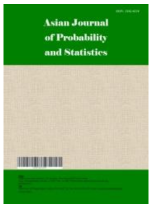

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Two-Parameter Nwikpe (TPAN) Distribution with Application

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Authors' contributions

This work was carried out in collaboration among all authors. Author BJN designed the study, derived the new distribution, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author IDE managed the analyses of the study. All authors read and approved the final manuscript

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Abstract

A new two-parameter continuous distribution called the Two-Parameter Nwikpe (TPAN) distribution is derived in this paper. The new distribution is a mixture of gamma and exponential distributions. A few statistical properties of the new probability distribution have been derived. The shape of its density for different values of the parameters has also been established. The first four crude moments, the second and third moments about the mean of the new distribution were derived using the method of moment generating function. Other statistical properties derived include; the distribution of order statistics, coefficient of variation and coefficient of skewness. The parameters of the new distribution were estimated using maximum likelihood method. The flexibility of the Two-Parameter Nwikpe (TPAN) distribution was shown by fitting the distribution to three real life data sets. The goodness of fit shows that the new distribution outperforms the one parameter exponential, Shanker and Amarendra distributions for the data sets used for this study.

Keywords: Probability distribution; TPAN distribution; statistical properties and goodness of fit.

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1 Introduction

Due to the deficiencies of some classical distribution in modelling real life data sets, deriving new distributions by modifying existing distributions using different generators has been proposed by statisticians in recent times. Such generalized family include the beta-generated family of distributions proposed by Eugene et al. [1], Kumaraswamy generalized family by Cordeiro and de Castro [2], the Transmuted family of distributions established by Shaw and Buckley [3] The Exponentiated Generalised (EG) family of distributions derived by Cordeiro et al. [4], the Gamm-G (Type I) family of distributions derived by Zografos and Balakrishnan [5], The Gamma-G (Type II) family of distribution given by Ristic and Balakrishnan [6], The Log-Gamma-G family of distribution proposed by Amini et al. [7], The Exponentiated T-X family of distributions introduced by Alzaghal et al. [8] and the Marshall-Olkin family of distributions proposed by Marshall and Olkin [9] amongst others. It has been established that most of the distributions obtained by generalization performs better than their baseline distribution(s). For instance, generalized the Gompertz distribution using Beta-generalized family, by applying the distribution to a data set on lifetime of fifty devices, it was revealed that the Beta Gompertz distribution is more flexible than its baseline distribution and other sub-models.

Nwikpe et al. [10] recently developed a new continuous probability distribution called the Nwikpe distribution. The Nwikpe distribution is a single parameter distribution derived by taken a two component additive mixture of gamma and exponential distributions. One of the short falls of the Nwikpe distribution is that the distribution has a single parameter. Ogutunde in [11] observed that introducing more parameters into an existing distribution enhances the flexibility of the distribution. Thus, the aim of this paper is to develop a new distribution with two parameters. The new distribution is obtained by taken a three component additive mixture of gamma and exponential distributions, the resultant mixed model is a two parameter probability distribution called the Two-Parameter Nwikpe (TPAN) distribution.

2 Two-Parameter Nwikpe (TPAN) Distribution

Let X denote a continuous random variable, X is said to follow the Two-Parameter Nwikpe distribution if its PDF is given by:

$$f(y;\theta,\alpha)=\frac{\theta^3}{(\theta^5+2\alpha+6)}(\theta y^3+\alpha y^2+\theta^3)e^{-\theta y}\quad y>0,\theta>0\,,\alpha\,\theta>0 \tag{1}$$

The Two-Parameter Nwikpe Distribution is a three-component mixture of gamma distribution with shape and scale parameters 3 and θ receptively, gamma$(4,\theta)$ and exponential distribution with parameter θ. Equation (1) is btained using:

$$f(y;\theta,\alpha)=f_1(y;\theta)p_1+f_2(y;\theta)p_2+f_3(y;\theta)p_3 \tag{2}$$

where

$$f_1(y;\theta)=\theta e^{-\theta y},f_2(y;\theta)=\frac{\theta^3y^2}{\Gamma(3)}e^{-\theta y},\;f_3(y;\theta)=\frac{\theta^4y^3}{\Gamma(4)}e^{-\theta y}$$

And $\frac{\theta^5}{(\theta^5+2\alpha+6)},\frac{2\alpha}{(\theta^5+2\alpha+6)}\;and\;\frac{6}{(\theta^5+2\alpha+6)}$ are the mixture proportion$s(p_i)$, i =1,2,3

3 The CDF of the Two-Parameter Nwikpe (TPAN) Distribution

For any random variable Y which follows the Two-parameter Nwikpe distribution in equation (1), its cumulative distribution function CDF is given as;

$$F(y;\theta,) = \left[1-\left(1+\frac{\theta^{3}y^{3}+(\alpha+3)\left(\theta^{2}y^{2}+2\theta y\right)}{(\theta^{5}+2\alpha+6)}\right)e^{-\theta y}\right] \quad (3)$$

By integrating equation (1), equation (3) is obtained.

Theorem: The Two-Parameter Nwikpe Distribution is a Proper PDF

This implies that

$$\int_{0}^{\infty}(y;\theta,\alpha) = 1$$

$$\int_{0}^{\infty} f(y;\theta,\alpha) = \frac{\theta^{3}}{(\theta^{5}+2\alpha+6)}\int_{0}^{\infty}(\theta^{3}+\alpha y^{2}+\theta\, y^{3})e^{-\theta y}\,dy$$

$$= \frac{\theta^{3}}{(\theta^{5}+2\alpha+6)}\left(\frac{\theta^{3}}{\theta}+\frac{\alpha\Gamma(3)}{\theta^{3}}+\frac{\theta\Gamma(4)}{\theta^{4}}\right)$$

$$= \frac{\theta^{3}}{(\theta^{5}+2\alpha+6)}\left(\theta^{2}+\frac{2\alpha}{\theta^{3}}+\frac{6}{\theta^{3}}\right)$$

$$= \frac{\theta^{3}}{(\theta^{5}+2\alpha+6)}\left(\frac{\theta^{5}+2\alpha+6}{\theta^{3}}\right) = 1$$

4 Graph of the CDF of the Two-Parameter Nwikpe distribution

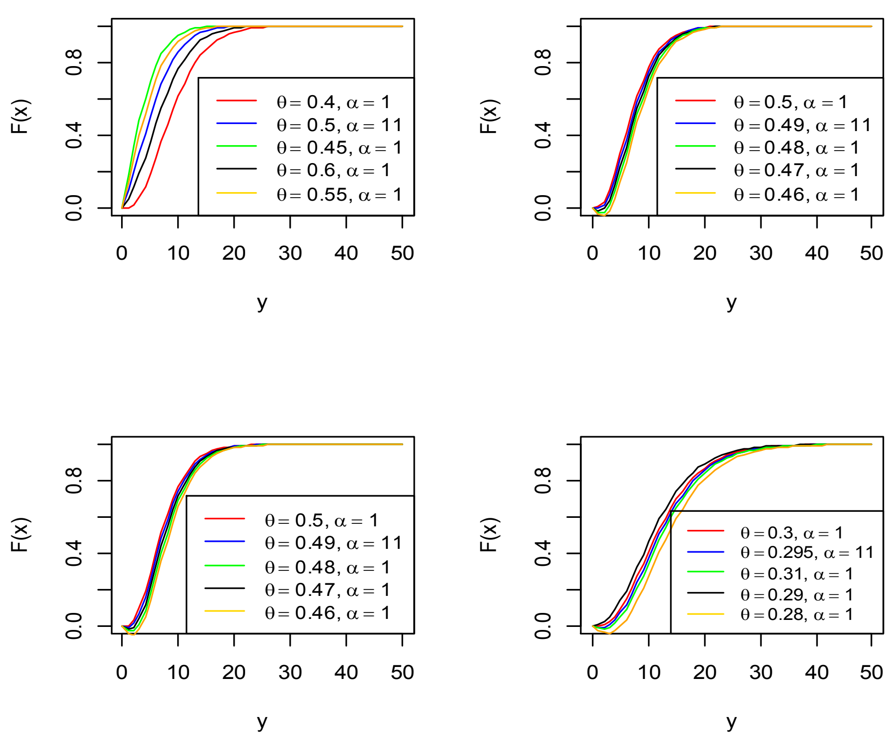

Fig. 1. Graphs of the CDF of the Two-Parameter Nwikpe Distribution

5 The Graphs of PDF of the TPAN Distribution

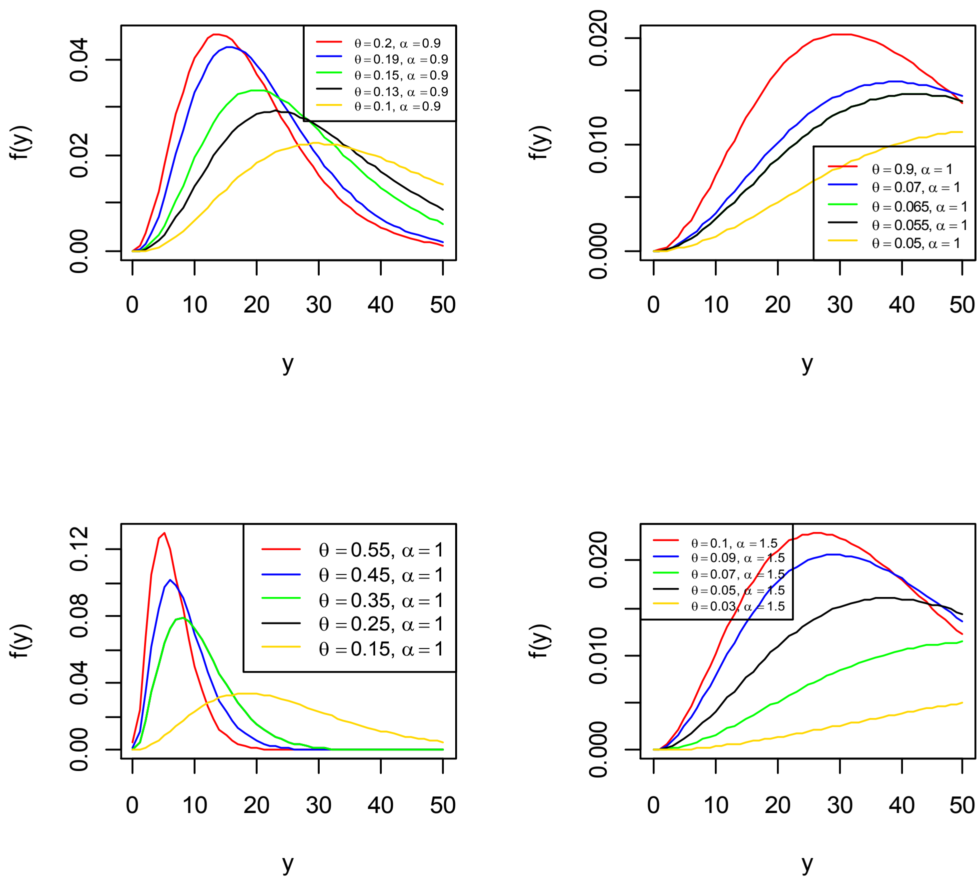

Fig. 2. Graphs of the PDF of the two-parameter Nwikpe distribution

6 Hazard Function or Failure Rate of the Two-Parameter Nwikpe Distribution

By definition, the hazard function of a random variable X isdefined as

$$h(x) = \frac{f(x)}{1 - F(x)}$$

For any random variable X which follows the Two-Parameter Nwikpe distribution, its hazard function is given as;

$$h(x) = \frac{\dfrac{\theta^3(\theta^3 + \alpha y^2 + \theta y^3)e^{-\theta y}}{(\theta^5 + 2\alpha + 6)}}{\left\{1 - \left[1 - \left(1 + \dfrac{\theta^3 y^3 + (\alpha + 3)(\theta^2 y^2 + 2\theta y)}{(\theta^5 + 2\alpha + 6)}\right) e^{-\theta y}\right]\right\}}$$

$$= \frac{\theta^3(\theta^3 + \alpha y^2 + \theta\, y^3)}{\theta^{\,3} y^3 + (\alpha + 3)\left(\theta^{\,2} y^2 + 2\theta y\right) + (\theta^5 + 2\alpha + 6)} \quad (3)$$

7 Graph of the Hazard Function of the Two-Parameter Nwikpe Distribution

Fig. 3 depict the graph of the two-parameter Nwikpe distribution for the different values of α and θ.

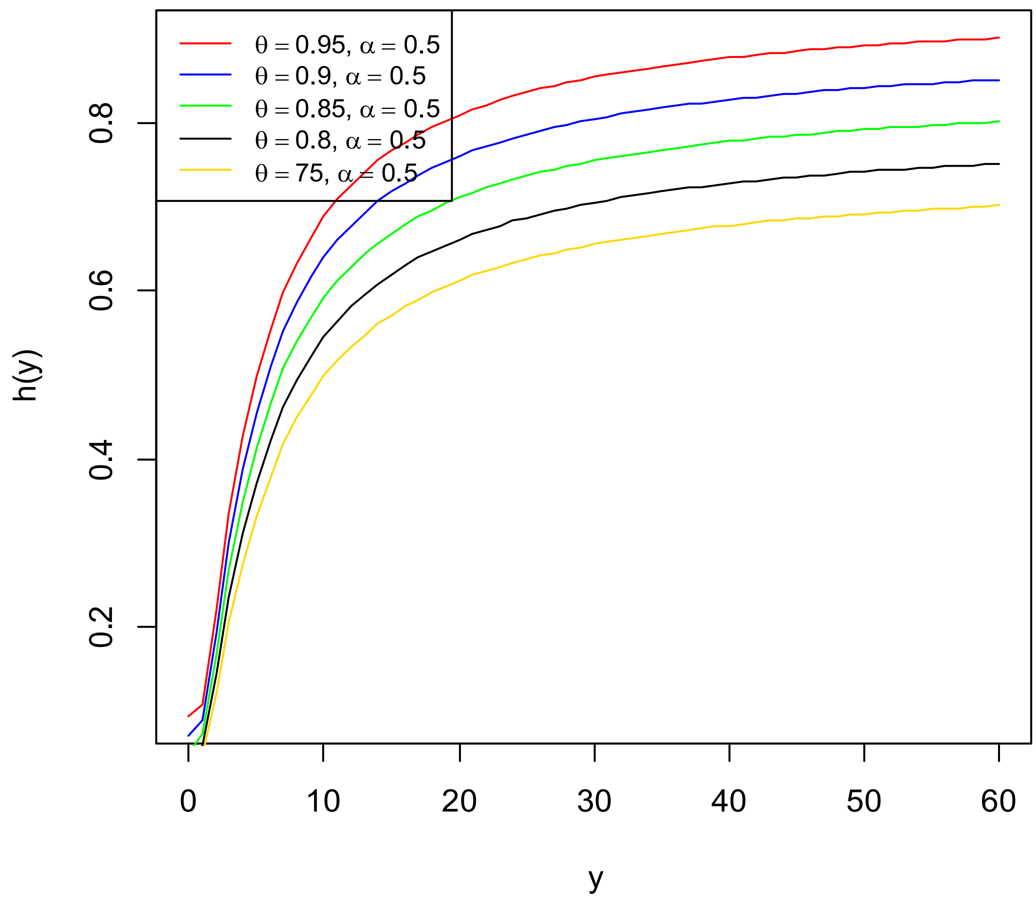

Fig. 3. Graph of the hazard function of the two-parameter Nwikpe distribution

8 The Moment Generating Function of the Two-Parameter Nwikpe Distribution

The moment generating function of a random variable X is defined as

$$M_X(t) = E(e^{xt}) = \int_{-\infty}^{\infty} e^x f(x)dx$$

For a random variable$X$, whose PDF is the PDF of the Two-Parameter Nwikpe distribution,

$$M_X(t) = \frac{\theta^3}{(\theta^5 + 2\alpha + 6)} \int_0^{\infty} (\theta^3 + \alpha y^2 + \theta\, y^3) e^{-\theta y} e^{yt} dy$$

$$= \frac{\theta^3}{(\theta^5 + 2\alpha + 6)} \left(\frac{\theta\Gamma(4)}{(\theta - t\,)^4} + \frac{\alpha\Gamma(3)}{(\theta - t\,)^3} + \frac{\theta^3}{(\theta - t\,)} \right)$$

$$= \frac{\theta^3}{(\theta^5 + 2\alpha + 6)} \left(\frac{6\theta}{(\theta - t\,)^4} + \frac{2\alpha}{(\theta - t\,)^3} + \frac{\theta^3}{(\theta - t\,)} \right)$$

$$= \frac{\theta^3}{(\theta^5 + 2\alpha + 6)} \left\{ \frac{6\theta}{\theta^4} \sum_{k=0}^{\infty} \binom{k+3}{k} \left(\frac{t}{\theta}\right)^k + \frac{2\alpha}{\theta^3} \sum_{k=0}^{\infty} \binom{k+2}{k} \left(\frac{t}{\theta}\right)^k + \frac{\theta^3}{\theta} \sum_{k=0}^{\infty} \left(\frac{t}{\theta}\right)^k \right\}$$

$$= \left(\frac{\theta^3}{(\theta^5 + 2\alpha + 6)} \right) \sum_{k=0}^{\infty} \left(\frac{(k+3)(k+2)(k+1)}{\theta^3} + \frac{\alpha(k+2)(k+1)k!}{\theta^3} + \theta^2 \right) \left(\frac{t}{\theta}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{(k+3)(k+2)(k+1) + \alpha(k+2)(k+1) + \theta^5}{(\theta^5 + 2\alpha + 6)} \right) \left(\frac{t}{\theta}\right)^k$$

The kth moments about the origin μ_k' are the coefficients of $\frac{t}{\theta}$ in the expression above.

$$\mu_k' = \frac{k!\,[(k+3)(k+2)(k+1) + \alpha(k+2)(k+1) + \theta^5]}{\theta^k(\theta^5 + 2\alpha + 6)} \tag{6}$$

From (6) the first four crude moments f the distribution is obtained as follows:

$$\mu_1' = \left(\frac{24 + 6\alpha + \theta^5}{\theta\,(\theta^5 + 2\alpha + 6)} \right), \mu_2' = \frac{120 + 24\alpha + 2\theta^5}{\theta^2\,(\theta^5 + 2\alpha + 6)} \quad \mu_3' = \frac{720 + 120\alpha + 6\theta^5}{\theta^3\,(\theta^5 + 2\alpha + 6)}\ and$$

$$\mu_4' = \frac{5040 + 720\alpha + 24\theta^5}{\theta^3\,(\theta^5 + 2\alpha + 6)}$$

9 The Second Moments about the Mean of the Two-Parameter Nwikpe Distribution

The kth corrected moment or the moment about the mean of the a random variable Y is defined as:

$$\mu_k = E(y - \mu)^k \tag{7}$$

Clearly, the first central moment is zero.

From equation (7), the second central moment of the Two-parameter N-E distribution isgiven as:

$$\mu_2 = Var(y) = E(Y^2) - E(Y)^2$$

$$= \frac{120 + 24\alpha + 2\theta^5}{\theta^2\,(\theta^5 + 2\alpha + 6)} - \left(\frac{24 + 6\alpha + \theta^5}{\theta\,(\theta^5 + 2\alpha + 6)} \right)^2$$

$$= \frac{\theta^{10}+8\ 4\theta^5+16\theta^5\alpha+12\alpha^2+96\alpha+144}{\theta^2\ (\theta^5+2\alpha+6)^2} \quad (8)$$

10 The Third Central Moment of the Two-Parameter Nwikpe Distribution

From definition, the third central moment of a random variable X which follows the Two-Parameter Nwikpe distribution is

$$\mu_3 = E(x-\mu)^3$$

$$= \mu_3' - 3\mu_1'\mu_2' + 2\mu_1^3$$

Where μ_1', μ_2' and μ_3' are as defined in section 7 above

$$\mu_3 = \frac{720+120\alpha+6\theta^5}{\theta^3\,(\theta^5+2\alpha+6)} - 3\left(\frac{120+24\alpha+2\theta^5}{\theta^2\,(\theta^5+2\alpha+6)}\right)\left(\frac{24+6\alpha+\theta^5}{\theta\,(\theta^5+2\alpha+6)}\right) + 2\left(\frac{24+6\alpha+\theta^5}{\theta\,(\theta^5+2\alpha+6)}\right)^3$$

$$= \frac{32\theta^{15}+756\theta^{10}+180\theta^{10}\alpha+592\theta^5\alpha^2+14328\theta^5\alpha+4\,8\alpha^3}{\theta^3(\theta^5+2\alpha+6)^3} + \frac{576\alpha^2+1728\theta^5+1728\alpha+1728}{\theta^3(\theta^5+2\alpha+6)^3} \quad (9)$$

11 Distribution of Order Statistics of the Two-Parameter Nwikpe Distribution

Let $X_{1,}X_2, X_3 \ldots X_{n,}$ be an n-dimensional random sample from a distribution whose PDF if $f(y)$, suppose thecorresponding order statistics obtained from the sample is $Y_{1:n}Y >_{2:n} > Y_{3:n} > \cdots > Y_{n:n}$. By definition, the density of the kth order statistics is given as:

$$f_{X:n}(\mathrm{y}) = \frac{n!}{(k-1)!\,(n-k)!}\sum_{i=o}^{n}\binom{n-k}{i}(-1)^i\big(F(y)\big)^{k-1+i}f(y)$$

If Y has the PDF of the Two-Parameter Nwikpe distribution then,

$$F(y;\theta,\alpha) = \left[1-\left(1+\frac{\theta^3y^3+(\alpha+3)(\theta^2y^2+2\theta y)}{(\theta^5+2\alpha+6)}\right)e^{-\theta y}\right]$$

and

$$f(y;\theta,\alpha) = \frac{\theta^3}{(\theta^5+2\alpha+6)}(\theta y^3+\alpha y^2+\theta^3)e^{-\theta y}$$

$$f_{y:n}(\mathrm{y}) = \frac{n!}{(k-1)!\,(n-k)!}\sum_{i=o}^{n}\binom{n-k}{i}(-1)^i \times$$

$$\left\{1-\left(1+\frac{\theta^{3}y^{3}+(\alpha+3)(\theta^{2}y^{2}+2\theta y)}{(\theta^{5}+2\alpha+6)}\right)e^{-\theta y}\right\}^{k-1+i}\times\left(\frac{\theta^{3}(\theta^{3}+\alpha y^{2}+\theta\, y^{3})e^{-\theta y}}{(\theta^{5}+2\alpha+6)}\right)$$

$$=\frac{n!\,\theta^{3}(\theta^{3}+\alpha y^{2}+\theta\, y^{3})e^{-\theta y}}{(k-1)!\,(n-k)!\,(\theta^{5}+2\alpha+6)}\sum_{i=1}^{n}\binom{n-k}{i}(-1)^{i}\sum_{i=1}^{m}\binom{k-1+i}{j}(-1)^{j}\times$$
$$\left(1+\frac{\theta^{3}y^{3}+(\alpha+3)(\theta^{2}y^{2}+2\theta y)}{(\theta^{5}+2\alpha+6)}\right)^{j}e^{-\theta yj}$$

Recall

$$(1+x)=\sum_{\mathrm{r=o}}^{\mathrm{p}}\binom{p}{r}\mathrm{x}^{\mathrm{r}}\text{, let } k-1+i=m$$

$$f_{y:n}(\mathrm{y})=\frac{n!\,\theta^{3}(\theta^{3}+y^{2}+\theta\, y^{3})e^{-\theta y}}{(k-1)!\,(n-k)!\,(\theta^{5}+2\alpha+6)}\sum_{i=0}^{n}\sum_{j=0}^{m}\binom{n-k}{i}\binom{m}{j}(-1)^{i+j}\times$$
$$\left(1+\frac{\theta^{3}y^{3}+(\alpha+3)\left(\theta^{2}y^{2}+2\theta y\right)}{(\theta^{5}+2\alpha+6)}\right)^{j}\qquad(10)$$

12 Coefficient of Variation of the Two-Parameter Nwikpe (TPAN) Distribution

The coefficient of variation of a random variable X, is given by

$$C.V=\frac{\sqrt{E(X-\mu)^{2}}}{E(X)}=\frac{\sigma}{\mu_{1}'}$$

Thus, for a random variable X , which follows the TPAN distribution, the coefficient of variation is:

$$C.V=\sqrt{\frac{\theta^{10}+84\theta^{5}+16\theta^{5}\alpha+12\alpha^{2}+96\alpha+144}{\theta^{2}\ (\theta^{5}+2\alpha+6)^{2}}}\div\frac{24+6\alpha+\theta^{5}}{\theta\,(\theta^{5}+2\alpha+6)}$$

$$=\frac{\sqrt{(\theta^{10}+84\theta^{5}+16\theta^{5}\alpha+12\alpha^{2}+96\alpha+144)}}{24+6\alpha+\theta^{5}}\qquad(11)$$

13 Coefficient of Skewness of the Two-Parameter Nwikpe (TPAN) Distribution

If X follows the two-parameter Nwikpe distribution its coefficient of skewness is computed as follows:

$$\beta_{2}=\frac{E(X-\mu)^{3}}{\sigma^{3}}=\frac{\mu_{3}}{((\mu_{2})^{1/2})^{3}}$$

Where μ_2 and μ_3 are the variance and third central moment of the two parameter N-E distribution respectively.

$$\beta_2 = \frac{\frac{(720+120\alpha+6\theta^5)(\theta^5+2\alpha+6)^2-3(120+24\alpha+2\theta^5)(24+6\alpha+\theta^5)(\theta^5+2\alpha+6)+2(24+6\alpha+\theta^5)^3}{\theta^3(\theta^5+2\alpha+6)^3}}{\left(\frac{\theta^{10}+84\theta^5+16\theta^5\alpha+12\alpha^2+96\alpha+144}{\theta^2\ (\theta^5+2\alpha+6)^2}\right)^{3/2}}$$

$$= \frac{(720+120\alpha+6\theta^5)(\theta^5+2\alpha+6)^2+2(24+6\alpha+\theta^5)^3}{(\theta^{10}+84\theta^5+16\theta^5\alpha+12\alpha^2+96\alpha+144)^{\frac{3}{2}}}$$

$$+\frac{-3(120+24\alpha+2\theta^5)(24+6\alpha+\theta^5)(\theta^5+2\alpha+6)}{(\theta^{10}+84\theta^5+16\theta^5\alpha+12\alpha^2+96\alpha+144)^{\frac{3}{2}}} \quad (12)$$

14 Index of Dispersion of the Two-Parameter Nwikpe Distribution

The variance to mean ratio of the Two-Parameter Nwikpe distribution is given as follows:

$$\beta_3 = \frac{\sigma^2}{\mu_1'}$$

$$= \frac{\theta^{10}+84\theta^5+16\theta^5\alpha+12\alpha^2+96\alpha+144}{\theta^2\ (\theta^5+2\alpha+6)^2} \div \left(\frac{24+6\alpha+\theta^5}{\theta\ (\theta^5+2\alpha+6)}\right)$$

$$= \frac{\theta^{10}+84\theta^5+16\theta^5\alpha+12\alpha^2+96\alpha+144}{\theta\ (\theta^5+2\alpha+6)\ (24+6\alpha+\theta^5)} \quad (13)$$

15 Parameter Estimation for theTwo-Parameter Nwikpe Distribution

Given a random sample $y_1, y_2, \dots, y_n$ of size n from the Two-Parameter Nwikpe distribution with PDF$f(y;\theta)$, the likelihood function, L is defined as

$$L(y,\theta,\alpha) = \prod_{i=1}^{n} f(y_i;\theta,\alpha) = \prod_{i=1}^{n}\left(\frac{\theta^3}{(\theta^5+2\alpha+6)}(\theta {y_i}^3+\alpha {y_i}^2+\theta^3)e^{-\theta y_i}\right)$$

$$= \left(\frac{\theta^3}{(\theta^5+2\alpha+6)}\right)^n \prod_{i=1}^{n}\left\{(\theta {y_i}^3+\alpha {y_i}^2+\theta^3)e^{-\theta\sum_{i=1}^{n} y_i}\right\}$$

By taking the log of both sides we get;

$$\log\left(L(y,\theta)\right) = n log\left(\frac{\theta^3}{(\theta^5+2\alpha+6)}\right) + \sum_{i=1}^{n} log(\theta {y_i}^3+\alpha {y_i}^2+\theta^3) - \theta\sum_{i=1}^{n} y_i$$

$$\frac{\partial lL(y,\theta)}{\partial\theta} = \frac{3n}{\theta} - \frac{5\theta^4 n}{\theta^5+2\alpha+6} - \sum_{i=1}^{n} y_i + \sum_{i=1}^{n}\frac{{y_i}^3+3\theta^2}{\theta {y_i}^3+\alpha {y_i}^2+\theta^3}$$

$$= \frac{4n}{\theta} - \frac{5\theta^4 n}{\theta^5+2\alpha+6} - \sum_{i=1}^{n} y_i + \sum_{i=1}^{n}\frac{{y_i}^3+3\theta^2}{\theta {y_i}^3+\alpha {y_i}^2+\theta^3} = 0 \quad (14)$$

$$\frac{\partial lL(y,\theta)}{\partial\alpha} = -\frac{2n}{\theta^5+2\alpha+6} + \sum_{i=1}^{n}\frac{{y_i}^2}{\theta {y_i}^3+\alpha {y_i}^2+\theta^3}$$

$$-\frac{2n}{\theta^5+2\alpha+6}+\sum_{i=1}^{n}\frac{{y_i}^2}{\theta {y_i}^3+\alpha {y_i}^2+\theta^3} \;=\; 0 \qquad (15)$$

The solution of equation (14) and (15) gives the maximum likelihood estimates of the parameters of the two-parameter Nwikpe distribution. However, the equations cannot be solved analytically thus, was solved numerically using R programming with some data set.

16 Application of the Two-Parameter Nwikpe (TPAN) Distribution

To determine the applicability, and flexibility of the Two- Parameter Nwikpe distribution,the TPAN distribution was applied to three data sets to determine its goodness of fit. The goodness of fit of the distribution was compared with exponential, Lindley, Akash, Amarendra, Sujatha and Shanker distributions using Akaike Information Criterion(AIC), Bayesian Information Criterion (BIC), and -2ln$L$. The distribution with the smallest AIC, BIC and -2InL is regarded as the most flexible and superior distribution. The results are shown in the tables below.

Data Set 1

The first data set represent the length of time (in years) that 81 randomly selected Nigerian graduates stayed without job before been employed by the universal basic education commission in 2011.

2,5,7,5,6,7,7,6,6,9,9,6,6,7,5,4,5,2,9,8,5,9,6,6,7,2,8,3,6,6,2,8,5,7,4,5,6,8,8,9,3,7,6,2,6,8,9,7,6,6,9,5,9,5,5,5,3,9, 8,6,6,6,7,9,4,4,6,9,7,8,8,9,4,6,3,5,4,7,6,6,5

Data Set 2

The third data is the duration data relating to relief times in minutes of patients receiving analgesics. The data set was given by Gross and Clark in Shanker [12,13], In recent time, the data has been used by to fit the Amerandra distribution. The data set consists of twenty (20) observations and it is as follows:

1.1,1.4,1.3,1.7,1.9,1.8,1.6,2.2, 1.7, 2.7,4.1,1.8,1.5,1.2,1.4,3,1.7,2.3,1.6,2

Data Set 3

The fifth data is the tensile strength, measured in GPa, of sixty-nine (69) carbon fibres tested under tension at gauge lengths of 20 mm. According to Shanker [12,13], the data was reported by Bader and Priest, in [12,13]. The data have been used by Shanker [12,13] to fit the sujatha distribution. The data set is given as follows:

1.312,1.314,1.479,1.552,1.700,1.803,1.861 1.865 1.944 1.958 1.966 1.997 2.006 2.021 .027 2.055 2.063 2.098 2.140 2.179 2.224 2.240 2.253 2.270 2.272 2.274 2.301 2.301 2.359 2.382 2.382 2.426 2.434 2.435 2.478 2.490 2.511 2.514 2.535 2.554 2.566 2.570 2.586 2.629 2.633 2.642 2.648 2.684 2.697 2.726 2.770 2.773 2.800 2.809 2.818 2.821 2.848 2.880 2.954 3.012 3.067 3.084 3.090 3.096 3.128 3.233 3.433 3.585 3.585

Table 1. Goodness of Fit of the two-parameter Nwikpe distribution for data set 1

Model	Parameter estimate	-2inL	AIC	BIC	AICC	Rank
Exponential	0.1639676	454.913	456.91	459.3044	456.959	7
TPAN	0.7315012 -1.2714104	358.8496	362.8496	362.666	363.003	1
Shanker	0.308451	408.922	410.9216	410.83	410.9722	5
Lindley	0.290978	418.578	420.578	420.478	420.6286	6
Amarendra	0.6015208	373.971	375.9707	375.879		2
Sujatha	0.4403537	392.3864	394.3863	394.2949	394.4369	4
Akash	0.460469	388.6078	390.6073	390.5162	390.56683	3

Table 2. Goodness of fit of the two-parameter Nwikpe distribution for data set 2

Model	Parameter estimate	-2inL	AIC	BIC	AICC	Rank
Exponential	0.5263	65.7	67.6777	68.7	67.9	7
TPAN	1.571863 1237.377732	-22.94263	49.88525	51.48755	52.1934	1
Shanker	0.8038668	59.78	61.78332	64.3852	65.091	6
Lindley	0.8161	60.50	62.50	63.49	62.72	5
Amarendra	1.4807	55.64	57.64	58.63	57.86	2
Sujatha	1.1367	57.50	59.50	60.49	59.72	3
Akash	1.1569	59.50	61.70	61.72	61.72	4

Table 3. Goodness of fit of the two-parameter Nwikpe distribution for data set 3

Model	Parameter estimate	-2inL	AIC	BIC	AICC	Rank
Exponential	0.407941	261.7432	263.7411	265.9655	263.80112	7
TPAN	1.222463 826.921036	184.77	188.6774	188.447	188.855	1
Shanker	0.6580296	233.0054	235.0054	237.2376	235.01	5
Lindley	0.65900001	238.3667	240.3659	242.6134	240.44	6
Amarendra	1.244256	207.947	209.947	209.786	210.01	2
Sujatha	0.9361194	221.6088	223.6088	225.8355	223.6688	3
Akash	0.9647255	224.2798	226.2797	228.51323	226.34234	4

17 Discussion and Conclusion

A new two-parameter, distribution called the Two-Parameter Nwikpe(TPAN) distribution is derived in this paper. The graph of the cumulative distribution function in Fig. 1 shows that the corresponding PDF is a proper PDF. The graph of its hazard rate function is given in Fig. 3 reveals that the new distribution has an increasing hazard function thus, suitable for modelling data set from real life situation characterized by increasing hazard rate. The graphs of the probability density function in Fig. 2 reveal that the distribution is asymmetric. Table 1 shows the goodness of fit of the TPAN distribution and other competing distributions for the first data set used in this study. The Table reveals that the TPAN distribution has the smallest AIC, BIC sand -2InL. Thus, considered to be more flexible and superior to the exponential, Lindley, Akash Sujatha, Amarendra and Shanker distributions for the data set. Similarly, it could be deduced from table Table 2 and Table 3 that the TPAN distribution has the smallest AIC, BICand -2InL for the second and third data sets respectively. This indicates that the TPAN distribution gave the best fit to all the data sets used in this study. Consequently, we conclude that the Two-Parameter Nwikpe (TPAN) distribution is the most flexible distributions for the data sets used in this study.

Competing Interests

Authors have declared that no competing interests exist.

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