1.0498E-13
6/07/2020	0.65202	0.00229	0.00965	0.00932	0.04301	0.04300	1.0309E-13
6/08/2020	0.65403	0.00251	0.00965	0.00931	0.04183	0.04183	1.1682E-13
6/09/2020	0.65676	0.00263	0.00948	0.00908	0.04045	0.04045	1.1289E-13
6/10/2020	0.66080	0.00282	0.00956	0.00897	0.03887	0.03887	1.2228E-13
6/11/2020	0.66595	0.00295	0.00956	0.00877	0.03706	0.03706	1.2212E-13
6/12/2020	0.11119	0.00248	0.03660	0.00853	0.03502	0.03502	1.8592E-13
6/13/2020	0.15701	0.00274	0.01637	0.00999	0.03274	0.03274	1.452E-14
6/14/2020	0.19134	0.00252	0.02119	0.01007	0.03022	0.03022	1.1653E-13
6/15/2020	0.21881	0.00235	0.02483	0.01014	0.02744	0.02744	1.0959E-13
6/16/2020	0.22837	0.00230	0.02610	0.01038	0.02439	0.02439	1.6866E-13
6/17/2020	0.26471	0.00206	0.03053	0.01041	0.02107	0.02106	2.6872E-13
6/18/2020	0.27918	0.00209	0.03225	0.01057	0.01746	0.01746	1.5037E-13
6/19/2020	0.28104	0.00213	0.03260	0.01093	0.01356	0.01356	5.1935E-13
6/20/2020	0.28404	0.00207	0.03321	0.01158	0.00935	0.00935	7.3114E-14
6/21/2020	0.29150	0.00195	0.03392	0.01270	0.00484	0.00484	1.1657E-13
6/22/2020	0.29742	0.00191	0.03487	0.01354	0.00000	0.00000	6.8686E-14
6/23/2020	0.28114	0.00192	0.03371	0.01552	-0.00517	-0.00517	1.1846E-13
6/24/2020	0.29667	0.00163	0.03591	0.01972	-0.01067	-0.01067	1.2246E-13
6/25/2020	0.36014	0.00107	0.04907	0.03099	-0.01653	-0.01653	6.6441E-13
6/26/2020	0.00010	0.67306	0.00006	0.00006	-0.02274	-0.02274	4.4667E-15
6/27/2020	0.00943	0.00661	0.08153	0.00535	-0.02931	-0.02932	2.3408E-14
6/28/2020	0.01488	0.00410	0.46890	0.00278	-0.03627	-0.03627	3.9346E-13
6/29/2020	0.00172	0.00005	0.83190	0.83190	-0.04361	-0.04361	3.1913E-13
6/30/2020	0.00000	0.12088	0.00032	0.00032	-0.05134	-0.05134	6.4196E-13

Table 3a. Summary of recovery and death rate per day

Date	Death rate δ_2	Recovery rate Θ
3/13/2020	0.00000	0.00000
3/14/2020	0.00000	0.00000
3/15/2020	0.00000	0.00000
3/16/2020	0.00000	0.00000
3/17/2020	0.00000	0.00000
3/18/2020	0.00000	0.00000
3/19/2020	0.00000	0.00000
3/20/2020	0.00000	0.00000
3/21/2020	0.00000	0.00000
3/22/2020	0.00000	0.00000
3/23/2020	0.00000	0.00000
3/24/2020	0.00000	0.00000
3/25/2020	0.00000	0.00000
3/26/2020	0.03226	0.00000
3/27/2020	0.03226	0.00000
3/28/2020	0.02632	0.00000
3/29/2020	0.02381	0.00000
3/30/2020	0.02000	0.00000
3/31/2020	0.01695	0.00000
4/01/2020	0.01235	0.00000
4/02/2020	0.02727	0.00000
4/03/2020	0.02459	0.00000
4/04/2020	0.02381	0.00000
4/05/2020	0.04225	0.00000
4/06/2020	0.03797	0.00000
4/07/2020	0.03488	0.04070
4/08/2020	0.03911	0.06704
4/09/2020	0.03804	0.06522
4/10/2020	0.03704	0.11640
4/11/2020	0.03665	0.12565
4/12/2020	0.04061	0.12690
4/13/2020	0.04327	0.19231
4/14/2020	0.04000	0.23556
4/15/2020	0.04701	0.22650
4/16/2020	0.04472	0.21545
4/17/2020	0.04962	0.22901
4/18/2020	0.05185	0.24815
4/19/2020	0.04982	0.24555
4/20/2020	0.04730	0.25000
4/21/2020	0.04620	0.27393

Table 3b. Summary of recovery and death rate per day (continued)

Date	Death Rate δ_2	Recovery Rate Θ
4/22/2020	0.04375	0.25938
4/23/2020	0.04167	0.27976
4/24/2020	0.04082	0.28571
4/25/2020	0.03944	0.29577
4/26/2020	0.03857	0.31405
4/27/2020	0.03743	0.33155
4/28/2020	0.03906	0.33594
4/29/2020	0.04293	0.36364
4/30/2020	0.05109	0.36496
5/01/2020	0.05353	0.36983
5/02/2020	0.05161	0.35054
5/03/2020	0.04898	0.35306
5/04/2020	0.04486	0.34019
5/05/2020	0.04475	0.32702
5/06/2020	0.04778	0.32455
5/07/2020	0.04670	0.32528
5/08/2020	0.04622	0.31895
5/09/2020	0.04762	0.35565
5/10/2020	0.04857	0.35857
5/11/2020	0.05035	0.36224
5/12/2020	0.05427	0.38128
5/13/2020	0.05541	0.37467
5/14/2020	0.05762	0.36364
5/16/2020	0.06024	0.36265
5/17/2020	0.05637	0.35287
5/18/2020	0.05482	0.36842
5/19/2020	0.05192	0.37175
5/20/2020	0.04859	0.35569
5/21/2020	0.04509	0.33814
5/22/2020	0.04307	0.32730
5/23/2020	0.04195	0.31879
5/24/2020	0.04201	0.31549
5/25/2020	0.03966	0.31260
5/26/2020	0.03858	0.30045
5/27/2020	0.03739	0.27736
5/28/2020	0.03585	0.26020
5/29/2020	0.03553	0.25100

Table 3c. Summary of recovery and death rate per day (continued)

Date	Death Rate δ_2	Recovery Rate Θ
5/30/2020	0.03337	0.24576
5/31/2020	0.03262	0.24363
6/1/2020	0.03414	0.23850
6/2/2020	0.03392	0.23841
6/3/2020	0.03339	0.24955
6/4/2020	0.03333	0.25299
6/5/2020	0.03193	0.25990
6/6/2020	0.03192	0.30346
6/7/2020	0.03036	0.30213
6/8/2020	0.02970	0.32635
6/9/2020	0.02944	0.32151
6/10/2020	0.02877	0.33872
6/11/2020	0.02862	0.33966
6/12/2020	0.02905	0.35219
6/13/2020	0.02893	0.35320
6/14/2020	0.02866	0.34864
6/15/2020	0.02817	0.34505
6/16/2020	0.02720	0.34404
6/17/2020	0.02646	0.33457
6/18/2020	0.02748	0.34273
6/19/2020	0.02721	0.35437
6/20/2020	0.02702	0.35418
6/21/2020	0.02595	0.33931
6/22/2020	0.02606	0.35022
6/23/2020	0.02585	0.35985
6/24/2020	0.02497	0.35017
6/25/2020	0.02452	0.34491
6/26/2020	0.02476	0.34430
6/27/2020	0.02426	0.33316
6/28/2020	0.02356	0.32471
6/29/2020	0.02326	0.32520
6/30/2020	0.02325	0.32030

3.2 Infection Rate or Incidence Rate

Infection rate or simply the incidence rate of any disease is the probability of an infection occurrence or risk of an infection in a given sample of individuals or population [20]. When an infection is transmitted to new individuals, it reproduces itself hence the use of reproduction number to explain infection rates. The formula for infection rate according to [20] is

$$Rate\ of\ infection = \frac{No\ of\ infection}{Population\ at\ risk}$$

The number of infection in our model represent the total confirmed cases of positive Covid-19 (n and i) and the population at risk is the total population tested for Covid-19 or under surveillance (Susceptible). From model formulation, one assumption was that the individuals in quarantine are non-infectious. The infection rate is presented below in Fig. 5. From the estimates of this study, maximum infection rate was equal 0.892999 recorded on 28/6/2020, average infection rate was 0.019958 and minimum 0.00012 on 26/6/2020.

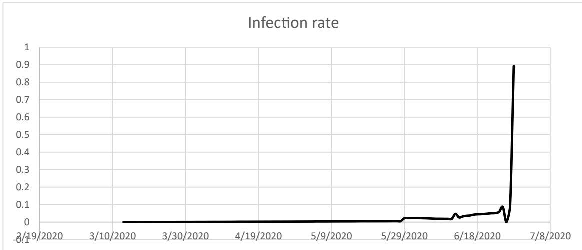


Fig. 5. Infection rate of Covid-19

4 SIMULATION AND DISCUSSION

Table 4. Min, Max and average values of h,γ, n and i

Parameter/variable	Min value	Max value	Average value
Θ	0	0.381275441	0.230174042
δ_2	0	0.060240964	0.032827283
h	3.10847×10^{-07}	0.999927126	0.671306343
γ	4.76914×10^{-05}	0.673061064	0.008914713
n	5.99067×10^{-05}	0.831896392	0.019320105
i	5.98555×10^{-05}	0.831896392	0.01124545

4.1 Simulation

We use Matlab inbuilt solver ode45 to perform our simulation. We consider the equation of the quarantine individuals in our simulations. Equation (16) in which its right hand side was transformed through numerical differentiation as polynomial equation in equation (17), was fitted using the recorded data in Ministry of health data in wordometer. Note that E in the table 2a. to table 2c, means ten raised to the power of the number given. Minimum, maximum and averages values of $\Theta, \delta_2, h, \gamma, n$ and i obtained are summarized below in table 4.

Equation (18) is an initial value problem. Date 30/06/2020 was considered as the initial conditions of the model with $q(0) = 0.00013384$. The parameter values in table 4 were chosen through inspection in such a way that they can produce maximum, average and minimal simulation of q.

4.2 Discussion

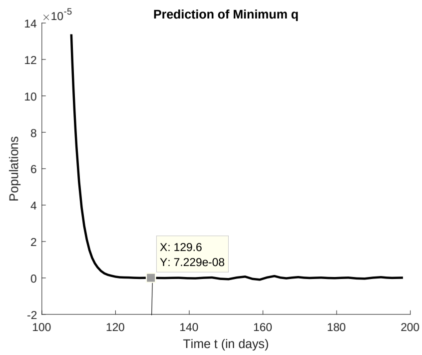
Fitting of the daily observed data to the model yielded Fig. 3 with total sum of least square error being 4.16669×10^{-11}. This indicated the model fitted well to observed data and therefore can be used for prediction.

Predictions in Fig. 5. were based on possible combination of parameters and state variables to yield maximum, minimum or average possible value of q.

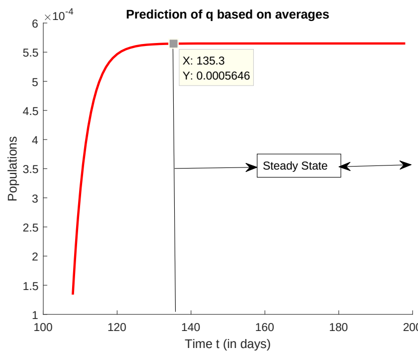
Clearly, through inspection of equation

$$\frac{dq}{dt} = h\gamma n + (1-h)\gamma i - (\theta + \delta_2)q, \tag{18}$$

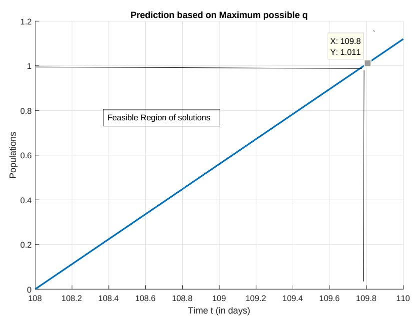
we note that during data fitting n was set to be greater than I therefore maximum q is obtained on following conditions: maximum $h, \gamma, , n$ and i and minimum Θ, δ_2. Hence minimum q is obtained on following conditions: minimum h, γ, n and i and maximum Θ, δ_2.



(e) Predicted q based on minimum



(f) Predicted q based on averages



(g) Predicted q based on maximum

Fig. 5. Predicted values for q

The result obtained in Fig. 5e. using possible minimum prediction of q yielded unrealistic results because it indicated that all individuals would clear from quarantine by 130^{th} day. The result obtained in Fig. 5f. using averages in prediction of q indicated that individuals in quarantine would continue to rise up to 0.0005646 by 136^{th} day and thereafter remains constant. Note that from subsection 2.2, $n \geq 1$ is not feasible in this model because $0 \leq n \leq 1$. The result obtained in Fig. 5g. using possible maximum prediction of q, that is $q = 1$ when $P = q$, yielded unrealistic results because it indicated that all individuals would be in quarantine by 110^{th} day.

5 CONCLUSION

a In the section (2), we described the model, rescaled the equations,got the boundedness and positivity of the model and analyzed the DFE AND EEP of the model.

b In the section 3, we estimated the parameters of our model. We got the recovery rate and death rate of Covid 19 (table 3). We also estimated the parameters h, γ, state variables n, i Observed data , predicted data and least square (table 2). We also estimated the maximum, minimum and averages of infection rate of Covid 19.

c In the section 4, we used matlab solver ode45 to estimate minimum and average values of h, γ, n, i (table 4).

The total number of infected is expected to rise exponentially up to about 26,855 individuals by 130^{th} day and remain constant up to 190^{th} day.

Ministry of Health recorded total COVID-19 cases is not a true picture on the ground, we are far much more than that. A case in point on 30/6/2020 where total cases were 6366 on ministry website. On same date our model estimation of individuals in latency and infectious state in community were 15,390 and 15390. There is a total of 30780 cases not yet traced.

The result from best model fit indicated daily variability in parameters and state estimates. This variability makes it difficult to precisely predict dynamics using deterministic model. Future studies should consider stochastic models. Also further studies should be considered for estimation of parameters of infected individuals and the exposed individuals and model analysis.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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