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The Fourier Multipliers of $p-$Fourier Spaces on Compact Groups

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Abstract

In this paper, we prove some properties of Fourier multipliers on compact groups. Mainly we obtain the invariance of $p-$Fourier spaces under the action of Fourier multipliers over compact groups.

Keywords: Compact group; Banach algebra; Fourier transform; $p-$Fourier space; Fourier multiplier
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1 Introduction

The Fourier transform has various applications for instance in Physics and Engineering. A recent use of Fourier transform in signal process can be found in [1] and [2]. The classical Fourier transform in $\mathbb{R}^n$ or in a general abelian group brought out some functions spaces called $p-$Fourier spaces. Classical $p-$Fourier spaces were previously considered by Figà-Talamanca et al. [3], Larsen [4] and Martin and Yap [5]. Thanks to the Fourier-Stieltjes transform of vector measures on compact groups introduced by Assiamoua and Olubummo in [6], two of the authors of the present paper defined the vector analogue of $p-$Fourier spaces and studied some of their topological properties [7]. The particular case where $p = 1$ gives the vector version of the notion of Fourier algebra of compact groups. The inversion formula allowed to introduce Fourier multipliers over compact groups in [8] following Pisier [9] who studied the case of abelian groups. The main goal of this paper is to study Fourier multipliers over $p-$Fourier spaces. We obtain among other results the important fact that $p-$Fourier spaces are invariant by the action of Fourier multipliers.
We have organized the article as follows. The section 2 is devoted to fix notations and to recall some properties of the $p-$Fourier spaces and related spaces which we may need. In section 3 we establish our main results.

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2 Preliminaries

Let G be a compact group with normalized Haar measure dg. Its dual space Σ is defined as the set of all unitary equivalence classes of irreducible representations of G. In each $\sigma \in \Sigma$, we choose an element U^σ and denote its hilbertian representation space by H_σ. In compact group analysis, it is well known that H_σ is of finite dimension d_σ [10]. Let $(\xi_1^\sigma, \ldots, \xi_{d_\sigma}^\sigma)$ be a basis of H_σ. The matrix elements of U^σ are given by

$$u_{ij}^\sigma(g) = \langle U_g^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle. \tag{2.1}$$

We denote by $\overline{U}^\sigma$ the contragredient of the representation U^σ, that is the representation whose matrix elements are the complex conjugate of those of U^σ. For further details on representations theory we refer to [10], [11] and [12].

Now let A be a complex Banach algebra (in fact A can just be taken as a Banach space in any situation where we do not need to multiply its elements). We denote by $L_1(G, A)$ the space of Haar-integrable $A-$valued functions on G in the Bochner sense. The Fourier transform of $f \in L_1(G, A)$ is defined in [6] by

$$\widehat{f}(\sigma)(\xi, \eta) = \int_G \langle \overline{U}_g^\sigma \xi, \eta \rangle f(g) dg \tag{2.2}$$

where $(\xi, \eta) \in H_\sigma \times H_\sigma$. In this case $\widehat{f}(\sigma)$ is interpreted as a sesquilinear mapping from $H_\sigma \times H_\sigma$ into A. The authors in [6] obtained, among other results, that the Fourier transformation $\mathcal{F} : f \to \mathcal{F}(f) := \widehat{f}$ is injective and that the reconstruction formula is given by

$$f = \sum_{\sigma \in \Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) u_{ij}^\sigma. \tag{2.3}$$

Now we set

$$\mathbb{S}(\Sigma, A) = \prod_{\sigma \in \Sigma} \mathbb{S}(H_\sigma \times H_\sigma, A) \tag{2.4}$$

where $\mathbb{S}(H_\sigma \times H_\sigma, A)$ is the space of continuous sesquilinear mappings from $H_\sigma \times H_\sigma$ into A. For $\varphi \in \mathbb{S}(\Sigma, A)$, we set

$$\|\varphi\|_\infty = \sup\{\|\varphi(\sigma)\| \, : \sigma \in \Sigma\} \tag{2.5}$$

where $\|\varphi(\sigma)\|$ is the usual norm of a continuous sesquilinear mapping :

$$\|\varphi(\sigma)\| = \sup\{\|\varphi(\sigma)(\xi, \eta)\| \, : \, \|\xi\| \leq 1, \, \|\eta\| \leq 1\}. \tag{2.6}$$

We consider the following subspaces of $\mathbb{S}(\Sigma, A)$:

$$\mathbb{S}_\infty(\Sigma, A) = \{\varphi \in \mathbb{S}(\Sigma, A) \, : \, \|\varphi\|_\infty < \infty\} \tag{2.7}$$

and for $1 \leq p < \infty$,

$$\mathbb{S}_p(\Sigma, A) = \{\varphi \in \mathbb{S}(\Sigma, A) \, : \sum_{\sigma \in \Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \|\varphi(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p < \infty\}. \tag{2.8}$$

Many fundamental properties of these spaces were studied in [13]. On the other hand, the vector version of $p-$Fourier spaces $\mathcal{A}_p(G, A)$ were defined and studied in [7]. We recall their definitions :

$$\mathcal{A}_p(G, A) = \{f \in L_1(G, A) \, : \, \widehat{f} \in \mathbb{S}_p(\Sigma, A)\}, \, 1 \leq p \leq \infty. \tag{2.9}$$

Each space $\mathbb{S}_p(\Sigma, A)$ is a Banach space if it is endowed with the norm

$$\|\varphi\|_{\mathbb{S}_\infty} = \sup\{\|\varphi(\sigma)\| \, : \sigma \in \Sigma\}, \text{ for } p = \infty \tag{2.10}$$

and

$$\|\varphi\|_{\mathbb{S}_p} = \left(\sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \|\varphi(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p \right)^{\frac{1}{p}}, \ 1 \le p < \infty. \tag{2.11}$$

Also each space $\mathcal{A}_p(G, A)$ is a Banach space if it is endowed with each one of the following norms

$$\|f\|_{\mathcal{A}_p} = \|f\|_{L_1} + \|\widehat{f}\|_{\mathbb{S}_p} \tag{2.12}$$

and

$$\|f\|^{\mathcal{A}_p} = \|\widehat{f}\|_{\mathbb{S}_p}. \tag{2.13}$$

We give now the following definition.

Definition 2.1. Let $\varphi : \Sigma \to \mathbb{C}$ be a function. A Fourier multiplier on $L_1(G, A)$ is a mapping $M_\varphi : L_1(G, A) \to L_1(G, A), f \mapsto M_\varphi f$ such that

$$M_\varphi f = \sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \varphi(\sigma)\widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma)u_{ij}^\sigma. \tag{2.14}$$

where $\widehat{f}$ is of finite support.

We recall the following result which we may need enormously. Its proof can be found in [8].

Theorem 2.1. *M_φ is a Fourier multiplier if and only if $\widehat{M_\varphi f} = \varphi\widehat{f}$.*

3 Main Results

We define the product $\times$ on $\mathbb{S}(\Sigma, A)$ as follows. If $\phi_1, \ \phi_2 \in \mathbb{S}(\Sigma, A)$ then $\phi_1 \times \phi_2$ is given by

$$(\phi_1 \times \phi_2)(\sigma)(\xi_j^\sigma, \xi_i^\sigma) = \sum_{k=1}^{d_\sigma} \phi_1(\sigma)(\xi_k^\sigma, \xi_i^\sigma)\phi_2(\sigma)(\xi_j^\sigma, \xi_k^\sigma). \tag{3.1}$$

More explicitly if we consider the matrices $(a_{i,j}^\sigma)_{1\le i,j\le d_\sigma}$ and $(b_{i,j}^\sigma)_{1\le i,j\le d_\sigma}$ defined by

$$a_{i,j}^\sigma = \phi_1(\sigma)(\xi_j^\sigma, \xi_i^\sigma), \ b_{i,j}^\sigma = \phi_2(\sigma)(\xi_j^\sigma, \xi_i^\sigma) \tag{3.2}$$

then the matrix associated with $(\phi_1 \times \phi_2)(\sigma)$ is the matrix product $(a_{i,j}^\sigma)(b_{i,j}^\sigma)$.

Theorem 3.1. *For $f, g \in L_1(G, A)$, we have $\widehat{(f * g)} = \widehat{f} \times \widehat{g}$ where $f * g$ denotes the convolution of f by g.*

Proof.

$$\begin{aligned}
\widehat{f*g}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) &= \int_G < \overline{U}_t^\sigma \xi_j^\sigma, \xi_i^\sigma > f*g(t)dt \\
&= \int_G < \overline{U}_t^\sigma \xi_j^\sigma, \xi_i^\sigma > \left(\int_G f(ts^{-1})g(s)ds\right) dt \\
&= \int_{G\times G} < \overline{U}_{ts}^\sigma \xi_j^\sigma, \xi_i^\sigma > f(t)g(s)dtds \\
&= \int_G g(s)ds \int_G < \overline{U}_t^\sigma \overline{U}_s^\sigma \xi_j^\sigma, \xi_i^\sigma > f(t)dt \\
&= \int_G \widehat{f}(\sigma)(\overline{U}_s^\sigma \xi_j^\sigma, \xi_i^\sigma)g(s)ds \\
&= \int_G \widehat{f}(\sigma)(\sum_k \overline{u}_{kj}^\sigma(s)\xi_k^\sigma, \xi_i^\sigma)g(s)ds \\
&= \sum_k \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \int_G \overline{u}_{kj}^\sigma(s)g(s)ds \\
&= \sum_k \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \int_G < \overline{U}_s^\sigma \xi_j^\sigma, \xi_k^\sigma > g(s)ds \\
&= \sum_k \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma)\widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) = (\widehat{f} \times \widehat{g})(\sigma)(\xi_j^\sigma, \xi_i^\sigma),
\end{aligned}$$

using in the computation the equalities

$$\overline{u}^{\sigma}_{kj}(s) = < \overline{U}^{\sigma}_{s}\xi^{\sigma}_{j}, \xi^{\sigma}_{k} > \text{ and } \overline{U}^{\sigma}_{s}\xi^{\sigma}_{j} = \sum_{k}^{d_\sigma} \overline{u}^{\sigma}_{kj}(s)\xi^{\sigma}_{k}.$$

Thus $\widehat{f * g} = \widehat{f} \times \widehat{g}$. □

We know how convolution is an important tool in Analysis. The following theorem links convolution and Fourier multipliers.

Theorem 3.2. *Let M_{φ_1}, M_{φ_2} be Fourier multipliers on $L_1(G, A)$, $f, g \in L_1(G, A)$. The following equalities hold.*

1. $M_{\varphi_1}(f * g) = (M_{\varphi_1} f) * g$.
2. $M_{\varphi_1} f * M_{\varphi_2} g = M_{\varphi_1\varphi_2}(f * g)$.

Proof. Let $(\xi, \eta) \in H_\sigma \times H_\sigma$ with $\xi = \sum_{j=1}^{d_\sigma} \alpha_j \xi^{\sigma}_{j}$ and $\eta = \sum_{i=1}^{d_\sigma} \beta_i \xi^{\sigma}_{i}$ in the canonical basis $(\xi^{\sigma}_{1}, \ldots, \xi^{\sigma}_{d_\sigma})$ of H_σ. The equality $\widehat{M_{\varphi_1} f} = \varphi_1 \widehat{f}$ leads to

$$(\widehat{M_{\varphi_1} f})(\sigma)(\xi, \eta) = \varphi_1(\sigma)\widehat{f}(\sigma)(\xi, \eta) = \varphi_1(\sigma)\widehat{f}(\sigma)(\sum_{j=1}^{d_\sigma} \alpha_j \xi^{\sigma}_{j}, \sum_{i=1}^{d_\sigma} \beta_i \xi^{\sigma}_{i})$$

$= \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \widehat{f}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{i})$. Thus we have:

$$1.)\ \mathcal{F}(M_{\varphi_1}(f * g))(\sigma)(\xi, \eta) = \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \widehat{f * g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{i})$$
$$= \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} (\widehat{f} \times \widehat{g})(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{i}) = \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \sum_{k} \widehat{f}(\sigma)(\xi^{\sigma}_{k}, \xi^{\sigma}_{i})\widehat{g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{k})$$
$$= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \sum_{k} \varphi_1(\sigma)\widehat{f}(\sigma)(\xi^{\sigma}_{k}, \xi^{\sigma}_{i})\widehat{g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{k}) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \sum_{k} \widehat{M_{\varphi_1} f}(\sigma)(\xi^{\sigma}_{k}, \xi^{\sigma}_{i})\widehat{g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{k})$$
$$= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} (\widehat{M_{\varphi_1} f} \times \widehat{g})(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{i}) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} (\widehat{M_{\varphi_1} f * g})(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{i})$$
$$= \mathcal{F}((M_{\varphi_1} f) * g)(\sigma)(\xi, \eta).$$

By injectivity of $\mathcal{F}$ we have : $M_{\varphi_1}(f * g) = (M_{\varphi_1} f) * g$.

$$2.)\ \mathcal{F}(M_{\varphi_1} f * M_{\varphi_2} g)(\sigma)(\xi, \eta) = (\widehat{M_{\varphi_1} f} \times \widehat{M_{\varphi_2} g})(\sigma)(\xi, \eta)$$
$$= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} (\widehat{M_{\varphi_1} f} \times \widehat{M_{\varphi_2} g})(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{i}) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \sum_{k} \widehat{M_{\varphi_1} f}(\sigma)(\xi^{\sigma}_{k}, \xi^{\sigma}_{i})\widehat{M_{\varphi_2} g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{k})$$
$$= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \sum_{k} \varphi_1(\sigma)\widehat{f}(\sigma)(\xi^{\sigma}_{k}, \xi^{\sigma}_{i})\varphi_2(\sigma)\widehat{g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{k})$$
$$= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \sum_{k} \varphi_1(\sigma)\varphi_2(\sigma)\widehat{f}(\sigma)(\xi^{\sigma}_{k}, \xi^{\sigma}_{i})\widehat{g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{k})$$
$$= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} \sum_{k} \widehat{M_{\varphi_1\varphi_2}} f(\sigma)(\xi^{\sigma}_{k}, \xi^{\sigma}_{i})\widehat{g}(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{k}) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta_i} (\widehat{M_{\varphi_1\varphi_2}} f \times \widehat{g})(\sigma)(\xi^{\sigma}_{j}, \xi^{\sigma}_{i})$$

$$= \sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}\alpha_j\overline{\beta_i}(\widehat{M_{\varphi_1\varphi_2}f} * g)(\sigma)(\xi_j^\sigma,\xi_i^\sigma) = \mathcal{F}(M_{\varphi_1\varphi_2}(f*g))(\sigma)(\xi,\eta).$$

Again by injectivity of $\mathcal{F}$, we conclude that
$M_{\varphi_1}f * M_{\varphi_2}g = M_{\varphi_1\varphi_2}(f*g)$. □

Before we state the next theorem, we discuss some examples of functions φ satisfying the condition

$$\inf\{|\varphi(\sigma)| \,:\, \sigma\in\Sigma\} > 0 \tag{3.3}$$

Let $G = \mathbb{T}$ be the one-dimensional torus, then $\Sigma = \mathbb{Z}$, the set of integers. Consider the two families $(\varphi_\theta)_{\theta\in[0,2\pi[}$ and $(\psi_\theta)_{\theta\in[0,2\pi[}$ of functions defined from $\mathbb{Z}$ into $\mathbb{C}$ by

$$\varphi_\theta(n) = e^{in\theta} \quad \text{and} \quad \psi_\theta(n) = \frac{e^{in\theta}}{n^2+1}. \tag{3.4}$$

Each mapping φ_θ satisfies the condition (3.3) whereas the functions ψ_θ do not satisfy it.

Theorem 3.3. *If φ is bounded and is such that $\inf\{|\varphi(\sigma)| \,:\, \sigma\in\Sigma\} > 0$, then*
$M_\varphi f \in \mathcal{A}_p(G,A)$ if and only if $f\in\mathcal{A}_p(G,A)$.

Proof.

$$\begin{aligned} M_\varphi f \in \mathcal{A}_p(G,A) &\implies \widehat{M_\varphi f} \in \mathbb{S}_p(\Sigma,A)\\ &\implies \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} < \infty. \end{aligned}$$

We have

$$\begin{aligned} \|\widehat{M_\varphi f}\|_{\mathbb{S}_p}^p &= \sum_{\sigma\in\Sigma} d_\sigma \sum_{i,j}\|\widehat{M_\varphi f}(\sigma)(\xi_j^\sigma,\xi_i^\sigma)\|^p\\ &= \sum_{\sigma\in\Sigma} d_\sigma \sum_{i,j}\|\varphi(\sigma)\widehat{f}(\sigma)(\xi_j^\sigma,\xi_i^\sigma)\|^p\\ &= \sum_{\sigma\in\Sigma} d_\sigma \sum_{i,j}|\varphi(\sigma)|^p\|\widehat{f}(\sigma)(\xi_j^\sigma,\xi_i^\sigma)\|^p\\ &= \sum_{\sigma\in\Sigma} d_\sigma|\varphi(\sigma)|^p \sum_{i,j}\|\widehat{f}(\sigma)(\xi_j^\sigma,\xi_i^\sigma)\|^p. \end{aligned}$$

Now, since $\inf\{|\varphi(\sigma)| \,:\, \sigma\in\Sigma\} > 0$ then there exists $C>0$ such that $C\le \inf\{|\varphi(\sigma)| \,:\, \sigma\in\Sigma\}$. Therefore

$$\|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \ge C\|\widehat{f}\|_{\mathbb{S}_p}.$$

Hence

$$\begin{aligned} \widehat{M_\varphi f} \in \mathcal{A}_p(G,A) &\implies \|\widehat{f}\|_{\mathbb{S}_p} < \infty\\ &\implies \widehat{f}\in\mathbb{S}_p(\Sigma,A)\\ &\implies f\in\mathcal{A}_p(G,A). \end{aligned}$$

Conversely, we have

$$\begin{aligned} f\in\mathcal{A}_p(G,A) &\implies \widehat{f}\in\mathbb{S}_p(\Sigma,A)\\ &\implies \|\widehat{f}\|_{\mathbb{S}_p} < \infty. \end{aligned}$$

From the boundedness of φ, there exist $C'>0$ such that

$$\sup\{|\varphi(\sigma)| \,:\, \sigma\in\Sigma\} \le C'.$$

Then

$$\|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \le C'\|\widehat{f}\|_{\mathbb{S}_p}.$$

So

$$\begin{aligned} f \in \mathcal{A}_p(G,A) &\implies \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} < \infty \\ &\implies \widehat{M_\varphi f} \in \mathbb{S}_p(\Sigma, A) \\ &\implies M_\varphi f \in \mathcal{A}_p(G,A). \end{aligned}$$

□

Hereafter are some inequalities involving Fourier multipliers.

Theorem 3.4. *Let M_φ be a bounded Fourier multiplier on $L_1(G,A)$. Then there exists two constants $C_1 > 0\,, C_2 > 0$ such that for each function f in $\mathcal{A}_p(G,A)$, we have:*

1. $\|M_\varphi f\|_{\mathcal{A}_p} \leq C_1\|f\|_{L_1} + C_2\|\widehat{f}\|_{\mathbb{S}_p}$.
2. $\|M_\varphi f\|^{\mathcal{A}_p} \leq C_2\|\widehat{f}\|_{\mathbb{S}_p}$.

Proof. Since M_φ is bounded on $L_1(G,A)$, there exists a constant $C_1 > 0$ such that $\forall f \in L_1(G,A),\ \|M_\varphi f\|_{L_1} \leq C_1\|f\|_{L_1}$. The boundedness of M_φ implies that φ is also bounded as a function on Σ. From the proof of Theorem 3.3, we get the existence of a constant $C' \geq 0$ such that $\|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \leq C'\|\widehat{f}\|_{\mathbb{S}_p}$. Setting $C_2 = C'$, we obtain:

$$\begin{aligned} 1.\ \|M_\varphi f\|_{\mathcal{A}_p} &= \|M_\varphi f\|_{L_1} + \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \leq C_1\|f\|_{L_1} + C_2\|\widehat{f}\|_{\mathbb{S}_p}. \\ 2.\ \|M_\varphi f\|^{\mathcal{A}_p} &= \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \leq C_2\|\widehat{f}\|_{\mathbb{S}_p}. \end{aligned}$$

□

As a consequence of the above inequalities, we have the next result which asserts that each bounded Fourier multiplier on $L_1(G,A)$ is also a bounded Fourier multiplier on the $p-$Fourier space.

Corollary 3.5. *If M_φ is a bounded Fourier multiplier on $L_1(G,A)$ then M_φ is also a bounded Fourier multiplier on $\mathcal{A}_p(G,A)$ endowed with each of the norms $\|\cdot\|_{\mathcal{A}_p}$ or $\|\cdot\|^{\mathcal{A}_p}$.*

Proof. According to Theorem 3.4 (part 1), there exists two positive constants C_1 and C_2 such that $\|M_\varphi f\|_{\mathcal{A}_p} \leq C_1\|f\|_{L_11} + C_2\|\widehat{f}\|_{\mathbb{S}_p}$. If we set $C = \max\{C_1, C_2\}$ then we have $\|M_\varphi f\|_{\mathcal{A}_p} \leq C(\|f\|_{L_1} + \|\widehat{f}\|_{\mathbb{S}_p})$, that is $\|M_\varphi f\|_{\mathcal{A}_p} \leq C\|f\|_{\mathcal{A}_p}$.

On the other hand, we know that $\|\widehat{f}\|_{\mathbb{S}_p} = \|f\|^{\mathcal{A}_p}$ by definition , so using Theorem 3.4 (part 2), we have $\|M_\varphi f\|^{\mathcal{A}_p} \leq C_2\|f\|^{\mathcal{A}_p}$. □

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Competing Interests

The authors declare that no competing interests exist.

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