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Fixed Point Theorems for Presic Type Mappings in G_p-Metric Spaces

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper, we introduce some fixed point theorems in Presic type mappings on G_p-metric spaces. The present results generalizes various known results in the related literature.

Keywords: Fixed point, G_p-metric space, Contractive mapping

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1 Introduction and Preliminaries

In 1922, Banach [1] established famous fundamental fixed point theorem, also known as Banach contraction principle. The Banach contraction principle is the simplest and one of the most versatile

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elementary results in fixed point theory. Over the years, various extensions and generalizations of this principle have appeared in the literature. Matthews [2], introduced the partial metric spaces and proved a fixed point theorem on this space. After that several fixed point results have been proved in this space, for more details see [3] [4] [5] [6] [7]. In 2006, Mustafa and Sims [8] introduced a new structure called G-metric space as a generalization of the usual metric spaces. Afterwards based on the notion of a G-metric space, many fixed point results for different contractive conditions have been presented, for more details see [9] [10] [11] [12] [13]. Recently, based on the two above metric spaces, Zand and Nezhad [14] introduced a new generalized metric spaces G_p as a both generalization of the partial metric space and G-metric spaces. Some of these works may be noted in [15] [16] [17].

Now, we mention briefly some fundamental definitions.

Definition 1.1. [14] Let X be a nonempty set and let $G_p : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:

($GP1$) $0 \leq G_p(x,x,x) \leq G_p(x,x,y) \leq G_p(x,y,z)$, all $x, y, z \in X$;

($GP2$) $G_p(x,y,z) = G_p(x,z,y) = G_p(y,z,x) \ldots$, (symmetry in all three variables);

($GP3$) $G_p(x,y,z) \leq G(x,a,a) + G_p(a,y,z) - G_p(a,a,a)$, for any $a, x, y, z \in X$, (rectangle inequality);

($GP4$) $x = y = z$ if $G_p(x,y,z) = G_p(x,x,x) = G_p(y,y,y) = G_p(z,z,z)$;

Then the pair (X, G_p) is called a G_p-metric space.

Proposition 1.1. *[14] Let (X, G_p) be a G_p-metric space. Then for any x, y, z and $a \in X$ the following relations are true.*

(i) $G_p(x,y,z) \leq G_p(x,x,y) + G_p(x,x,z) - G_p(x,x,x)$;

(ii) $G_p(x,y,y) \leq 2G_p(x,x,y) - G_p(x,x,x)$;

(iii) $G_p(x,y,z) \leq G_p(x,a,a) + G_p(y,a,a) + G_p(z,a,a) - 2G_p(a,a,a)$;

(iv) $G_p(x,y,z) \leq G_p(x,a,z) + G_p(a,y,z) - G_p(a,a,a)$.

Definition 1.2. [14] Let (X, G_p) be a G_p-metric space and a sequence $\{x_n\}$ is called a G_p convergent to $x \in X$ if

$$\lim_{n,m\to\infty} G_p(x, x_n, x_m) = G_p(x,x,x).$$

A point $x \in X$ is said to be limit point of the sequence $\{x_n\}$ and written $x_n \to x$.

Thus if $x_n \to x$ in a G_p metric space (X, G_p), then for any $\epsilon > 0$, there exists $\ell \in \mathbb{N}$ such that $|G_p(x, x_n, x_m) - G_p(x,x,x)| < \epsilon$, for all $n, m > \ell$.

Proposition 1.2. *[14] Let (X, G_p) be a G_p-metric space, then for any sequence $\{x_n\}$ in X, the following are equivalent that*

(i) $\{x_n\}$ *is* G_p *convergent to* x;

(ii) $G_p(x_n, x_n, x) \to G_p(x,x,x)$ *as* $n \to \infty$;

(iii) $G_p(x_n, x, x) \to G_p(x,x,x)$ *as* $n \to \infty$.

Definition 1.3. [18]

(1) The sequence $\{x_n\}_{n\in\mathbb{N}}$ in a G_p-metric space (X, G_p) is said to be a G_p Cauchy sequence if there exists $r \in \mathbb{R}$ such that $\lim_{n,m\to\infty} G_p(x_n, x_m, x_m) = r$.

(2) (X, G_p) is said to be G_p-complete if for every G_p Cauchy sequence $\{x_n\}_{n\in\mathbb{N}}$ there exists $x \in X$ such that

$$\lim_{n,m\to\infty} G_p(x_n, x_m, x_m) = \lim_{n,m\to\infty} G_p(x_n, x_m, x) = G_p(x, x, x).$$

Lemma 1.1. *[15] Let (X, G_p) be a G_p-metric space. Then*

(i) If $G_p(x, y, z) = 0$ then $x = y = z$,

(ii) If $x \neq y$ then $G_p(x, y, y) > 0$.

Proposition 1.3. *[14] Every G_p-metric space (X, G_p) defines a metric space (X, d_{G_p}) as follows: $d_{G_p(x,y)} = G_p(x, y, y) + G_p(y, x, x) - G_p(x, x, x) - G_p(y, y, y)$, for all $x, y \in X$.*

2 Main Results

Considering the convergence of certain sequences S. B. Presic [19] generalized Banach contraction principle as follows:

Theorem 2.1. *[19] Let (X, d) be a complete metric space, k a positive integer and $T : X^k \to X$ a mapping satisfying the following contractive type condition*

$$d(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1})) \leq q_1 d(x_1, x_2) + q_2 d(x_2, x_3) + ... + q_k d(x_k, x_{k+1}) \tag{2.1}$$

for every $x_1, x_2, ..., x_{k+1}$ in X where $q_1, q_2, ..., q_k$ are non negative constants such that $q_1 + q_2 + ... + q_k < 1$. Then there exist a point x in X such that $T(x, x, ..., x) = x$. Moreover, if $x_1, x_2, ..., x_k$, are arbitrary points in X and for $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, ..., x_{n+k-1}) \quad (n = 1, 2, ...)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim x_n = T(\lim x_n, \lim x_n, ..., \lim x_n).$$

Remark that condition (2.1) in the case $k = 1$ reduces to the well-known Banach contraction mapping principle. So, Theorem 2.1 is a generalization of the Banach fixed point theorem.

Ćirić and Presic [20], generalized Theorem 2.1 as follows:

Theorem 2.2. *[20] Let (X, d) be a complete metric space, k a positive integer and $T : X^k \to X$ a mapping satisfying the following contractive type condition*

$$d(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1})) \leq \lambda \max_{1\leq i\leq k} \{d(x_i, x_{i+1})\} \tag{2.2}$$

where $\lambda \in (0, 1)$ is constant and $x_1, x_2, ..., x_{k+1}$ in X. Then there exist a point x in X such that $T(x, x, ..., x) = x$. Moreover, if $x_1, x_2, ..., x_k$, are arbitrary points in X and for $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, ..., x_{n+k-1}) \quad (n = 1, 2, ...)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim x_n = T(\lim x_n, \lim x_n, ..., \lim x_n).$$

If in addition we suppose that on a diagonal $\triangle \subset X^k$

$$d(T(u, u, ..., u), T(v, v, ..., v)) < d(u, v) \tag{2.3}$$

holds for all $u, v \in X$, with $u \neq v$, then x is the unique point in X with $T(x, x, ..., x) = x$.

Nazır and Abbas [21], proved common fixed point theorems of Presic type in partial metric space. Also, Dhasmana [22] showed a unique common fixed point theorem is obtained in settings of G-metric spaces by using the concept of Presic fixed point theorem. Further, Gairola and Dhasmana [23] proved common fixed point theorems of Presic type in G-metric space which extends the result of Ćirić-Presic [20], Dhasmana [22] and George-Khan [24].

We will carry this idea to G_p-metric spaces, which is a generalization of partial metric spaces.

Theorem 2.3. *Let (X, G_p) be complete G_p-metric spaces, k a positive integer and $T : X^k \to X$ a mapping satisfying the following contractive type condition*

$$G_p(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1}), T(x_3, x_4, ..., x_{k+2})) \leq \lambda \max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\} \quad (2.4)$$

where $\lambda \in (0, 1)$ is constant and $x_1, x_2, ..., x_k$, are arbitrary elements in X. Then there exists a point x in X such that $T(x, x, ..., x) = x$. Moreover, if $x_1, x_2, ..., x_{k+2}$ are arbitrary points in X and $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, ..., x_{n+k-1}) \quad (n = 1, 2, ...) \quad (2.5)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim_{n\to\infty} x_n = T(\lim x_n, \lim x_n, ..., \lim x_n).$$

If in addition we suppose that

$$G_p(T(u, u, ..., u), T(v, v, ..., v), T(w, w, ..., w)) < G_p(u, v, w) \quad (2.6)$$

holds for all $u, v, w \in X$, with $u \neq v \neq w$, then x is the unique point in X with $T(x, x, ..., x) = x$.

Proof. $x_1, x_2, ..., x_k$, be k arbitrary in X. Using these points define a sequence (x_n) as follows:

$$x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1}), \qquad (n = 1, 2, \dots).$$

For simplicity set $\gamma_n = G_p(x_n, x_{n+1}, x_{n+2})$. We shall prove by induction that for each $n \in \mathbb{N}$;

$$\gamma_n \leq M\theta^n \quad (2.7)$$

where $\theta = \lambda^{\frac{1}{k}}$ and $M = \max\{\frac{\gamma_1}{\theta}, \frac{\gamma_2}{\theta^2}, ..., \frac{\gamma_k}{\theta^k}\}$. According to the definition of M we can writing for $n = 1, 2, ..., k$

$$\gamma_n \leq M\theta^n, \quad \gamma_{n+1} \leq M\theta^{n+1}, ..., \gamma_{n+k-1} \leq M\theta^{n+k-1}.$$

Then we have:

$$\begin{aligned}\gamma_{n+k} &= G_p(x_{n+k}, x_{n+k+1}, x_{n+k+2})\\ &= G_p(T(x_n, x_{n+1}, ..., x_{n+k-1}), T(x_{n+1}, x_{n+2}, ..., x_{n+k}), T(x_{n+2}, x_{n+3}, ..., x_{n+k+1})).\end{aligned}$$

By (2.4)

$$\begin{aligned}\gamma_{n+k} &= G_p(T(x_n, x_{n+1}, ..., x_{n+k-1}), T(x_{n+1}, x_{n+2}, ..., x_{n+k}), T(x_{n+2}, x_{n+3}, ..., x_{n+k+1}))\\ &\leq \lambda \max\{\gamma_n, \gamma_{n+1}, \gamma_{n+2}, ..., \gamma_{n+k-1}\}\\ &\leq \lambda \max\{M\theta^n, M\theta^{n+1}, ..., M\theta^{n+k-1}\}\end{aligned}$$

as $\theta = \lambda^{\frac{1}{k}}$

$$\begin{aligned}\gamma_{n+k} &\leq \lambda M\theta^n \quad (\text{as } 0 < \theta < 1)\\ &= M\theta^{n+k}\end{aligned}$$

and the inductive proof of (2.7) is complete. Next using (2.7) for any $n, m \in \mathbb{N}$ we have the following argument:

$$\begin{aligned} G_p(x_n, x_m, x_m) \leq & G_p(x_n, x_{n+1}, x_{n+1}) + G_p(x_{n+1}, x_{n+2}, x_{n+2}) + ... \\ & + G_p(x_{m-1}, x_m, x_m) - \{G_p(x_{n+1}, x_{n+1}, x_{n+1}) + G_p(x_{n+2}, x_{n+2}, x_{n+2}) + \\ & ... + G_p(x_{m-1}, x_{m-1}, x_{m-1})\} \\ \leq & G_p(x_n, x_{n+1}, x_{n+2}) + G_p(x_{n+1}, x_{n+2}, x_{n+3}) + ... + G_p(x_{m-2}, x_{m-1}, x_m) \\ = & \gamma_n + \gamma_{n+1} + ... + \gamma_{m-2} \\ \leq & M\theta^n + M\theta^{n+1} + ... + M\theta^{m-2} \\ \leq & \frac{M\theta^n}{1-\theta} \end{aligned}$$

by which we conclude that (x_n) is a G_p Cauchy sequence. Since (X, G_p) is complete G_p-metric space, there exists $x \in X$ such that $\{x_n\}$ sequence converges $x \in X$. So,

$$\lim_{n,m\to\infty} G_p(x_n, x_m, x_m) = \lim_{n,m\to\infty} G_p(x_n, x_m, x) = G_p(x, x, x) = 0.$$

Then for any integer n we have

$$\begin{aligned} G_p(x_{n+k}, x_{n+k}, T(x, x, ..., x)) = & G_p(T(x, x, ..., x), T(x_n, x_{n+1}, ..., x_{n+k-1}, T(x_n, x_{n+1}, ..., x_{n+k-1})) \\ \leq & G_p(T(x, x, ..., x), T(x, ..., x, x_n), T(x, ..., x, x_n)) + \\ & G_p(T(x, ..., x, x_n), T(x, ..., x, x_n, x_{n+1}), T(x, ..., x, x_n, x_{n+1})) + \\ & G_p(T(x, ..., x, x_n, x_{n+1}), T(x, ..., x, x_n, x_{n+1}, x_{n+2}), T(x, ..., x, x_n, x_{n+1}, x_{n+2})) \\ & + ... + G_p(T(x, x_n, ..., x_{n+k-2}), T(x_n, x_{n+1}, ..., x_{n+k-1}), T(x_n, x_{n+1}, ..., x_{n+k-1})) \\ & - \{G_p(T(x, ..., x, x_n), T(x, ..., x, x_n), T(x, ..., x, x_n)) + \\ & G_p(T(x, ..., x, x_n, x_{n+1}), T(x, ..., x, x_n, x_{n+1}), T(x, ..., x, x_n, x_{n+1})) + \\ & ... + G_p(T(x, x_n, ..., x_{n+k-2}), T(x, x_n, ..., x_{n+k-2}), T(x, x_n, ..., x_{n+k-2}))\} \\ \leq & G_p(T(x, x, ..., x), T(x, ..., x, x_n), T(x, ..., x, x_n)) + \\ & G_p(T(x, ..., x, x_n), T(x, ..., x, x_n, x_{n+1}), T(x, ..., x, x_n, x_{n+1})) + \\ & G_p(T(x, ..., x, x_n, x_{n+1}), T(x, ..., x, x_n, x_{n+1}, x_{n+2}), T(x, ..., x, x_n, x_{n+1}, x_{n+2})) \\ & + ... + G_p(T(x, x_n, ..., x_{n+k-2}), T(x_n, x_{n+1}, ..., x_{n+k-1}), T(x_n, x_{n+1}, ..., x_{n+k-1})) \\ \leq & \lambda \max\{G_p(x, x, x), G_p(x, x_n, x_n)\} + \lambda \max\{G_p(x, x, x), G_p(x, x_n, x_n), G_p(x_n, x_{n+1}, x_{n+1})\} + \\ & + ... + \lambda \max\{G_p(x, x, x), G_p(x, x_n, x_n), G_p(x_n, x_{n+1}, x_{n+1}), ..., G_p(x_{n+k-2}, x_{n+k-1}, x_{n+k-1})\}. \end{aligned}$$

Taking the limit when n tends to infinity we obtain

$$G_p(x, x, T(x, x, ..., x)) \leq \lambda G_p(x, x, x)$$

that is,

$$G_p(x, x, T(x, x, ..., x)) \leq 0$$

which implies

$$T(x, x, ..., x) = x.$$

Thus we proved that;

$$\lim x_n = T(\lim x_n, \lim x_n, ..., \lim x_n).$$

Now suppose that (2.6) holds. To prove the uniqueness of the fixed point, let us assume that for some $y, z \in X$, $x \neq y \neq z$ we have

$$T(y, y, ..., y) = y, \qquad T(z, z, ..., z) = z$$

Then by (2.6),

$$G_p(x, y, z) = G_p(T(x, x, ..., x), T(y, y, ..., y), T(z, z, ..., z)) < G_p(x, y, z), \tag{2.8}$$

which is a contraction. So, x is the unique point in X such that $T(x, x, ..., x) = x$. □

Example 2.4. *Let $X = [0, 2]$ and $G_p : X \times X \times X \to \mathbb{R}^+$ defined by*

$$G_p(x, y, z) = \begin{cases} |x-y| + |y-z| + |x-z|, & \text{if } x, y, z \in [0, 1) \\ \max\{x, y, z\}, & \text{otherwise} \end{cases}$$

(X, G_p) is a complete G_p metric space. Let $k \in Z^+$ and $T : X^k \to X$ be the mapping defined by

$$T(x_1, x_2, ..., x_k) = \begin{cases} \frac{x_1+x_k}{4k}, & \text{if } x_1, x_2, ..., x_k \in [0, 1) \\ 0, & \text{otherwise} \end{cases}$$

Now $x_1, x_2, ..., x_k, x_{k+1}, x_{k+2} \in [0, 1)$ and $\lambda = \frac{1}{2}$. Thus, we obtain

$$\begin{aligned}
&G_p(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1}), T(x_3, x_4, ..., x_{k+2}))\\
&=\left|\frac{x_1+x_k}{4k} - \frac{x_2+x_{k+1}}{4k}\right| + \left|\frac{x_2+x_{k+1}}{4k} - \frac{x_3+x_{k+2}}{4k}\right| + \left|\frac{x_1+x_k}{4k} - \frac{x_3+x_{k+2}}{4k}\right|\\
&\leq\frac{1}{4k}[|x_1-x_2| + |x_2-x_3| + |x_1-x_3| + |x_k-x_{k+1}| + |x_{k+1}-x_{k+2}| + |x_k-x_{k+2}|]\\
&=\frac{1}{4k}|G_p(x_1, x_2, x_3) + G_p(x_k, x_{k+1}, x_{k+2})|\\
&\leq\frac{1}{2k}\max\{G_p(x_1, x_2, x_3), G_p(x_k, x_{k+1}, x_{k+2})\}\\
&\leq\frac{1}{2k}\max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\}\\
&\leq\frac{1}{2}\max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\}\\
&=\lambda\max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\}.
\end{aligned}$$

If $x_1, x_2, ..., x_k \in [0, 1)$ and $x_{k+1}, x_{k+2} \in [1, 2]$ then

$$\begin{aligned}
G_p(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1}), T(x_3, x_4, ..., x_{k+2}))) &= \frac{x_1+x_k}{4k}\\
&\leq \frac{1}{2k}x_k\\
&\leq \frac{1}{2}\max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\}\\
&= \lambda\max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\}.
\end{aligned}$$

If $x_1, x_2, ..., x_k, x_{k+1} \in [0, 1)$ and $x_{k+2} \in [1, 2]$ then

$$\begin{aligned}
G_p(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1}), T(x_3, x_4, ..., x_{k+2}))) &= \left|\frac{x_1+x_k}{4k} - \frac{x_2+x_{k+1}}{4k}\right|\\
&\leq \frac{1}{2k}\max\{x_k, x_{k+1}\}\\
&\leq \frac{1}{2}\max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\}\\
&= \lambda\max_{1\leq i\leq k}\{G_p(x_i, x_{i+1}, x_{i+2})\}.
\end{aligned}$$

Similarly, if $x_1, x_2, ..., x_k, x_{k+2} \in [0, 1)$ *and* $x_{k+1} \in [1, 2]$ *then*

$$\begin{aligned} G_p(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1}), T(x_3, x_4, ..., x_{k+2}))) &= |\frac{x_1 + x_k}{4k} - \frac{x_3 + x_{k+2}}{4k}| \\ &\leq \frac{1}{2k} \max\{x_k, x_{k+2}\} \\ &\leq \frac{1}{2} \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \\ &= \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\}. \end{aligned}$$

When some $x_j \in [1, 2]$ *and* $x_1, x_2, ..., x_{j-1}, x_{j+1}, ..., x_k, x_{k+1}, x_{k+2} \in [0, 1)$ *or* $x_1, x_2, ..., x_k, x_{k+1}, x_{k+2} \in [1, 2]$ *then we obtain*

$$\begin{aligned} G_p(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1}), T(x_3, x_4, ..., x_{k+2}))) &= 0 \\ &\leq \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\}. \end{aligned}$$

Thus T *satisfies (2.4) with* $\lambda = \frac{1}{2}$ *and we have* $T(x, x, ..., x) = x$. *Moreover for all* $x, y, z \in X$ *with* $x \neq y \neq z$

$$G_p(T(x, x, ..., x), T(y, y, ..., y), T(z, z, ..., z)) < G_p(x, y, z).$$

Thus all required hypotheses of Theorem (2.3) are satisfied. Furthermore, for any arbitrary points $x_1, x_2, ..., x_k \in X$, *the sequence* (x_n) *defined by (2.5) converges to* $x = 0$, *the unique fixed point of* T.

Corollary 2.5. *Let* (X, G_p) *be complete* G_p*-metric spaces,* $k \in Z^+$ *and* $T : X^k \to X$ *a mapping satisfying the following contractive type condition*

$$G_p(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1}), T(x_3, x_4, ..., x_{k+2}))) \leq \sum_{i=1}^{k} \lambda_i G_p(x_i, x_{i+1}, x_{i+2}) \qquad (2.9)$$

where $\lambda_1, \lambda_2, \ldots, \lambda_k$ *are non-negative constants,* $\sum_{i=1}^{k} \lambda_i \in (0, 1)$ *and* $x_1, x_2, ..., x_k$, *are arbitrary elements in* X. *Then there exists a point* x *in* X *such that* $T(x, x, ..., x) = x$. *Moreover, if* $x_1, x_2, ..., x_{k+2}$ *are arbitrary points in* X *and* $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, ..., x_{n+k-1}) \quad (n = 1, 2, ...) \qquad (2.10)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ *is convergent and*

$$\lim_{n \to \infty} x_n = T(\lim x_n, \lim x_n, ..., \lim x_n).$$

If in addition we suppose that

$$G_p(T(u, u, ..., u), T(v, v, ..., v), T(w, w, ..., w)) < G_p(u, v, w) \qquad (2.11)$$

holds for all $u, v, w \in X$, *with* $u \neq v \neq w$, *then* x *is the unique point in* X *with* $T(x, x, ..., x) = x$.

Remark 2.1. Theorem 2.3 is generalization of corollary 2.5, as the condition (2.9) implies the conditions (2.4) and (2.6). Actually,

$$\begin{aligned} &\lambda_1 G_p(x_1, x_2, x_3) + \lambda_2 G_p(x_2, x_3, x_4) + \cdots + \lambda_k G_p(x_i, x_{i+1}, x_{i+2}) \\ &\qquad \leq (\lambda_1 + \lambda_2 + \cdots + \lambda_k) \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \qquad (2.12) \end{aligned}$$

and $\lambda_1 + \lambda_2 + \cdots + \lambda_k < 1$. Beside, for any $u, v, w \in X$, with $u \neq v \neq w$, from (2.9) we have

$$\begin{aligned}
G_p(T(u,u,...,u),&T(v,v,...,v),T(w,w,...,w)) \leq G_p(T(u,u,...,u),T(u,u,...,u,v),T(u,u,...,u,v))+\\
&G_p(T(u,u,...,u,v),T(u,u,...,u,v,v),T(u,u,...,u,v,v)) + ...+\\
&G_p(T(v,w,...,w),T(w,w,...,w),T(w,w,...,w))-\\
&\{G_p(T(u,u,...,u,v),T(u,u,...,u,v),T(u,u,...,u,v))+\\
&G_p(T(u,u,...,u,v,v),T(u,u,...,u,v,v),T(u,u,...,u,v,v)) + ...+\\
&G_p(T(u,w,...,w),T(u,w,...,w),T(u,w,...,w))\}\\
\leq &G_p(T(u,u,...,u),T(u,u,...,u,v),T(u,u,...,u,v))+\\
&G_p(T(u,u,...,u,v),T(u,u,...,u,v,v),T(u,u,...,u,v,v)) + ...+\\
&G_p(T(v,w,...,w),T(w,w,...,w),T(w,w,...,w))\\
\leq &\lambda_1 G_p(u,v,w) + \lambda_2 G_p(u,v,w) + \cdots + \lambda_k G_p(u,v,w)\\
= &(\lambda_1 + \lambda_2 + \cdots + \lambda_k) G_p(u,v,w) < G_p(u,v,w)
\end{aligned}$$

and, as a result, (2.9) implies (2.6).

3 Conclusion

Nazır and Abbas [21], proved common fixed point theorems of Presic type in partial metric space. Further Dhasmana [22], showed fixed point theorem by using Presic type mapping in G-metric spaces. Our works generalizes several similar results in the literature.

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Competing Interests

Authors have declared that no competing interests exist.

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