Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 1550332, 8 pages
https://doi.org/10.1155/2022/1550332

Research Article

On Some New Common Fixed Point Results for Finite Number of Mappings in Fuzzy Metric Spaces

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Received 24 April 2022; Revised 27 June 2022; Accepted 29 July 2022; Published 17 August 2022

Academic Editor: Ranjan Kumar

We essentially suggest the concept of mutual sequences and Cauchy mutual sequence and utilize the same to prove the existence and uniqueness of common fixed point results for finite number of self- and non-self-mappings using fuzzy $\mathbb{Z}^*$-contractive mappings in fuzzy metric spaces. Our main result was obtained under generalized contractive condition in the fuzzy metric spaces. We provide examples to vindicate the claims and usefulness of such investigations. In this way, the present results generalize and enrich the several existing literature of the fuzzy metric spaces.

1. Introduction

In 1975, Kramosil and Michalek [1] introduced the notion of fuzzy metric spaces using the theory of fuzzy sets, which generalizes the metric spaces. Later on, many authors have introduced the notion of fuzzy metric spaces in different ways (see [2–5]). The widely accepted definition is given by George and Veeramani [6]. They presented slight modification on the definition of fuzzy metric spaces initiated by the respective authors by obtaining Hausdorff topology on the same setting. Utilizing the notion of the fuzzy metric, many authors proved various interesting common fixed point result for self- and non-self-mappings using different contraction in this setting. In 1984, Hadžić [7] proved some common fixed point theorems for family of mapping. After that, Bari and Vetro [8] also proved theorems for family of mappings in fuzzy metric spaces. In 1994, Subrahmanyam [9] generalized Jungck's theorem [10] in the setting of fuzzy metric spaces introduced by Kramosil and Michalek [1]. Vasuki [11] proved common fixed point theorems in the same setting. In 2002, Rhoades [12] proved common fixed point theorems for non-self-mappings using quasicontraction.

Jungck and Rhoades [13] introduced the concept of weak compatibility in metric spaces, which was further studied by Singh and Jain [14] in the fuzzy metric settings. Sedghi et al. [15] proved common fixed point theorems for four weakly compatible mappings. For the common fixed point using the notion of common limit range property, we refer common fixed point theorems by Chauhan et al. [16]. In this continuation, Imdad et al. [17] proved common fixed point theorems in fuzzy metric spaces using common property (E.A) and Prasad et al. [18] presented some coincidence point theorems via contractive mappings.

Recently, Roldan and Sintunavarat [19] introduced an important concept of fuzzy metric spaces on the product space $\mathcal{X}^{\mathcal{N}}$ which is induced by a simple fuzzy metric structure and compare the convergence, Cauchy, and completeness between these two structures. They also proved common fixed point results using CLRg property in the same metric setting. On the other hand, Shukla et al. [20] unify classes of different fuzzy contractive mappings presented in [21–24] and introduced a new class of fuzzy $\mathcal{Z}$-contractive mapping and notions of properties S and S' to prove fixed point results in the fuzzy metric spaces.

In this paper, firstly we define the mutual sequences and Cauchy mutual sequences. The idea behind defining Cauchy mutual sequence is to collect those Cauchy sequences which are converging to the same limit. After that, we utilize this idea to find common fixed points. Indeed, Cauchy mutual sequences in a fuzzy metric space $(\mathcal{X}, \mathcal{M}, *)$ are the special type of Cauchy sequences in fuzzy metric space $(\mathcal{X}^{\mathcal{N}}, \mathcal{M}^{\mathcal{N}}, *)$ which converge to the same limit $\varrho \in \mathcal{X}$, if they are convergent. We also generalize the $\mathbb{Z}$-contraction for finite number of mappings; using these contractive mappings, we will prove some unique common fixed point theorems in fuzzy metric spaces. The main aim of this paper is to prove unique common fixed point theorems using $\mathbb{Z}^*$-contraction (which is the extension of $\mathbb{Z}$-contraction for finite number of mappings) in fuzzy metric spaces for self- and non-self-mappings with the help of mutual sequences. We also give examples for validity of our claims. In this way, our results generalize and improve several existing results.

2. Preliminaries

Throughout this paper, $\mathcal{N}, n, m$ are natural numbers, $i \in (1, 2, .., \mathcal{N})$; $\mathcal{X}^{\mathcal{N}}$ will denote Cartesian product of $\mathcal{N}$-copies of $\mathcal{X}$ and $\mathcal{X}$ is any nonempty set. In the sequel, sometimes $\mathcal{T}(\varrho)$ will be denoted by $\mathcal{T}\varrho$.

Definition 1 (see [6]). An ordered triple $(\mathcal{X}, \mathcal{M}, *)$ is called a fuzzy metric space if $\mathcal{X}$ is a (nonempty) set, $\mathcal{M}$ is a fuzzy set on $\mathcal{X}^2 \times (0, \infty)$, and $*$ is a continuous t-norm satisfying the following conditions, for all $\varrho, \rho, z \in \mathcal{X}$ and $t, s > 0$:

(1) $\mathcal{M}(\varrho, \rho, t) > 0$

(2) $\mathcal{M}(\varrho, \rho, t) = 1$, if and only if $\varrho = \rho$

(3) $\mathcal{M}(\varrho, \rho, t) = \mathcal{M}(\rho, \varrho, t)$

(4) $\mathcal{M}(\varrho, z, t + s) \geq \mathcal{M}(\varrho, \rho, t) * \mathcal{M}(\rho, z, s)$

(5) $\mathcal{M}(\varrho, \rho, .)$: $(0, \infty) \longrightarrow (0, 1]$ is continuous

Definition 2 (see [6]).

(i) Let $(\mathcal{X}, \mathcal{M}, *)$ be a fuzzy metric space. A sequence (ϱ_n) is said to be converge to ϱ in $\mathcal{X}$ if and only if $\lim_{n \longrightarrow \infty} \mathcal{M}(\varrho_n, \varrho, t) = 1$ for all $t > 0$, i.e., for each $r \in (0, 1)$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{M}(\varrho_n, \varrho, t) > 1 - r$ for all $n \geq n_0$

(ii) A sequence (ϱ_n) in a fuzzy metric space $(\mathcal{X}, \mathcal{M}, *)$ is a Cauchy sequence if and only if for each $\epsilon > 0$, $t > 0$ there exists $n_0 \in \mathcal{N}$ such that $\mathcal{M}(\varrho_n, \varrho_m, t) > 1 - \epsilon$ for all $n, m > n_0$. On the other hand, (ϱ_n) is called a Cauchy sequence if $\lim_{n \longrightarrow \infty} \mathcal{M}(\varrho_n, \varrho_{n+m}, t) = 1$ for all $m \in \mathbb{N}$ and $t > 0$

(iii) A fuzzy metric space $(\mathcal{X}, \mathcal{M}, *)$ is said to be complete if every Cauchy sequence in $\mathcal{X}$ is convergent to some $\varrho \in \mathcal{X}$

Lemma 3 (see [19]). *Let $(\mathcal{X}, \mathcal{M}, *)$ be a fuzzy metric space and $\mathcal{X}^{\mathcal{N}} = \underbrace{\mathcal{X} \times \mathcal{X} \times \cdots \times \mathcal{X}}_{\mathcal{N}\ times}$, where $\mathcal{N} \in \mathbb{N}$ and define a fuzzy set $\mathcal{M}^{\mathcal{N}}$ on $\mathcal{X}^{\mathcal{N}} \times \mathcal{X}^{\mathcal{N}} \times [0, \infty) \longrightarrow [0, 1]$ such that*

$$\mathcal{M}^{\mathcal{N}}(\mathcal{P}, \mathcal{Q}, t) = *_{i=1}^{\mathcal{N}} \mathcal{M}(p_i, q_i, t), \quad \text{for all } \mathcal{P} = (p_1, p_2, \cdots, p_{\mathcal{N}}),\ \mathcal{Q} = (q_1, q_2, \cdots, q_{\mathcal{N}}) \in \mathcal{X}^{\mathcal{N}}, t > 0. \tag{1}$$

Then, the following hold:

(i) $(\mathcal{X}^{\mathcal{N}}, \mathcal{M}^{\mathcal{N}}, *)$ is also a fuzzy metric space

(ii) Let $(\mathcal{P}_n = (p_n^1, p_n^2, \cdots, p_n^{\mathcal{N}}))$ be a sequence on $\mathcal{X}^{\mathcal{N}}$ and $\mathcal{P} = (p^1, p^2, \cdots, p^{\mathcal{N}}) \in \mathcal{X}^{\mathcal{N}}$; then, sequence $(\mathcal{P}_n)$ converges to $\mathcal{P}$ on $(\mathcal{X}^{\mathcal{N}}, \mathcal{M}^{\mathcal{N}}, *)$ if and only if all sequences (p_n^i) converge to (p_i) on $(\mathcal{X}, \mathcal{M}, *)$, for all $i \in (1, 2, \cdots, \mathcal{N})$

(iii) Let $(\mathcal{P}_n = (p_n^1, p_n^2, \cdots, p_n^{\mathcal{N}}))$ be a sequence on $\mathcal{X}^{\mathcal{N}}$; then, $(\mathcal{P}_n)$ is Cauchy sequence on $(\mathcal{X}^{\mathcal{N}}, \mathcal{M}^{\mathcal{N}}, *)$ if and only if (p_n^i) is Cauchy sequence on $(\mathcal{X}, \mathcal{M}, *)$, for all $i \in (1, 2, \cdots, \mathcal{N})$

(iv) $(\mathcal{X}, \mathcal{M}, *)$ is complete if and only if $(\mathcal{X}^{\mathcal{N}}, \mathcal{M}^{\mathcal{N}}, *)$ is complete

Definition 4 (see [20]). Let $\mathcal{Z}$ denote the family of all functions $\zeta : (0, 1] \times (0, 1] \longrightarrow \mathbb{R}$ satisfying the following condition: $\zeta(t, s) > s$, for all $t, s \in (0, 1)$.

Definition 5 (see [20]). Let $\mathcal{T}$ be a self-mapping and $(\mathcal{X}, \mathcal{M}, *)$ a fuzzy metric space. If there exists $\zeta \in \mathcal{Z}$ such that

$$\mathcal{M}(\mathcal{T}\varrho, \mathcal{T}\rho, t) \geq \zeta(\mathcal{M}(\mathcal{T}\varrho, \mathcal{T}\rho, t), \mathcal{M}(\varrho, \rho, t)), \tag{2}$$

for all $\varrho, \rho \in \mathcal{X}$, $\mathcal{T}\varrho \neq \mathcal{T}\rho$, $t > 0$, then $\mathcal{T}$ is called a fuzzy $\mathcal{Z}$-contractive mapping with respect to the function $\zeta \in \mathcal{Z}$.

Definition 6 (see [20]). Let $\mathcal{T}$ be any self-mapping in $\mathcal{X}$, $\zeta \in \mathcal{Z}$ and $(\mathcal{X}, \mathcal{M}, *)$ a fuzzy metric space then quadruplet $(\mathcal{X}, \mathcal{M}, \mathcal{T}, \zeta)$ has the property (S'), if there exists $\varrho_n \in \mathcal{X}$ such that $\varrho_n = \mathcal{T}^n \varrho$, for all $n \in \mathbb{N}$ and $\inf_{m>n} \mathcal{M}(\varrho_n, \varrho_m, t) \leq \inf_{m>n} \mathcal{M}(\varrho_{n+1}, \varrho_{m+1}, t)$, for all $n \in \mathbb{N}$, $t > 0$ and $0 < \lim_{n \longrightarrow \infty} \inf_{m>n} \mathcal{M}(\varrho_n, \varrho_m, t) < 1$, for all $t > 0$ implies that

$$\lim_{n \longrightarrow \infty} \inf_{m>n} \zeta(\mathcal{M}(\varrho_{n+1}, \varrho_{m+1}, t), \mathcal{M}(\varrho_n, \varrho_m, t)) = 1, \quad \text{for all } t > 0. \tag{3}$$

3. Proposed Results

For brevity, we observe that Definitions 4 and 6 can be unified as follows.

Definition 7. Let $\mathbb{Z}^*$ denote the set of all functions $\zeta : (0,1] \times (0,1]$ to $\mathbb{R}$ satisfying the following conditions:

(i) $\zeta(p,q) > q$, for all~$p, q \in (0,1)$

(ii) Let (p_n) and (q_n) be two sequences in $(0,1]$ such that $p_n \leq q_n$, for all $n \in \mathbb{N}$ and $\lim_{n\longrightarrow\infty} p_n \in (0,1]$ and then $\lim_{n\longrightarrow\infty} \inf_{m>n} \zeta(q_n, p_n) = 1$

Example 1. Define $\zeta : (0,1] \times (0,1] \longrightarrow \mathbb{R}$ such that $\zeta(p,q) = q/p$ and $q_n = (1/2) - (1/3n)$, $p_n = (1/2) + (1/3n)$, we observe that $\zeta \in \mathbb{Z}^*$ as $\lim_{n\longrightarrow\infty} ((1/2) - (1/3n))/((1/2) + (1/3n)) = 1$.

Definition 8. Let $(\mathscr{X}, \mathscr{M}, *)$ be a fuzzy metric space and $\mathscr{T}_1, \mathscr{T}_2, \cdots, \mathscr{T}_\mathscr{N}$ are $\mathscr{N}$-mapping on $\mathscr{X}$ satisfying following condition:

$$\begin{aligned}&\mathscr{M}^\mathscr{N}\left(\left(\mathscr{T}_1\varrho^1, \mathscr{T}_2\varrho^2, \cdots, \mathscr{T}_\mathscr{N}\varrho^\mathscr{N}\right), \left(\mathscr{T}_2\rho^2, \mathscr{T}_3\rho^3, \cdots, \mathscr{T}_\mathscr{N}\rho^\mathscr{N}, \mathscr{T}_1\rho^1\right), t\right)\\ &\geq \zeta\left(\mathscr{M}^\mathscr{N}\left(\left(\mathscr{T}_1\varrho^1, \mathscr{T}_2\varrho^2, \cdots, \mathscr{T}_\mathscr{N}\varrho^\mathscr{N}\right), \left(\mathscr{T}_2\rho^2, \mathscr{T}_3\rho^3, \cdots, \mathscr{T}_\mathscr{N}\rho^\mathscr{N}, \mathscr{T}_1\rho^1\right), t\right), \mathscr{M}^\mathscr{N}\right.\\ &\left.\quad\times\left(\left(\varrho^1, \varrho^2, \cdots, \varrho^\mathscr{N}\right), \left(\rho^2, \rho^3, \cdots, \rho^\mathscr{N}, \rho^1\right), t\right)\right),\end{aligned} \tag{4}$$

for all $t > 0$, $(\varrho^1, \varrho^2, \cdots, \varrho^\mathscr{N}) \neq (\rho^1, \rho^2, \cdots, \rho^\mathscr{N}) \in \mathscr{X}$, $(\mathscr{T}_1\varrho^1, \mathscr{T}_2\varrho^2, \cdots, \mathscr{T}_\mathscr{N}\varrho^\mathscr{N}) \neq (\mathscr{T}_1\rho^1, \mathscr{T}_2\rho^2, \cdots, \mathscr{T}_\mathscr{N}\rho^\mathscr{N})$, where $\mathscr{N} \in \mathbb{N}$, $\zeta \in \mathbb{Z}^*$ and $(\mathscr{X}^\mathscr{N}, \mathscr{M}^\mathscr{N}, *)$ is a fuzzy metric space induced by $(\mathscr{X}, \mathscr{M}, *)$. Then, $\mathscr{T}_1, \mathscr{T}_2, \cdots, \mathscr{T}_\mathscr{N}$ are said to be fuzzy $\mathbb{Z}^*$-contractive mappings.

Observe that for "$\mathscr{N} = 1$," Definition 5 is a particular case of Definition 8.

Definition 9. A sequence $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}) \in \mathscr{X}^\mathscr{N} = \underbrace{\mathscr{X} \times \mathscr{X} \times \cdots \times \mathscr{X}}_{\mathscr{N}\text{ times}}$ is said to be a mutual sequence in a fuzzy metric space $(\mathscr{X}, \mathscr{M}, *)$.

Definition 10. Let $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}) \in \mathscr{X}^\mathscr{N} = \underbrace{\mathscr{X} \times \mathscr{X} \times \cdots \times \mathscr{X}}_{\mathscr{N}\text{ times}}$ be a mutual sequence in a fuzzy metric space $(\mathscr{X}, \mathscr{M}, *)$ and all sequences (ϱ_n^i), $i \in (1, 2, \cdots \mathscr{N})$ converge, then the sequence $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N})$ is said to be convergent mutual sequence. If the sequence (ϱ_n^i), for all $i \in (1, 2, \cdots, \mathscr{N})$ converge to the unique limit $\varrho \in \mathscr{X}$, then the mutual sequence is said to be coconvergent mutual sequence and limit ϱ is said to be the mutual limit.

Definition 11. A mutual sequence $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}) \in \mathscr{X}^N = \underbrace{\mathscr{X} \times \mathscr{X} \times \cdots \times \mathscr{X}}_{\mathscr{N}\text{ times}}$ is said to be a Cauchy mutual sequence in a fuzzy metric space $(\mathscr{X}, \mathscr{M}, *)$, if for each $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0 (n, m \in \mathbb{N})$, we have

$$\mathscr{M}^\mathscr{N}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}\right), \left(\varrho_m^2, \varrho_m^3, \cdots, \varrho_m^\mathscr{N}, \varrho_m^1\right), t\right) > 1 - \epsilon, \tag{5}$$

for all $t > 0$, i.e., a mutual sequence is said to be a Cauchy mutual sequence if as $n, m \longrightarrow \infty$,

$$\mathscr{M}^\mathscr{N}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}\right), \left(\varrho_m^2, \varrho_m^3, \cdots, \varrho_m^\mathscr{N}, \varrho_m^1\right), t\right) \longrightarrow 1, \tag{6}$$

for all $t > 0$, where $(\mathscr{X}^\mathscr{N}, \mathscr{M}^\mathscr{N}, *)$ is fuzzy metric spaces induced by $(\mathscr{X}, \mathscr{M}, *)$.

Now, we are considering the following lemmas to prove the existence and uniqueness of common fixed points.

Lemma 12. *Every Cauchy mutual sequence in $(\mathscr{X}, \mathscr{M}, *)$ is a Cauchy sequence in $(\mathscr{X}^\mathscr{N}, \mathscr{M}^\mathscr{N}, *)$.*

Proof. Let $\{(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N})\}$ be a mutual sequence. By Definition 1 and Lemma 3, we have

$$\begin{aligned}&\mathscr{M}^\mathscr{N}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}\right), \left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^\mathscr{N}\right), t\right)\\ &\geq \mathscr{M}^\mathscr{N}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}\right), \left(\varrho_k^2, \varrho_k^3, \cdots, \varrho_k^\mathscr{N}, \varrho_k^1\right), \frac{t}{2}\right)\\ &\quad * \mathscr{M}^\mathscr{N}\left(\left(\varrho_k^2, \varrho_k^3, \cdots, \varrho_k^\mathscr{N}, \varrho_k^1\right), \left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^\mathscr{N}\right), \frac{t}{2}\right).\end{aligned} \tag{7}$$

Now, we have sequence $\{(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N})\}$ a Cauchy mutual sequence. So, as $n, m, k \longrightarrow \infty$, we get

$$\mathscr{M}^\mathscr{N}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N}\right), \left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^\mathscr{N}\right), t\right) \longrightarrow 1. \tag{8}$$

Hence, $\{(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^\mathscr{N})\}$ is a Cauchy sequence in $(\mathscr{X}^\mathscr{N}, \mathscr{M}^\mathscr{N}, *)$.

The following example shows that the converse of Lemma 12 may not be true. □

Example 2. Let $\mathscr{X} = [1, \infty)$ define a fuzzy metric space $(\mathscr{X}, \mathscr{M}, *)$, where

$$\mathscr{M}(\varrho, \rho, t) = \begin{cases} 1, & \text{if } \varrho = \rho \\ \dfrac{2}{2 + \max\{\varrho, \rho\}}, & \text{otherwise} \end{cases} \quad \text{for all } \varrho, \rho \in \mathbb{R}^+,\ t > 0, \tag{9}$$

and $*$ be a continuous t-norm defined as $a * b = \min\{a, b\}$. Consider a mutual sequence $\{(1 - (1/n), 2 - (1/n), 3 - (1/n))\}$ on fuzzy metric space $(\mathscr{X}^\mathscr{N}, \mathscr{M}^\mathscr{N}, *)$, then mutual sequence is a Cauchy sequence in $(\mathscr{X}^\mathscr{N}, \mathscr{M}^\mathscr{N}, *)$ but not

Cauchy mutual sequence because as $n, m \longrightarrow \infty$, we have

$$\mathscr{M}^3\left(\left(1-\frac{1}{n}, 2-\frac{1}{n}, 3-\frac{1}{n}\right), \left(2-\frac{1}{m}, 3-\frac{1}{m}, 1-\frac{1}{m}\right), t\right) \longrightarrow \frac{2}{5}, \tag{10}$$

for all $t > 0$.

Lemma 13. *In a fuzzy metric space, every convergent Cauchy mutual sequence is coconvergent.*

Proof. Let $\{(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}})\}$ be a convergent mutual sequence which converges to $(\varrho^1, \varrho^2, \cdots, \varrho^{\mathscr{N}})$, where $\varrho^i \in \mathscr{X}$, for $i \in (1, 2, \cdots, \mathscr{N})$. Since $\{(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}})\}$ is a convergent Cauchy mutual sequence, as $n, m \longrightarrow \infty$, we have

$$\mathscr{M}^{\mathscr{N}}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}\right), \left(\varrho_m^2, \varrho_m^3, \cdots, \varrho_m^{\mathscr{N}}, \varrho_m^1\right), t\right) \longrightarrow 1, \tag{11}$$

for all $t > 0$, which implies that

$$\mathscr{M}^{\mathscr{N}}\left(\left(\varrho^1, \varrho^2, \cdots, \varrho^{\mathscr{N}}\right), \left(\varrho^2, \varrho^3, \cdots, \varrho^{\mathscr{N}}, \varrho^1\right), t\right) = 1, \tag{12}$$

for all $t > 0$. Hence, $\varrho^1 = \varrho^2 = \cdots = \varrho^{\mathscr{N}}$. □

Theorem 14. *Let $(\mathscr{X}, \mathscr{M}, *)$ be a complete fuzzy metric space; $\mathscr{T}_1, \mathscr{T}_2, \cdots, \mathscr{T}_{\mathscr{N}}$ are $\mathscr{N}$-self mappings on $\mathscr{X}$ satisfying*

(a) fuzzy $\mathbb{Z}^*$-contraction

(b) $\lim_{n \longrightarrow \infty} \inf_{m > n} \mathscr{M}^{\mathscr{N}}((\mathscr{T}_1^m(\varrho), \mathscr{T}_2^m(\varrho), \cdots, \mathscr{T}_{\mathscr{N}}^n(\varrho)), (\mathscr{T}_2^m(\varrho), \mathscr{T}_3^m(\varrho), \cdots, \mathscr{T}_{\mathscr{N}}^n(\varrho), \mathscr{T}_1^m(\varrho)), t) > 0$, for all $t > 0$, $\varrho \in \mathscr{X}$

Then, $\mathscr{T}_1, \mathscr{T}_2, \cdots, \mathscr{T}_{\mathscr{N}}$ have unique common fixed point.

Proof. Let $\varrho_0^i \in \mathscr{X}$ and $\mathscr{T}_i(\varrho_n^i) = \varrho_{n+1}^i$, for all $n \in \mathbb{N} \cup \{0\}$ and $i \in (1, 2, \cdots \mathscr{N})$. We get $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}})$ as a mutual sequence on $(\mathscr{X}, \mathscr{M}, *)$ and according to Lemma 3, $(\mathscr{X}^{\mathscr{N}}, \mathscr{M}^{\mathscr{N}}, *)$ is also a fuzzy metric space.

If $\varrho_n^i = \varrho_{n-1}^i$, for all $i \in (1, 2, \cdots, \mathscr{N})$ and for any $n \in \mathbb{N}$, then $\mathscr{T}_i(\varrho_n^i) = \varrho_{n-1}^i = \varrho_n^i$, i.e., ϱ_n^i is the fixed point of $\mathscr{T}_{i'}s$, for every i and a fixed n. Suppose that $\varrho_n^1 \neq \varrho_n^2 \neq \cdots \neq \varrho_n^{\mathscr{N}}$, for fixed n. From, Definitions 8 and 7

$$\begin{aligned}
&\mathscr{M}^{\mathscr{N}}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}\right), \left(\varrho_{n-1}^2, \varrho_{n-1}^3, \cdots, \varrho_{n-1}^{\mathscr{N}}, \varrho_{n-1}^1\right), t\right) \\
&\quad = \mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_n^1, \mathscr{T}_2\varrho_n^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}\right), \left(\mathscr{T}_2\varrho_{n-1}^2, \mathscr{T}_3\varrho_{n-1}^3, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_{n-1}^{\mathscr{N}}, \mathscr{T}_1\varrho_{n-1}^1\right), t\right) \\
&\quad \geq \zeta\left(\mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_n^1, \mathscr{T}_2\varrho_n^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}\right), \left(\mathscr{T}_2\varrho_{n-1}^2, \mathscr{T}_3\varrho_{n-1}^3, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_{n-1}^{\mathscr{N}}, \mathscr{T}_1\varrho_{n-1}^1\right), t\right), \mathscr{M}^{\mathscr{N}}\right. \\
&\qquad \times \left.\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}\right), \left(\varrho_{n-1}^2, \varrho_{n-1}^3, \cdots, \varrho_{n-1}^{\mathscr{N}}, \varrho_{n-1}^1\right), t\right)\right) \\
&\quad > \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}\right), \left(\varrho_{n-1}^2, \varrho_{n-1}^3, \cdots, \varrho_{n-1}^{\mathscr{N}}, \varrho_{n-1}^1\right), t\right),
\end{aligned} \tag{13}$$

for all $t > 0$. So, $\varrho_n^1 = \varrho_n^2 = \cdots = \varrho_n^{\mathscr{N}} = \varrho\ (\text{say})$ is a common fixed point of $\mathscr{T}_{i'}s$.

Now, assume that no consecutive terms of the sequence $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}})$ are the same and $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}) = (\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}})$ for some $n < m$, i.e.,

$$\varrho_{n+1}^i = \mathscr{T}_i\varrho_n^i = \mathscr{T}_i\varrho_m^i = \varrho_{m+1}^i, \tag{14}$$

for some $n < m$ and for all i. From Definition 8 and 7, we have

$$\begin{aligned}
&\mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{n+2}^1, \varrho_{n+2}^2, \cdots, \varrho_{n+2}^{\mathscr{N}}\right), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathscr{N}}, \varrho_{n+1}^1\right), t\right) \\
&\quad = \mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_{n+1}^1, \mathscr{T}_2\varrho_{n+1}^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_{n+1}^{\mathscr{N}}\right),\right. \\
&\qquad \times \left.\left(\mathscr{T}_2\varrho_n^2, \mathscr{T}_3\varrho_n^3, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}, \mathscr{T}_1\varrho_n^1\right), t\right) \\
&\quad \geq \zeta\left(\mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_{n+1}^1, \mathscr{T}_2\varrho_{n+1}^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_{n+1}^{\mathscr{N}}\right),\right.\right. \\
&\qquad \times \left.\left(\mathscr{T}_2\varrho_n^2, \mathscr{T}_3\varrho_n^3, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}, \mathscr{T}_1\varrho_n^1\right), t\right), \mathscr{M}^{\mathscr{N}} \\
&\qquad \times \left(\left(\varrho_{n+1}^1, \varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right) \\
&\quad > \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{n+1}^1, \varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right),
\end{aligned} \tag{15}$$

for all $t > 0$. Similarly, we get

$$\begin{aligned}
&\mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{n+1}^1, \varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right) \\
&\quad < \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{n+2}^1, \varrho_{n+2}^2, \cdots, \varrho_{n+2}^{\mathscr{N}}\right), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathscr{N}}, \varrho_{n+1}^1\right), t\right) \\
&\quad < \cdots < \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{m+1}^1, \varrho_{m+1}^2, \cdots, \varrho_{m+1}^{\mathscr{N}}\right), \left(\varrho_m^2, \varrho_m^3, \cdots, \varrho_m^{\mathscr{N}}, \varrho_m^1\right), t\right),
\end{aligned} \tag{16}$$

for all $t > 0$, which is a contradiction in light of the inequality (14). Therefore, $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}) \neq (\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}})$, for some $n < m$.

Now consider, $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}) \neq (\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}})$, for all $n \neq m (\in \mathbb{N})$. Then, from Definitions 8 and 7, we have

$$\begin{aligned}
&\mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{m+1}^1, \varrho_{m+1}^2, \cdots, \varrho_{m+1}^{\mathscr{N}}\right), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathscr{N}}, \varrho_{n+1}^1\right), t\right) \\
&\quad = \mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_m^1, \mathscr{T}_2\varrho_m^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_m^{\mathscr{N}}\right),\right. \\
&\qquad \times \left.\left(\mathscr{T}_2\varrho_n^2, \mathscr{T}_3\varrho_n^3, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}, \mathscr{T}_1\varrho_n^1\right), t\right) \\
&\quad \geq \zeta\left(\mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_m^1, \mathscr{T}_2\varrho_m^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_m^{\mathscr{N}}\right),\right.\right. \\
&\qquad \times \left.\left(\mathscr{T}_2\varrho_n^2, \mathscr{T}_3\varrho_n^3, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}, \mathscr{T}_1\varrho_n^1\right), t\right)\mathscr{M}^{\mathscr{N}} \\
&\qquad \times \left.\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right)\right) \\
&\quad > \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right),
\end{aligned} \tag{17}$$

for all $t > 0$ and $n < m$. Taking infimum (over $m > n$) in the above inequality, we have

$$\inf_{m>n} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right) \leq \inf_{m>n} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{m+1}^1, \varrho_{m+1}^2, \cdots, \varrho_{m+1}^{\mathscr{N}}\right), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathscr{N}}, \varrho_{n+1}^1\right), t\right), \tag{18}$$

for all $t > 0$. Therefore, $(\inf_{m>n} \mathscr{M}^{\mathscr{N}}((\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}), (\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1), t))$ is a monotonic and bounded sequence, for all $t > 0$. So, there exist some $s(t) \leq 1$ such that

$$\lim_{n \longrightarrow \infty} \inf_{m>n} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right) = s(t), \tag{19}$$

for all $t > 0$.

Denote

$$p_n = \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right), \tag{20}$$

$$q_n = \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{m+1}^1, \varrho_{m+1}^2, \cdots, \varrho_{m+1}^{\mathscr{N}}\right), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathscr{N}}, \varrho_{n+1}^1\right), t\right), \tag{21}$$

for all $t > 0$.

Now, our claim is $s(t) = 1$, for every $t > 0$. Letting on contrary that $s(t_1) < 1$, for some $t_1 > 0$. In light of (3), we have $p_n \leq q_n$ and by condition (b), $\lim_{n \longrightarrow \infty} p_n \in (0, 1]$. Applying Definition 7, we get

$$\lim_{n \longrightarrow \infty} \inf_{m>n} \zeta\left(\mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t_1\right), \mathscr{M}^{\mathscr{N}} \times \left(\left(\varrho_{m+1}^1, \varrho_{m+1}^2, \cdots, \varrho_{m+1}^{\mathscr{N}}\right), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathscr{N}}, \varrho_{n+1}^1\right), t_1\right)\right) = 1. \tag{22}$$

From (2), we have

$$\begin{aligned}
&\inf_{m>n} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{m+1}^1, \varrho_{m+1}^2, \cdots, \varrho_{m+1}^{\mathscr{N}}\right), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathscr{N}}, \varrho_{n+1}^1\right), t_1\right) \\
&\quad \geq \inf_{m>n} \zeta\left(\mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_m^1, \mathscr{T}_2\varrho_m^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_m^{\mathscr{N}}\right),\right.\right. \\
&\quad \times \left.\left(\mathscr{T}_2\varrho_n^2, \mathscr{T}_3\varrho_n^3, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}, \mathscr{T}_1\varrho_n^1\right), t_1\right), \mathscr{M}^{\mathscr{N}} \\
&\quad \times \left.\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t_1\right)\right) \\
&\quad > \inf_{m>n} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t_1\right).
\end{aligned} \tag{23}$$

By (22) and as $n \longrightarrow \infty$, we get

$$\inf_{m>n} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t_1\right) = s(t_1) = 1, \tag{24}$$

which is a contradiction. We conclude that

$$\lim_{n \longrightarrow \infty} \lim_{n,m \longrightarrow \infty} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathscr{N}}\right), \left(\varrho_n^2, \varrho_n^3, \cdots, \varrho_n^{\mathscr{N}}, \varrho_n^1\right), t\right) = 1, \tag{25}$$

for all $t > 0$. Hence, the mutual sequence $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}})$ is a Cauchy mutual sequence. Completeness of $\mathscr{X}$ and Lemma 12 ensure that there exists $(\varrho^1, \varrho^2, \cdots, \varrho^{\mathscr{N}}) \in \mathscr{X}$ such that

$$\lim_{n \longrightarrow \infty} \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}\right), \left(\left(\varrho^1, \varrho^2, \cdots, \varrho^{\mathscr{N}}\right), t\right) = 1, \tag{26}$$

for all $t > 0$. From Lemma 13, sequence $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}})$ is coconvergent sequence, i.e., $\varrho^1 = \varrho^2 = \cdots = \varrho^{\mathscr{N}} = \varrho$ (say). Now, we have to prove that ϱ is a common fixed point of $\mathscr{T}_1, \mathscr{T}_2, \cdots, \mathscr{T}_{\mathscr{N}}$. Suppose that $\mathscr{T}_i\varrho \neq \varrho$, $i \in (1, 2, \cdots, \mathscr{N})$. Without loss of generality, let us assume that $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}) \neq (\varrho, \varrho, \cdots, \varrho)$ and $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}) \neq (\mathscr{T}_2\varrho, \mathscr{T}_3\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho, \mathscr{T}_1\varrho)$, for all $n \in \mathscr{N}$. So, there exists $t_1 > 0$ such that $\mathscr{M}^{\mathscr{N}}((\varrho, \varrho, \cdots, \varrho), (\mathscr{T}_2\varrho, \mathscr{T}_3\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho, \mathscr{T}_1\varrho), t_1) < 1$, $\mathscr{M}^{\mathscr{N}}((\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}), (\varrho, \varrho, \cdots, \varrho), t_1) < 1$ and

$$\begin{aligned}
&\mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_n^1, \mathscr{T}_2\varrho_n^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}\right), (\mathscr{T}_2\varrho, \mathscr{T}_3\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho, \mathscr{T}_1\varrho), t_1\right) \\
&\quad = \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{n+1}^1, \varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathscr{N}}\right), (\mathscr{T}_2\varrho, \mathscr{T}_3\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho, \mathscr{T}_1\varrho), t_1\right) < 1,
\end{aligned} \tag{27}$$

for all $n \in \mathbb{N}$. Then, we have

$$\begin{aligned}
&\mathscr{M}^{\mathscr{N}}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}\right), (\varrho, \varrho, \cdots, \varrho), t_1\right) < \zeta\left(\mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_n^1, \mathscr{T}_2\varrho_n^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}\right),\right.\right. \\
&\quad \times \left.(\mathscr{T}_2\varrho, \mathscr{T}_3\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho, \mathscr{T}_1\varrho, t_1), \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathscr{N}}\right), (\varrho, \varrho, \cdots, \varrho), t_1\right)\right) \\
&\quad = \mathscr{M}^{\mathscr{N}}\left(\left(\mathscr{T}_1\varrho_n^1, \mathscr{T}_2\varrho_n^2, \cdots, \mathscr{T}_{\mathscr{N}}\varrho_n^{\mathscr{N}}\right), (\mathscr{T}_2\varrho, \mathscr{T}_3\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho, \mathscr{T}_1\varrho), t_1\right) \\
&\quad = \mathscr{M}^{\mathscr{N}}\left(\left(\varrho_{n+1}^1, \varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathscr{N}}\right), (\mathscr{T}_2\varrho, \mathscr{T}_3\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho, \mathscr{T}_1\varrho), t_1\right),
\end{aligned} \tag{28}$$

as $n \longrightarrow \infty$; from (5) and Lemma 13, we get

$$\mathscr{M}^{\mathscr{N}}((\mathscr{T}_1\varrho, \mathscr{T}_2\varrho, \cdots \mathscr{T}_{\mathscr{N}}\varrho), (\varrho, \varrho, \cdots, \varrho), t_1) \geq 1, \tag{29}$$

which is a contradiction. Hence, $\mathscr{M}^{\mathscr{N}}((\mathscr{T}_1\varrho, \mathscr{T}_2\varrho, \cdots \mathscr{T}_{\mathscr{N}}\varrho), (\varrho, \varrho, \cdots, \varrho), t) = 1$, for all $t > 0$. Hence, ϱ is the common fixed point of $\mathscr{T}_{i'}s$, for all $i \in (1, 2, \cdots \mathscr{N})$.

Now, we have to prove the uniqueness of the common fixed point of $\mathscr{T}_{i'}s$. Assume on contrary that $\varrho, \rho \in \mathscr{X}$ be two distinct common fixed points of $\mathscr{T}_{i'}s$, for all $i \in (1, 2, \cdots, \mathscr{N})$ and there exists $t_1 > 0$ such that $\mathscr{M}^{\mathscr{N}}((\varrho, \varrho, \cdots, \varrho), (\rho, \rho, \cdots, \rho), t_1) < 1$. Then, from Definitions 8 and 7, we get

$$\begin{aligned}
&\mathscr{M}^{\mathscr{N}}((\varrho, \varrho, \cdots, \varrho), (\rho, \rho, \cdots, \rho), t_1) \\
&\quad = \mathscr{M}^{\mathscr{N}}((\mathscr{T}_1\varrho, \mathscr{T}_2\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho), (\mathscr{T}_2\rho, \mathscr{T}_3\rho, \cdots, \mathscr{T}_{\mathscr{N}}\rho, \mathscr{T}_1\rho), t_1) \\
&\quad = \zeta\left(\mathscr{M}^{\mathscr{N}}((\mathscr{T}_1\varrho, \mathscr{T}_2\varrho, \cdots, \mathscr{T}_{\mathscr{N}}\varrho), (\mathscr{T}_2\rho, \mathscr{T}_3\rho, \cdots, \mathscr{T}_{\mathscr{N}}\rho, \mathscr{T}_1\rho), t_1), \mathscr{M}^{\mathscr{N}}\right. \\
&\quad \times \left.((\varrho, \varrho, \cdots, \varrho), (\rho, \rho, \cdots, \rho), t_1)\right) > \mathscr{M}^{\mathscr{N}}((\varrho, \varrho, \cdots, \varrho), (\rho, \rho, \cdots, \rho), t_1),
\end{aligned} \tag{30}$$

implying thereby $\mathscr{M}^{\mathscr{N}}((\varrho, \varrho, \cdots, \varrho), (\rho, \rho, \cdots, \rho), t) = 1$, for all $t > 0$. Hence, $\varrho = \rho$. □

Remark 15. On putting $\mathscr{N} = 1$, in Theorem 14, it reduces to Theorem 3.19 presented in [20].

Example 3. Let $\mathcal{X}=\{0,1/5,1/3,1/2,1,2,12,17,31,45,60,71,91,100,111\}$ and $(\mathcal{X},\mathcal{M},*)$ be a fuzzy metric space in which $\mathcal{M}$ is a fuzzy set defined on $\mathcal{X}^2\times(0,\infty)$ such that $\mathcal{M}=t/(t+|\varrho-\rho|)$ for all $\varrho,\rho\in\mathcal{X}$ and $t>0$, $*$ is a continuous t-norm defined as $\varrho*\rho=\varrho.\rho$, then $(\mathcal{X},\mathcal{M},*)$ is a complete metric space. Now, let us define 5-maps $\mathcal{T}_1,\mathcal{T}_2,\cdots\mathcal{T}_5:\mathcal{X}\longrightarrow\mathcal{M}$ as

$$\begin{aligned}
\mathcal{T}_1(\varrho)&=\begin{cases}1, & \text{if } \varrho\in\{12,17,31\},\\ 0, & \text{otherwise,}\end{cases}\\
\mathcal{T}_2(\varrho)&=\begin{cases}\dfrac{1}{2}, & \text{if } \varrho\in\{45,60,71\},\\ 0, & \text{otherwise,}\end{cases}\\
\mathcal{T}_3(\varrho)&=\begin{cases}2, & \text{if } \varrho\in\{91,100,111\},\\ 0, & \text{otherwise,}\end{cases}\\
\mathcal{T}_4(\varrho)&=\begin{cases}0, & \varrho\in\left\{1,\dfrac{1}{2},\dfrac{1}{3},\dfrac{1}{5},0\right\},\\ \dfrac{1}{3}, & \text{otherwise,}\end{cases}\\
\mathcal{T}_5(\varrho)&=\begin{cases}0, & \text{if } \varrho\in\left\{\dfrac{1}{2},\dfrac{1}{3},\dfrac{1}{5},0\right\},\\ \dfrac{1}{5}, & \text{otherwise,}\end{cases}
\end{aligned}\tag{31}$$

and a function $\zeta:(0,1]\times(0,1]\longrightarrow\mathbb{R}$ such that $\zeta(p,q)=q/p$ for all $p,q\in(0,1]$.

Let $(\varrho_0^1,\varrho_0^2,\varrho_0^3,\varrho_0^4,\varrho_0^5)=(31,60,91,1,2)$ and $\mathcal{T}_i(\varrho_n^i)=\varrho_{n+1}^i$ for all $n\in\mathbb{N}\cup\{0\}$, we get $\{(31,60,91,1,2),(1,1/2,2,0,1/5),(0,0,0,0,0),\cdots\}$ as a mutual sequence. We can easily observe that conditions (a) and (b) of Theorem 14 are satisfied. Hence, 0 is the unique common fixed point of $\mathcal{T}_1,\mathcal{T}_2,\cdots\mathcal{T}_5$.

Now, we present the fixed point theorem for non-self-mappings.

Theorem 16. *Let $(\mathcal{X},\mathcal{M},*)$ be a fuzzy metric space, $\mathcal{Y}_1,\mathcal{Y}_2,\cdots,\mathcal{Y}_\mathcal{N}$ are $\mathcal{N}$ subset of $\mathcal{X}$. Let $\mathcal{T}_1:\mathcal{Y}_1\longrightarrow\mathcal{Y}_2,\mathcal{T}_2:\mathcal{Y}_2\longrightarrow\mathcal{Y}_3,\cdots,\mathcal{T}_{\mathcal{N}-1}:\mathcal{Y}_{\mathcal{N}-1}\longrightarrow\mathcal{Y}_\mathcal{N}$ and $\mathcal{T}_\mathcal{N}:\mathcal{Y}_\mathcal{N}\longrightarrow\mathcal{Y}_1$ are $\mathcal{N}$ mappings satisfying the following conditions:*

(i) $\mathcal{T}_i(\mathcal{Y}_i)$ are complete subspace of $\mathcal{X}$

(ii) $\mathcal{M}^\mathcal{N}((\mathcal{T}_1\varrho^1,\mathcal{T}_2\varrho^2,\cdots,\mathcal{T}_\mathcal{N}\varrho^\mathcal{N}),(\mathcal{T}_2\rho^2,\mathcal{T}_3\rho^3,\cdots,\mathcal{T}_\mathcal{N}\rho^\mathcal{N},\mathcal{T}_1\rho^1),t)\ge\zeta(\mathcal{M}^\mathcal{N}((\mathcal{T}_1\varrho^1,\mathcal{T}_2\varrho^2,\cdots,\mathcal{T}_\mathcal{N}\varrho^\mathcal{N}),(\mathcal{T}_2\rho^2,\mathcal{T}_3\rho^3,\cdots,\mathcal{T}_\mathcal{N}\rho^\mathcal{N},\mathcal{T}_1\rho^1),t),\mathcal{M}^\mathcal{N}((\varrho^1,\varrho^2,\cdots,\varrho^\mathcal{N}),(\rho^2,\rho^3,\cdots,\rho^\mathcal{N},\rho^1),t))$ for all $t>0$, $\varrho^i\ne\rho^i(\in\mathcal{Y}_i)$, $i\in(1,2,\cdots\mathcal{N})$, $(\mathcal{T}_1\varrho^1,\mathcal{T}_2\varrho^2,\cdots,\mathcal{T}_\mathcal{N}\varrho^\mathcal{N})\ne(\mathcal{T}_1\rho^1,\mathcal{T}_2\rho^2,\cdots,\mathcal{T}_\mathcal{N}\rho^\mathcal{N})$, where $\mathcal{N}\in\mathbb{N},\zeta\in\mathbb{Z}^*$ and $(\mathcal{X}^\mathcal{N},\mathcal{M}^\mathcal{N},*)$ is fuzzy metric spaces induced by $(\mathcal{X},\mathcal{M},*)$

(iii) $\lim_{n\longrightarrow\infty}\inf_{m>n}\mathcal{M}^\mathcal{N}((\mathcal{T}_1^m(\varrho^1),\mathcal{T}_2^m(\varrho^2),\cdots,\mathcal{T}_\mathcal{N}^n(\varrho^\mathcal{N})),(\mathcal{T}_2^m(\varrho^2),\mathcal{T}_3^m(\varrho^3),\cdots,\mathcal{T}_\mathcal{N}^n(\varrho^\mathcal{N}),\mathcal{T}_1^m(\varrho^1)),t)>0$ for all $t>0$, $\varrho^i\in\mathcal{Y}_i$, $i\in(1,2,\cdots,\mathcal{N})$

Then, $\mathcal{T}_1,\mathcal{T}_2,\cdots,\mathcal{T}_\mathcal{N}$ have unique common fixed point.

Proof. Let $\varrho_1^1\in\mathcal{Y}_1$ and $\mathcal{T}_1(\varrho_n^1)=\varrho_n^2$, $\mathcal{T}_2(\varrho_n^2)=\varrho_n^3,\ldots,\mathcal{T}_{\mathcal{N}-1}(\varrho_n^{\mathcal{N}-1})=\varrho_n^\mathcal{N}$ and $\mathcal{T}_\mathcal{N}(\varrho_n^\mathcal{N})=\varrho_{n+1}^1$, for all $n\in\mathbb{N}$. We get $(\varrho_m^1,\varrho_m^2,\cdots,\varrho_m^\mathcal{N})\in\mathcal{X}^\mathcal{N}$ as a mutual on $(\mathcal{X},\mathcal{M},*)$.

If $\varrho_n^i=\varrho_{n+1}^i$, for all $1\le i\le\mathcal{N}\in\mathbb{N}$ and for any $n\in\mathbb{N}$, then $\mathcal{T}_1(\varrho_n^1)=\varrho_n^2=\varrho_{n+1}^2$, $\mathcal{T}_2(\varrho_n^2)=\varrho_n^3=\varrho_{n+1}^3$, ..., $\mathcal{T}_{\mathcal{N}-1}(\varrho_n^{\mathcal{N}-1})=\varrho_n^\mathcal{N}=\varrho_{n+1}^\mathcal{N}$ and $\mathcal{T}_\mathcal{N}(\varrho_n^\mathcal{N})=\varrho_{n+1}^1=\varrho_{n+1}^1$. Now, from Lemma 3, Definition 7, and condition (ii), we have

$$\begin{aligned}
&\mathcal{M}^\mathcal{N}\left(\left(\varrho_n^1,\varrho_n^2,\cdots,\varrho_n^\mathcal{N}\right),\left(\varrho_{n+1}^2,\varrho_{n+1}^3,\cdots,\varrho_{n+1}^\mathcal{N},\varrho_{n+1}^1\right),t\right)\\
&=\mathcal{M}^\mathcal{N}\left(\left(\varrho_{n+1}^1,\varrho_n^2,\cdots,\varrho_n^\mathcal{N}\right),\left(\varrho_{n+1}^2,\varrho_{n+1}^3,\cdots,\varrho_{n+1}^\mathcal{N},\varrho_{n+1}^1\right),t\right)\\
&=\mathcal{M}^\mathcal{N}\left(\left(\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N},\mathcal{T}_1\varrho_n^1,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_n^{\mathcal{N}-1}\right),\right.\\
&\quad\times\left.\left(\mathcal{T}_1\varrho_{n+1}^1,\mathcal{T}_2\varrho_{n+1}^2,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_{n+1}^{\mathcal{N}-1},\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N}\right),t\right)\\
&\ge\zeta\left(\mathcal{M}^\mathcal{N}\left(\left(\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N},\mathcal{T}_1\varrho_n^1,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_n^{\mathcal{N}-1}\right),\right.\right.\\
&\quad\times\left.\left(\mathcal{T}_1\varrho_{n+1}^1,\mathcal{T}_2\varrho_{n+1}^2,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_{n+1}^{\mathcal{N}-1},\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N}\right),t\right),\mathcal{M}^\mathcal{N}\\
&\quad\times\left.\left(\left(\varrho_n^\mathcal{N},\varrho_n^1,\cdots,\varrho_n^{\mathcal{N}-1}\right),\left(\varrho_{n+1}^1,\varrho_{n+1}^2,\cdots,\varrho_{n+1}^{\mathcal{N}-1},\varrho_n^\mathcal{N}\right),t\right)\right)\\
&>\mathcal{M}^\mathcal{N}\left(\left(\varrho_n^\mathcal{N},\varrho_n^1,\cdots,\varrho_n^{\mathcal{N}-1}\right),\left(\varrho_{n+1}^1,\varrho_{n+1}^2,\cdots,\varrho_{n+1}^{\mathcal{N}-1},\varrho_n^\mathcal{N}\right),t\right)\\
&=\mathcal{M}^\mathcal{N}\left(\left(\varrho_n^1,\varrho_n^2,\cdots,\varrho_n^\mathcal{N}\right),\left(\varrho_{n+1}^2,\varrho_{n+1}^3,\cdots,\varrho_{n+1}^\mathcal{N},\varrho_{n+1}^1\right),t\right),
\end{aligned}\tag{32}$$

for all $t>0$, a contradiction, which implies that $\mathcal{M}^\mathcal{N}((\varrho_n^1,\varrho_n^2,\cdots,\varrho_n^\mathcal{N}),(\varrho_{n+1}^2,\varrho_{n+1}^3,\cdots,\varrho_{n+1}^\mathcal{N},\varrho_{n+1}^1),t)=1$, i.e., $\varrho_n^1=\varrho_n^2=\cdots=\varrho_n^\mathcal{N}=\varrho$ (say) is a common fixed point of $\mathcal{T}_i$'s.

From Lemma 3, Definition 7, and condition (ii), for all $t>0$, we have

$$\begin{aligned}
&\mathcal{M}^\mathcal{N}\left(\left(\varrho_{n+2}^1,\varrho_{n+2}^2,\cdots,\varrho_{n+2}^\mathcal{N}\right),\left(\varrho_{n+1}^2,\varrho_{n+1}^3,\cdots,\varrho_{n+1}^\mathcal{N},\varrho_{n+1}^1\right),t\right)\\
&=\mathcal{M}^\mathcal{N}\left(\left(\mathcal{T}_\mathcal{N}\varrho_{n+1}^\mathcal{N},\mathcal{T}_1\varrho_{n+2}^1,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_{n+2}^{\mathcal{N}-1}\right),\right.\\
&\quad\times\left.\left(\mathcal{T}_1\varrho_{n+1}^1,\mathcal{T}_2\varrho_{n+1}^2,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_{n+1}^{\mathcal{N}-1},\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N}\right),t\right)\\
&\ge\zeta\left(\mathcal{M}^\mathcal{N}\left(\left(\mathcal{T}_\mathcal{N}\varrho_{n+1}^\mathcal{N},\mathcal{T}_1\varrho_{n+2}^1,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_{n+2}^{\mathcal{N}-1}\right),\right.\right.\\
&\quad\times\left.\left(\mathcal{T}_1\varrho_{n+1}^1,\mathcal{T}_2\varrho_{n+1}^2,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_{n+1}^{\mathcal{N}-1},\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N}\right),t\right),\mathcal{M}^\mathcal{N}\\
&\quad\times\left.\left(\left(\varrho_{n+1}^\mathcal{N},\varrho_{n+2}^1,\cdots,\varrho_{n+2}^{\mathcal{N}-1}\right),\left(\varrho_{n+1}^1,\varrho_{n+1}^2,\cdots,\varrho_{n+1}^{\mathcal{N}-1},\varrho_n^\mathcal{N}\right),t\right)\right)\\
&>\mathcal{M}^\mathcal{N}\left(\left(\varrho_{n+1}^\mathcal{N},\varrho_{n+2}^1,\cdots,\varrho_{n+2}^{\mathcal{N}-1}\right),\left(\varrho_{n+1}^1,\varrho_{n+1}^2,\cdots,\varrho_{n+1}^{\mathcal{N}-1},\varrho_n^\mathcal{N}\right)t\right)\\
&=\mathcal{M}^\mathcal{N}\left(\left(\mathcal{T}_{\mathcal{N}-1}\varrho_{n+1}^{\mathcal{N}-1},\mathcal{T}_\mathcal{N}\varrho_{n+1}^\mathcal{N},\cdots,\mathcal{T}_{\mathcal{N}-2}\varrho_{n+2}^{\mathcal{N}-2}\right),\right.\\
&\quad\times\left.\left(\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N},\mathcal{T}_1\varrho_{n+1}^1,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_n^{\mathcal{N}-1}\right),t\right)\\
&\ge\zeta\left(\mathcal{M}^\mathcal{N}\left(\left(\mathcal{T}_{\mathcal{N}-1}\varrho_{n+1}^{\mathcal{N}-1},\mathcal{T}_\mathcal{N}\varrho_{n+1}^\mathcal{N},\cdots,\mathcal{T}_{\mathcal{N}-2}\varrho_{n+2}^{\mathcal{N}-2}\right),\right.\right.\\
&\quad\times\left.\left(\mathcal{T}_\mathcal{N}\varrho_n^\mathcal{N},\mathcal{T}_1\varrho_{n+1}^1,\cdots,\mathcal{T}_{\mathcal{N}-1}\varrho_n^{\mathcal{N}-1}\right),t\right),\mathcal{M}^\mathcal{N}\\
&\quad\times\left.\left(\left(\varrho_{n+1}^{\mathcal{N}-1},\varrho_{n+1}^\mathcal{N},\cdots,\varrho_{n+2}^{\mathcal{N}-2}\right),\left(\varrho_n^\mathcal{N},\varrho_{n+1}^1,\cdots,\varrho_{n+1}^{\mathcal{N}-2},\varrho_n^{\mathcal{N}-1}\right),t\right)\right)\\
&>\mathcal{M}^\mathcal{N}\left(\left(\varrho_{n+1}^{\mathcal{N}-1},\varrho_{n+1}^\mathcal{N},\cdots,\varrho_{n+2}^{\mathcal{N}-2}\right),\left(\varrho_n^\mathcal{N},\varrho_{n+1}^1,\cdots,\varrho_{n+1}^{\mathcal{N}-2},\varrho_n^{\mathcal{N}-1}\right),t\right)\\
&>\cdots>\mathcal{M}^\mathcal{N}\left(\left(\varrho_{n+1}^1,\varrho_{n+1}^2,\cdots,\varrho_{n+1}^\mathcal{N}\right),\left(\varrho_n^2,\varrho_n^3,\cdots,\varrho_n^\mathcal{N},\varrho_n^1\right),t\right).
\end{aligned}\tag{33}$$

In the light of inequality (6), we observe that the behavior of mutual sequence in the proof of Theorem 14 and mutual sequence as above is alike. The proof of sequence $(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathcal{N}})$ to be a Cauchy mutual sequence is immediate from Theorem 14.

From (i) and Lemma 12, there exist $\varrho^i \in \mathcal{T}_i(\mathcal{Y}_i)$ such that $(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathcal{N}})$ converges to $(\varrho^1, \varrho^2, \cdots, \varrho^{\mathcal{N}})$, and from Lemma 13, sequence $(\varrho_m^1, \varrho_m^2, \cdots, \varrho_m^{\mathcal{N}})$ is coconvergent to some point $\varrho \in \mathcal{X}$.

Now, we have to prove that ϱ is a common fixed point of $\mathcal{T}_i$'s. Without loss of generality, let us assume that $(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathcal{N}}) \neq (\varrho, \varrho, \cdots, \varrho)$ and

$$\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathcal{N}}\right) \neq (\mathcal{T}_2\varrho, \mathcal{T}_3\varrho, \cdots, \mathcal{T}_{\mathcal{N}}\varrho, \mathcal{T}_1\varrho), \quad \text{for all } n \in \mathcal{N}. \tag{34}$$

So, there exists $t_1 > 0$ such that $\mathcal{M}^{\mathcal{N}}((\varrho, \varrho, \cdots, \varrho), (\mathcal{T}_2\varrho, \mathcal{T}_3\varrho, \cdots, \mathcal{T}_{\mathcal{N}}\varrho, \mathcal{T}_1\varrho), t_1) < 1$,

$$\mathcal{M}^{\mathcal{N}}\left(\left(\varrho_n^1, \varrho_n^2, \cdots, \varrho_n^{\mathcal{N}}\right), (\varrho, \varrho, \cdots, \varrho), t_1\right) < 1, \tag{35}$$

$$\begin{aligned}&\mathcal{M}^{\mathcal{N}}\left((\mathcal{T}_1\varrho, \mathcal{T}_2\varrho, \cdots, \mathcal{T}_{\mathcal{N}}\varrho), \left(\mathcal{T}_2\varrho_{n+1}^2, \mathcal{T}_3\varrho_{n+1}^3, \cdots, \mathcal{T}_{\mathcal{N}}\varrho_{n+1}^{\mathcal{N}}, \mathcal{T}_1\varrho_n^1\right), t_1\right)\\ &= \mathcal{M}^{\mathcal{N}}\left((\mathcal{T}_1\varrho, \mathcal{T}_2\varrho, \cdots, \mathcal{T}_{\mathcal{N}}\varrho), \left(\varrho_{n+1}^1, \varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathcal{N}}\right), t_1\right) < 1,\end{aligned} \tag{36}$$

for all $n \in \mathbb{N}$. Then, we have

$$\begin{aligned}&\mathcal{M}^{\mathcal{N}}\left((\varrho, \varrho, \cdots, \varrho), \left(\varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathcal{N}}, \varrho_n^1\right), t_1\right)\\ &< \zeta\Big(\mathcal{M}^{\mathcal{N}}\left((\mathcal{T}_1\varrho, \mathcal{T}_2\varrho, \cdots, \mathcal{T}_{\mathcal{N}}\varrho), \left(\mathcal{T}_2\varrho_{n+1}^2, \mathcal{T}_3\varrho_{n+1}^3, \cdots, \mathcal{T}_{\mathcal{N}}\varrho_{n+1}^{\mathcal{N}}, \mathcal{T}_1\varrho_n^1\right), t_1\right), \mathcal{M}^{\mathcal{N}}\\ &\quad \times \left((\varrho, \varrho, \cdots, \varrho), \left(\varrho_{n+1}^2, \varrho_{n+1}^3, \cdots, \varrho_{n+1}^{\mathcal{N}}, \varrho_n^1\right), t_1\right)\Big)\\ &= \mathcal{M}^{\mathcal{N}}\left((\mathcal{T}_1\varrho, \mathcal{T}_2\varrho, \cdots, \mathcal{T}_{\mathcal{N}}\varrho), \left(\mathcal{T}_2\varrho_{n+1}^2, \mathcal{T}_3\varrho_{n+1}^3, \cdots, \mathcal{T}_{\mathcal{N}}\varrho_{n+1}^{\mathcal{N}}, \mathcal{T}_1\varrho_n^1\right), t_1\right)\\ &= \mathcal{M}^{\mathcal{N}}\left((\mathcal{T}_1\varrho, \mathcal{T}_2\varrho, \cdots, \mathcal{T}_{\mathcal{N}}\varrho), \left(\varrho_{n+1}^1, \varrho_{n+1}^2, \cdots, \varrho_{n+1}^{\mathcal{N}}\right), t_1\right),\end{aligned} \tag{37}$$

as $n \longrightarrow \infty$, and from (5), we get $\mathcal{M}^{\mathcal{N}}((\mathcal{T}_1\varrho, \mathcal{T}_2\varrho, \cdots \mathcal{T}_{\mathcal{N}}\varrho), (\varrho, \varrho, \cdots, \varrho), t_1) \geq 1$, a contradiction. Hence, $\mathcal{M}^{\mathcal{N}}((\mathcal{T}_1\varrho, \mathcal{T}_2\varrho, \cdots \mathcal{T}_{\mathcal{N}}\varrho), (\varrho, \varrho, \cdots, \varrho), t) = 1$, for all $t > 0$. Therefore, ϱ is the common fixed point of $\mathcal{T}_i$'s, for all $i \in (1, 2, \cdots \mathcal{N})$.

The proof of uniqueness of common fixed point runs similar to the proof of Theorem 14. Hence, we are through. □

Example 4. Let $\mathcal{X} = (0, 1]$ and $(\mathcal{X}, \mathcal{M}, *)$ be a fuzzy metric space where

$$\mathcal{M}(\varrho, \rho, t) = \begin{cases} 1, & \text{if } \varrho = \rho \\ \min\{\varrho, \rho\}, & \text{otherwise} \end{cases} \quad \text{for all } \varrho, \rho \in \mathcal{X},\ t > 0, \tag{38}$$

$*$ is continuous t-norm defined as $a * b = \min\{a, b\}$. Let

$$\begin{aligned}\mathcal{Y}_1 &= \left\{\frac{1}{1000}, \frac{1}{600}, \frac{1}{200}, \frac{1}{70}, 1\right\},\\ \mathcal{Y}_2 &= \left\{\frac{1}{900}, \frac{1}{500}, \frac{1}{100}, \frac{1}{50}, 1\right\},\\ \mathcal{Y}_3 &= \left\{\frac{1}{800}, \frac{1}{400}, \frac{1}{90}, \frac{1}{30}, 1\right\},\\ \mathcal{Y}_4 &= \left\{\frac{1}{700}, \frac{1}{300}, \frac{1}{80}, \frac{1}{20}, 1\right\},\end{aligned} \tag{39}$$

be subset of $\mathcal{X}$; we define $\mathcal{T}_1 : \mathcal{Y}_1 \longrightarrow \mathcal{Y}_2, \mathcal{T}_2 : \mathcal{Y}_2 \longrightarrow \mathcal{Y}_3, \mathcal{T}_3 : \mathcal{Y}_3 \longrightarrow \mathcal{Y}_4$ and $\mathcal{T}_4 : \mathcal{Y}_4 \longrightarrow \mathcal{Y}_1$ such that

$$\begin{aligned}&\mathcal{T}_1(1) = \mathcal{T}_2(1) = \mathcal{T}_3(1) = \mathcal{T}_4(1) = 1,\\ &\mathcal{T}_1\left(\frac{1}{1000}\right) = \frac{1}{900}, \mathcal{T}_1\left(\frac{1}{600}\right) = \frac{1}{500}, \mathcal{T}_1\left(\frac{1}{200}\right) = \frac{1}{100}, \mathcal{T}_1\left(\frac{1}{70}\right) = \frac{1}{50},\\ &\mathcal{T}_2\left(\frac{1}{900}\right) = \frac{1}{800}, \mathcal{T}_2\left(\frac{1}{500}\right) = \frac{1}{400}, \mathcal{T}_2\left(\frac{1}{100}\right) = \frac{1}{90}, \mathcal{T}_2\left(\frac{1}{50}\right) = \frac{1}{30},\\ &\mathcal{T}_3\left(\frac{1}{800}\right) = \frac{1}{700}, \mathcal{T}_3\left(\frac{1}{400}\right) = \frac{1}{300}, \mathcal{T}_3\left(\frac{1}{90}\right) = \frac{1}{80}, \mathcal{T}_3\left(\frac{1}{30}\right) = \frac{1}{20},\\ &\mathcal{T}_4\left(\frac{1}{700}\right) = \frac{1}{600}, \mathcal{T}_4\left(\frac{1}{300}\right) = \frac{1}{200}, \mathcal{T}_4\left(\frac{1}{80}\right) = \frac{1}{70}, \mathcal{T}_4\left(\frac{1}{20}\right) = 1.\end{aligned} \tag{40}$$

Now, define function ζ similar to Example 3. We can easily verify that all conditions of Theorem 16 are satisfied. If $\varrho_1^1 = 1/1000$ and $\mathcal{T}_1(\varrho_n^1) = \varrho_n^2$, $\mathcal{T}_2(\varrho_n^2) = \varrho_n^3, \ldots, \mathcal{T}_{\mathcal{N}-1}(\varrho_n^{\mathcal{N}-1}) = \varrho_n^{\mathcal{N}}$ and $\mathcal{T}_{\mathcal{N}}(\varrho_n^{\mathcal{N}}) = \varrho_{n+1}^1$ for all $n \in \mathcal{N}$. We get $\{(1/1000, 1/900, 1/800, 1/700), (1/600, 1/500, 1/400, 1/300), \cdots\}$ as mutual sequence, 1 as unique common fixed point of $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_3$, and $\mathcal{T}_4$.

4. Conclusion

In this paper, the concept of mutual sequences in $(\mathcal{X}, \mathcal{M}, *)$ is given, and with the help of induced fuzzy metric structure $(\mathcal{X}^{\mathcal{N}}, \mathcal{M}^{\mathcal{N}}, *)$, we define Cauchy mutual sequences in simple fuzzy metric structure $(\mathcal{X}, \mathcal{M}, *)$. For brevity, Definitions 2.6 and 2.4 (presented in [20]) are unified as Definition 7. We also present $\mathbb{Z}^*$ contraction, which is an extension of $\mathbb{Z}$-contraction for finite number of mappings. With the help of mutual sequences, we proved unique common fixed point theorems for finite numbers of mappings using $\mathbb{Z}^*$ contraction. We also provide many examples to show that our results are meaningful and to support our theorems. The given results generalize and extend several results in the existing literature. As perspectives, it would be interesting that the results presented in this paper proved for other contractive conditions and extend to other nonclassical metric structure, like bipolar fuzzy metric spaces [5] and relational fuzzy metric spaces [25].

Data Availability

No data were used to support this study.

Conflicts of Interest

All the authors declare that they have no conflict of interest.

Authors' Contributions

Each author contributed equally and significantly to every part of this article. All authors read and approved the final version of the paper.

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