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Modeling an N – warehouse Stock Allocation via Dynamic Programming Technique

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Authors' contributions

This work was carried out in collaboration between both authors. Authors CRC and CEE designed the study, wrote the protocol and wrote the first draft of the manuscript. Author CEE managed literature searches, analyses of the study, collected the data and analysis of the data. Author CRC proof read and complemented it. Author CEE managed the analyses of the study and this was supervised by author CRC. Both authors read and approved the final manuscript.

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ABSTRACT

Stock allocation is a system used to ensure that goods and services reach the ultimate users through efficient stocking in warehouses close to the consumers. The dire need for optimum distribution of goods to both retailers and consumers has caused a reasonable drift from ordinary allocation to developing a mathematical model that ensures efficient allocation of goods and services. Allocation of stock to warehouses is a complex problem that is broken down into simpler sub problems. Dynamic programming problem is a linear optimization method that obtains optimum solution of a multivariable problem by decomposing it into sub problems. A recursive equation links the different stages of the problem such that the optimum feasible solution of each stage is guaranteed to be the optimum feasible solution for the entire problem. This work will use the dynamic programming technique to develop a stock allocation model that would ensure optimum allocation of goods and services for maximum returns.
Relevant related literature are presented and reviewed with the aim of using this research to improve stock allocation processes. A manufacturing company that has at least six distribution

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outlets is used as a case study. The model is applied to data collected from the firm to obtain an enhanced stock allocation.

Keywords: Stock allocation; dynamic programming; optimum distribution; complex problem.

1. INTRODUCTION

The quest for optimal distribution of goods to both retailers and consumers has given rise to reasonable drift from ordinary allocation to developing a mathematical model that enhances steady and efficient allocation. Stock is the supply of goods for sale kept in the store for business. The growing global economy has caused a dramatic shift in stock management in the twentieth century from a mere approach to scientific approach."One of the related problems is that as the complexity and specialization in an organization increases, it becomes more difficult to allocate rationally and reasonably the available resources to various sections of the organization" [1]. The proper allocation of resources in both manufacturing and distribution industries is of paramount significance to the society since the chain of distribution is complete only when the goods get to consumers. The allocation process that minimizes cost and maximizes profit is always the desire of every organization. The need to obtain such a process is the aim of this work. In this regard, a lot of methods in operations research are available. Methods such as linear programming model; integer programming, goal programming, dynamic programming models etc can be used to ascertain optimum allocation of goods. This work will use the dynamic programming model to obtain an optimum allocation of resources that would provide the solution to the desire of any organization.

The dynamic programming is a linear optimization method that obtains optimum solution of a multivariable problem by decomposition of the problem into sub problems [2]. Dynamic programming is an approach of optimizing multistage decision processes with a recursive equation. The different stages of the problem are linked such that the optimum feasible solution of each stage is guaranteed to be the optimum feasible solution for the entire problem. The dynamism of demand makes it necessary to keep goods in stock and the act of maintaining stock has its associated costs. The act of stocking goods to satisfy future demand gives rise to the problem of designing a very efficient allocation technique that minimizes cost and maximizes profit. This involves minimizing an appropriate cost function that balances the total cost resulting from overstocking or under-stocking [3]. Dynamic Programming was first used by Richard Bellman in 1940 to describe a process of solving problems where one needs to find the best decisions one after the other [4]. This work uses the dynamic programming method to develop a stock allocation model that ensures optimum allocation of goods and services for maximum returns. If $r_i(Q_i)$ is the total return from the ith activity with the resource Q_i then we seek to maximize.

$$R(Q_1, Q_2, \ldots, Q_n) = r_1(Q_1) + r_2(Q_2) + \cdots + r_n(Q_n) \text{ Given that}$$

$$Q = Q_i \geq 0, \ i = 1, \quad 2, \ldots, n$$

2. REVIEW OF RELATED LITERATURE

A lot of work have been done in this area and a few are stated here. Stock allocation is an important part of any manufacturing organization and the availability of goods as at and when due is a sign of preparedness and efficient stock management which retain customers [5]. The problem of effective stock allocation is one that Should be handled properly to minimize cost. Customer service has become an important dimension of competition along with price and quality. In order to maintain a company's current customers and acquire new one. Prompt services is always considered for which the first requirement is to have goods readily available. [6]. Optimal stock allocation policy generally require comprehensive knowledge of the nature of demand of goods in an environment [3]. Stocking is an act of keeping goods in a store or warehouse / depot so as to make it available on demand to users. The act of stocking goods to satisfy future demand is vital to the manufacturing and distribution organizations [7,8]. This act has its associated costs both for keeping and not keeping stock. This work aims at developing the appropriate cost function that balances the total cost appropriate cost function resulting from overstocking or under stocking. The major objective of stock allocation models is to obtain an inventory level that minimizes the sum of the storage cost, holding cost and and other associated costs Dynamic Programming is one of the numerous linear optimization methods. It is a method for solving complex problems by breaking it down into simpler sub-problems [9]. It determines the optimum solution of a multivariable problem by decomposing it into stages with each stage comprising a single variable sub-problem. It is a recursive equation that links the different stages of the problem in a manner that guarantees that the optimal feasible solution of each stage is also optimal and feasible for the entire problem [10,11,9]. Optimal stock allocation policy generally require comprehensive knowledge of the nature of demand of goods in an environment [12]. Dynamic Programming is one of the numerous linear optimization methods. It is a method for solving complex problems by breaking it down into simpler sub-problems [13]. It determines the optimum solution of a multivariable problem by decomposing it into stages with each stage comprising a single variable sub-problem. It is a recursive equation that links the different stages of the problem in a manner that guarantees that the optimal feasible solution of each stage is also optimal and feasible for the entire problem [7,14] According to [15], dynamic programming is applicable to problems exhibiting the properties of overlapping sub-problems and optimal structure and the approach is especially useful when the number of repeating sub-problems grow exponentially as a function of the size of the inputs. The work of Bellman implies that the process refers to the act of supplying a decision through breaking down the problem into a sequence of decision steps which is done by defining a sequence of value function $v_1, v_2, \ldots, v_n$ with an argument y representing the state of the system at times i from 1 to n. Dynamic programming is guaranteed to give a mathematically optimal solution and the equations for the stages are written as follows:

Let $f_i(x)$ be the shortest distance to node x_i at stage i;

Define $d(x_{i-1}, x_i)$ as the distance from node x_{i-1} to x_i.

The f_i is computed from f_{i-1}, by the following recursive equation

$$f_i(x_i) = \min_{\text{All feasible } ((x_{i-1}, x_i)\text{nodes}} \{ d\,(x_{i-1},\ x_i) + f_{i-1}(x_{i-1})\}, i = 1,2,3 \ldots \text{[8]}$$

The computation in dynamic programming is done recursively so that the optimum solution of one sub-problem is used as an input to the next sub-program and by the time the last sub-program is solved, the optimum solution for the entire problem is ascertained [16]. [17] used the Chebyshev's polynomial approximation to obtain the optimum allocation of a given problem. It was shown that the polynomial P_m is the best approximation of the function since $|\lambda_m| \leq \|f - p_m\|$. Considering the multiplication separate return function and Single additive constant model of Dynamic Programming under certaintly, the allocation problem is expressed according to [18]. As

Maximize $\prod_{i-1}^{n} u_i$

Subject to $\sum u_i = Q,, \ u_i \geq 0, \ \ i = 1, 2, \ldots n.$

The model deals with the division of the given quantity into a given number of parts and each part is considered as a stage with the recursion formula as

$$f_i(x_i) = \max_{u_i = x_i} \{u_i\}$$

$$f_i(x_i) = \max_{0 < u_i \leq x_i} \{u_i\, f_{1-i}\,(x_i - u_i)\} = \max_{u_i} \{u_i(x_i - u_i)\},$$

3. MATHEMATICAL MODEL

Let Q be a certain quantity of resource that will be distributed among n-number of depots. Let R be the return which depends on the quantity of resource allotted to the depots.

The objective is to optimize total return

If $r_i(Q_i)$ denotes the return from the ith activity with resource Q_i, then the total return is given as

$$R(Q_1, Q_2, \ldots, Q_n) = r_1(Q_1) + r_2(Q_2) + \cdots + r_n(Q_n) \quad (1)$$

The quantity of resource Q is limited hence this gives rise to the constant.

$$Q = Q_1 + Q_2 + \cdots + Q_n\,, Q_i \geq \ 0, \ i, = 1,2, \ldots \ldots, n \quad (2)$$

Hence the problem is therefore sated as:

Optimize $R(Q_1, Q_2, \ \ldots, Q_n) = R(Q_1) + R(Q_2) + \cdots + R\,(Q_n)$

Given that $Q = Q_1 + Q_2 + \cdots + Q_n, \ Q_i \geq 0, \ i = 1, 2, \ldots, n$

If $f_n(Q) = \max_{0 \le Q_1 \le Q_n} [R(Q_1, Q_2, \ldots, Q_n] = \max_{0 \le Q_1 \le Q_n} [r_i(Q_1) + r_2(Q_2) + \cdots + r_n(Q_n)]$ (3)

$f_n(Q)$ is the maximum return from the distribution of the resource Q to the n activities/depots.

Resources is then allocated to the activities (stage) to get the expression $f_n(Q)$ and $f_{n-1}(Q)$ for arbitrary values of Q. A continuation of this process yields a total return for $r_n(Q_n) + f_{n-1}(Q - Q_n)$ for $(n-1)$ activities and $f_{n-1}(Q - Q_n)$ return.

Here an optimal choice of Q_n will maximize the function above and thus the dynamic programming model is expressed as:

$$f_n(Q) = \max_{0 \le Q_i \le Q_n} [r_n(Q_n) + f_{n-1}(Q - Q_n)], \ n = 2,3 \quad (4)$$

$$f_1(Q) = \max_{0 \le Q_i \le Q_n} r_1(Q_1)$$

i.e. $f_1(Q) = r_1(Q)$ (5)

i.e. all the resources Q is allotted to this activity with $f_1(Q)$ known, equation (4) provides a relation to evaluate $f_2(Q), f_3(Q), \ldots, f_{n-1}(Q), f_n(Q)$.

More specifically, given a quantity of stock b, divided b into n-parts so as to maximize their product i.e. let $f_n(b)$ be the maximum value.

Then

$f_1(b) = b$ and $f_n(b) = \max_{0 \le z \le b}\{z\, f_{n-1}(b - z)\}$

Now

Let x_i be the ith entry of the quantity $b\ (i = 1, 2, \ldots, n)$,

The problem is then:

Max $f_n(b) = x_1 . x_2 \ldots . x_n$

Subject to $x_1 + x_2 + \cdots + x_n = b$, $x_1 > 0$, $i = 1, 2, \ldots n$ where each part $x_i (i = 1, 2, \ldots, n)$ of b is regarded as a stage. Since x_1 may assume any positive value satisfying the given condition that $x_1 + x_2 + \cdots + x_n = b$, alternatives at each stage are infinite. Thus x_i is considered continuous variable.

∴ The recursive equation of the problem for all values of n can each be obtained as follows. For n = 1, $f_1(b) = x$, for n = 2, b is divided into two parts

say $x_1 = z$, $x_2 = z - b$ then

$f_2(b) = = \max_{0 \le z \le b} \{z f_1(b - z)\} = \max\ (x_1, x_2)$
= max $\{z(b - z)\}$ since $f_1(b - z) = b - z$

for n = 3, divide b into three parts giving as the initial choice and (b –z) to be divided into 2 parts

∴ By the principle of optimality, we have

$$f_3(b) = \max_{0 \le z \le b} \{z f_2(b - z)\}$$

Continuing in this manner gives the equation for general value of n as

$$f_n(b) = \max_{0 \le z \le b} \{z f_{n-1}(b - z)\} \quad (*)$$

(*) is the recursive equation.

4. ILLUSTRATION

In the work, six depots A, B, C, D, E, F with the products X,Y and Z are used. The data collected from a company for three years is represented in Tables 1 and 2. Table 3 represents another form of Table 2 while Table 4 summarizes and approximates in thousands the entries in the earlier tables. The corresponding returns from the products / warehouses are presented in Tables 5 and 6. The application of the Dynamic Programming technique is then used on table 6 to produce Tables 7 to 19. Data for another two years of the firms allocation to the six deports are shown in tables 20,21 and 22 and the Dynamic Programming technique also applied to get Table 23.

The return from each zone depends upon the sales of the three products in the zone. The returns for different products for the past five years (2008 – 2-012) are given in the following table. Ai, Bi, Ci Di, Ei Fi with i =1,2,3 represents the product in a given depot

Allocaion of products to six depos

Table 1. Allocation to products to depots per product per year for 5 years

Year		X	Y	Z
	A_1	564768	382752	427536
	B_1	564768	382752	427536
2	C_1	564768	382752	427536
	D_1	451815	306202	342029
	E_1	451814	306202	342028
	F_1	481814	306201	342029
3	A_2	451815	306203	342029
	B_2	451816	306204	342030
	C_2	451814	306204	342030
	D_2	335528	229652	250522
	E_2	335528	229651	256522
	F_2	335527	229651	256521
	A_3	225907	153101	171017
4	B_3	225908	153101	171016
	C_3	225907	153101	171017
	D_3	225910	153103	171003
	E_3	225910	153104	171002
	F_3	225910	153104	171002

Table 2. The allocation summed up for 3 years

	X	Y	Z
1	1,694,304	1,148,256	1,282,608
2	1,355,443	918,605	1,026,086
3	1,355,445	918,610	1,026,089
4	1,016,583	688,954	769,565
5	677,722	459,303	513,050
6	677,730	459,310	513,008

Beginning from zone 1, we have the following tables;

Table 3. The allocation summed up for 3 years

	1	2	3	4	5	6
X	1,694,304	1,355,443	1,355,445	1,016,583	677,722	677,730
Y	1,148,256	918,605	918,610	688,954	459,303	459,310
Z	1,282,608	1,026,086	1,026,089	769,565	513,050	513,008

Table of profit in thousands of crates

Table 4. Table of profit in thousands of crates

	A	B	C	D	E	F
X	1,694	1,355	1,355	1,017	678	678
Y	1,148	919	919	686	459	459
X	1,283	1,026	1,026	770	513	513

The corresponding table of returns (profits) in millions of naira is as follows

Table 5. The corresponding table of returns (profits) in millions of naira $(x_o x_1)$

Product	0	1	2	3	4	5	6
X	1	1.7	1.4	1.4	1.0	0.7	0.7
Y	2	1.2	0.9	0.9	0.7	0.5	0.5
Z	3	1.3	1.0	1.0	0.8	0.5	0.5

Stage 1

The stipulated profits corresponding to different products allocated to X are given in table $x_1 x_1$.

Table 6. ($x_1 x_1$)

Product	0	1	2	3
Profit	0	1.7	1.2	1.3

Stage 2

Table 7. Profits for possible combinations

		0	1	2	3
		0	1.7	1.2	1.3
0	0	0	1.7	1.2	1.3
1	1.4	1.4	1.3	2.5	
2	0.9	0.9	2.6		
3	1.0	1.0			

Thus the optimal profit and corresponding allocations of products to the two zones are given by

Table 8. Table of returns (profits) in millions of naira

Products	0	1	2	3
$f_2(x_2) + f_1(x_1)$	0	1.7	3.1	2.6
$x_2 + x_1$	0+0	0+1	1+1	2+1

Table 9. x_0 x_(1) returns from depots per product)

Products	0	1	2	3	4	5	6
Coke	1	1. 7	1.4	1.4	1.0	0.7	0.7
Fanta	2	1.2	0.9	0.9	0.7	0.5	0.5
Sprite	3	1.3	1.0	1.0	0.8	0.5	0.5

Stage 1

The estimated profits corresponding to different products allocated to I are given in table $x_0 x_1$ and are reproduced in table $x_1 x_1$

Table 10. 10 (x_1 x_(1)) estimated profit / product

Products	0	1	2	3
$f_1(x_1)$	0	1.7	1.2	1.3

Stage 2

Table 11. Profits for possible combinations

		0	1	2	3
		0	1.7	1.2	1.3
0	0	0	1.7	1.2	1.3
1	1.4	1.4	1.3	2.5	
2	0.9	0.9	2.6		
3	1.0	1.0			

Thus the optimal profit and corresponding allocations of products to the two zones are given by

Table 12. Optimal profit allocation

Products	0	1	2	3
$f_2(x_2) + f_1(x_1)$	0	1.7	3.1	2.6
$x_2 + x_1$	0+0	0+1	1+1	2+1

Stage 3

Zone product $(x_2 + x_1)$		0	1	2	3
$1+2$ $f_2(x_2) + f_1(x_1)$					
Zone 3		0	1.7	1.2	1.3
$x_3 + f_3(x_3)$					
0	0	0	1.7	1.2	1.3
1	1.4	1.4	1.3	2.5	
2	0.9	0.9	2.6		
3	1.0	1.0			

∴ The optimal profit and corresponding allocation of products to the three zones are given as

Table 13. Optimal profit allocation

Products	0	1	2	3
$f_3(x_3) + f_2(x_2) + f_1(x)$	0	1.7	3.1	4.5
$x_3 + (x_2 + x_1)$	$0+0$	0+1	1+1 or 0+2	1+2

Stage 4

Table 14. Profits for possible combinations

Product $(x_3 + x_2 + x_1)$		0	1	2	3
$1+3$ $f_3(x_3) + f_2(x_2) + f_1(x_1)$					
$x_4 + f_4(x_4)$		0	1.7	3.1	4.5
0	0	0	1.7*	3.1*	4.5*
1	1.4	1.0	2.7	2.4	
2	0.9	0.9	2.4		
3	1.0	0.8			

∴ The optimal profits and corresponding allocation of products to the four zones are given as

Table 15. Optimal profit allocation

Products:	0	1	2	3
$f_3(x_3) + f_2(x_2) + f_1(x)$:	0	1.7	3.1	4.5
$x_3 + (x_2 + x_1)$:	$0+0$	0+1	0+2	0+3

Stage 5

Table 16. Profits for possible combinations

Product $(x_4 + x_3 + x_2 + x_1)$		0	1	2	3
Zone 1+2+3+4 $f_4(x_4) + \cdots + f_1(x_1)$					
Zone 5 x_5	$f_5(x_5)$	0	1.7	1.2	1.3
0	0	0	1.7*	3.1*	4.5
1	0.7	0.7	2.4	3.8	
2	0.5	0.5	2.4		
3	0.5	0.5			

Optimal profits

Table 17. Optimal profit allocation

Products:	**0**	**1**	**2**	**3**
$f_5(x_5) + f_4(x_4) + \cdots + f_1(x_1)$:	0	1.7	3.1	4.5
$x_5 + (x_4 + x_3 + x_2 + x_1)$:	$0 + 0$	0+1	0+2	0+3

Stage 6

Table 18. Profits for possible combination

Product $\sum_{j=1}^{5} x_1$:		**0**	**1**	**2**	**3**
$\sum_{j=1}^{5} f_1(x_1)$		**0**	**1.7**	**1.2**	**1.3**
x_0	$f_6(x_6)$				
0	0	0	1.7*	3.1*	4.5*
1	0.7	0.7	2.4	3.8	
2	0.5	0.5	2.4		
3	0.5	0.5			

Optimal profits

Table 19. Optimal profit allocation

Products:	**0**	**1**	**2**	**3**
$f_6(x_6) + f_5(x_5) + \cdots + f_1(x_1)$:	0	1.7	3.1	4.5
$x_6 + (x_5 + x_4 + \cdots + x_1)$:	$0 + 0$	0+1	0+2	0+3

Thus the maximum profit is 4.7 if $x_6 = x_5 = x_4 = 0,\ x_3 = x_2 = x_1 = 1$. Hence maximum profit can be attained if the three products are allocated to the three zones (Ontisha, Warri and Benin) only on equal basis.

Table 20. Table of profit in thousands of crates

	A	**B**	**C**	**D**	**E**	**F**
X	816,152	652922	652922	489693	326462	326463
Y	1,003,406	802724	802724	601924	401362	401362
Z	902,758	722206	722206	541655	361104	361104
X	816	653	653	490	327	327
Y	1003	803	803	602	401	401
Z	903	722	722	542	361	361
X	0.8	0.7	o.7	0.5	0.3	0.3
Y	1.0	0.8	0.8	0.6	0.4	0.4
Z	9.0	0.7	0.7	0.5	0.4	0.4

For the remaining 2 years, we have the following

Table 21. Returns from depots per product

	Onisha	**Warri**	**Benin**	**Auch**	**Asaba**	**Lokoja**
Coke	0.8	0.7	0.7	0.5	0.3	0.3
Fanta	1.0	0.8	0.8	0.6	0.4	0.4
Sprite	9.0	0.7	0.7	0.5	0.5	0.4

The estimated profits corresponding to different products allocated to zone 1 are given below.

Table 22. Profits corresponding to the products

Product	**0**	**1**	**2**	**3**
Profit $f_1(x_1)$	0	0.8	1.0	9.0

Stage 2

Table 23. Profits for possible combinations

		0	**1**	**2**	**3**
		0	**0.8**	**1.0**	**9.0**
0		0	0.8*	1.0	9.0
1	0				
2	0.7	0.7	1.5*	1.7*	
3	0.7	0.8	1.6		
		0.7			

Thus the optimal profits and corresponding allocation of products to the two zones are given by

Products:	**0**	**1**	**2**	**3**
$f_2(x_2) + f_1(x_1)$:	0	0.8	1.5	1.7
$x_2 + x_1$:	0+0	0+1	1+1	2+2

Stage 3

Zone product $(x_2 + x_1)$1+2 $f_2(x_2) + f_1(x_1)$ x_2		**0**	**1**	**2**	**3**
		0	**0.8**	**1.5**	**1.7**
0		0	0.8*	1.5*	1.7
1	0	0.7	1.5*	2.2*	
2	0.7	0.8	1.6		
3	0.7	0.7			

∴ The optimal profits and corresponding allocation of products to the three zones are

Products:	**0**	**1**	**2**	**3**
$f_3(x_3) + f_2(x_2) + f_1(x_1)$:	0	0.8	1.5	2.2
$x_3 + (x_2 + x_1)$:	0+0	0+1	0+2	1+2

Stage 4

Product $(x_3 + x_2 + x_1)$		**0**	**1**	**2**	**3**
$f_3(x_3) + f_2(x_2) + f_1(x_1)$		**0**	**0.8**	**1.5**	**1.7**
x_4	$f_4(x_4)$				
0	0	0	0.8*	1.5*	2.2*
1	0.5				
2	0.6	0.5	1.3*	2.0*	
3	0.5	0.6	1.4		
		0.5			

∴ The optimal profits and corresponding allocation of products to the four zones are

Products:	**0**	**1**	**2**	**3**
$f_4(x_4) + f_3(x_3) + \cdots + f_1(x_1)$:	0	0.8	1.5	2.2
$x_4 + (x_3 + x_2 + x_1)$:	0+0	0+1	0+2	0+3

Stage 5

Product $(\mathbf{x_4+x_3+x_2+x_1})$		**0**	**1**	**2**	**3**
Zone 1 + 2+3+4 $\mathbf{f_4(x_4)+\cdots+f_1(x_1)}$		**0**	**0.8**	**1.5**	**1.7**
$\mathbf{x_5}$	$\mathbf{f_5(x_5)}$				
0	0	0	0.8*	1.5*	2.2*
1	0.3	0.3	1.5*	1.8	
2	0.4	0.4	1.2		
3	0.4	0.4			

The optimal profits is given by

Products:	**0**	**1**	**2**	**3**
$f(x_5)+f_4(x_4)+\cdots+f_1(x_1)$:	0	0.8	1.5	2.2
$x_5+(x_4+x_3+x_2+x_1)$:	0+0	0+1	0+2	0+3

Stage 6

$\mathbf{x_6}$	$\mathbf{f_6(x_6)}$	**0**	**1**	**2**	**3**
		0	**0.8**	**1.5**	**2.2**
0	0	0	0.8*	1.5*	2.2*
1	0.3				
2	0.4	0.3	1.1	1.8	
3	0.4	0.4	1.2		
		0.4			

Optimal profits

Products:	**0**	**1**	**2**	**3**
$f_6(x_6)+f_5(x_5)+\cdots+f_1(x_1)$:	0	0.8	1.5	2.2
$x_6+(x_5+x_4+\cdots+x_1)$:	0+0	0+1	0+2	0+3

∴ the maximum profit is 2. 2 if $x_6 = x_5 = x_4 = x_3 = 0$

$x_2 = x_1 = 1$. Hence maximum profit can be obtained if the three products are allocated to the three zones only on equal basis.

5. SUMMARY/ CONCLUSION

The proper allocation of resources in manufacturing and distribution firm is of paramount importance to the company and the society. The ability to obtain an allocation that optimizes the returns of a company puts the firm on a good footing. The act of stocking to satisfy future demand has its associated costs. There is therefore the need to design a model that minimizes the appropriate cost function that balances the total cost resulting from overstocking or under-stocking. Dynamic programming, an approach for optimizing multistage decision process is used on the stock allocation problem to obtain a better return. This research work has employed the dynamic programming technique to stock allocation to obtain optimum solution. The stock allocation process is partitioned into smaller sub-problems and the dynamic programming method which is an approach for optimizing multistage decision process is applied. With a recursive equation, the different stages of the problem are linked such that the optimum feasible solution of each Stage is guaranteed to be the optimum feasible solution for the entire problem. With the illustration for n = 6 used, the optimum profits and the corresponding product allocation to the warehouses is ascertained. It is plausible to allocate the stock on equal basis to the warehouses. It has therefore been shown that this method is good for obtaining optimal stock allocation.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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