Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 7057639, 12 pages
https://doi.org/10.1155/2022/7057639

Research Article

A Novel Decision-Making Process in the Environment of Generalized Version of Fuzzy Sets for the Selection of Energy Source

Joseph David Madasi ,[1] Salma Khan ,[2] Nasreen Kausar ,[3] Dragan Pamucar ,[4] Gezahagne Mulat Addis ,[5] and Muhammad Gulistan [2]

[1]*College of Mathematics and Computer Science, Zhejiang Normal University, China*
[2]*Department of Mathematics and Statistics Hazara University Mansehra, Pakistan*
[3]*Department of Mathematics, Faculty of Arts and Sciences, Yildiz Technical University, Esenler, 34210 Istanbul, Turkey*
[4]*Faculty of Organizational Sciences, University of Belgrade, Belgrade, Serbia*
[5]*Department of Mathematics, University of Gondar, P.O. Box: 196, Gondar, Ethiopia*

Correspondence should be addressed to Gezahagne Mulat Addis; gezahagne412@gmail.com

Received 4 July 2022; Revised 24 July 2022; Accepted 29 July 2022; Published 23 August 2022

Academic Editor: Ranjan Kumar

In this study, we focus our attention on a kind of generalized fuzzy set. This generalized fuzzy set is known as neutrosophic octahedron sets (NOSs). NOSs are a combination of neutrosophic, intuitionistic fuzzy, and octahedron sets that provide a better platform for dealing with imprecise and ambiguous data. First of all, we analyze uncertainty, for this purpose, we need neutrosophic octahedron set that can also reduce the loss of information about ambiguity and uncertainty. We use NOS over TOPSIS method (technique to order the performance by similarity with the ideal solution). It is a most suitable technique for describing uncertain data in the TOPSIS method in order to allow more imprecision than the neutrosophic, intuitionistic fuzzy, and octahedron set. Thus, the TOPSIS method of NOSs in decision making is used to overcome the problems that arise during decision-making. We use this proposed structure to implement the selection of the energy source by a numerical example as an application. As a result, this model is valuable for decision-making and can be used to choose the most environmentally friendly energy source. Finally, we present an example to demonstrate the validity and effectiveness of the proposed strategy.

1. Introduction

Decision-making is a beneficial method in human activities to consider the appropriate option among alternatives with the highest degree of membership from a group of available possibilities in terms of parameters. In decision-making problems, the evaluated values of alternatives considering the evaluated attribute are often imprecise. The theme of uncertainty and vagueness is difficult, to understand and implement in different areas. So, Zadeh, the developer of fuzzy set theory [1], introduces fuzzy sets in this area to solve the complications and make it more usable. Fuzzy set theory can be applied to evaluate the elements of a set defined by a membership (MM) function in a closed interval $[0, 1]$. After fuzzy set theory, Zadeh [2] also introduced the theme of interval valued fuzzy set in 1975. Atanasove developed the intuitionistic fuzzy set [3] in 1986, with MM and nonmembership (NMM) degrees such that their sum is less than or equal to one. In 1989, Atanassov and Gargov [4] developed a new them with the help of intuitionistic fuzzy set which is known as interval valued intuitionistic fuzzy set. Lee et al. [5] in 2020 introduced octahedron set by combining interval-valued fuzzy set, an intuitionistic fuzzy set and a fuzzy set. The theme of neutrosophic sets developed by Smarandache [6–8] by expanding Atanasove's ideas. He created the term "neutrosophic" because "neutrosophy" is etymologically related to "neutrosophic." On the other hand, Lupiáñez [9] developed the structure of neutrosophic sets and their topology with basic algebraic operations. In 2005, Wang et al. [10] developed the structure of interval neutrosophic sets. In 2009, Bhowmik and Pal [11] distinguished between truth-based intuitionist neutrosophic sets and intuitionist neutrosophic sets. They established that all intuitionistic neutrosophic sets are neutrosophic sets, but not all neutrosophic sets are intuitionistic neutrosophic sets. In addition, some new INS operations have been defined, as well as illustrations of how the operations could be implemented in real scenarios. Maji [12] developed the structure of neutrosophic soft sets by using Smarandache's idea of neutrosophic sets and also introduced some basic definitions and operation. In 2015, Alkhazaleh and Uluçay [13] initiated the theme of neutrosophic soft expert set with basic operations and also discussed real-life application. Alias et al. [14] established the theme of rough neutrosophic multisets in 2017 with basic operations and properties. The fuzzy sets discussed above are incapable of handling imprecise, uncertain, inconsistent, and incomplete periodic information. To overcome this challenge, Ali and Smarandache [15] expand the idea of neutrosophic sets and developed the structure of complex neutrosophic set. There are some many other applications for solving these uncertainty like [16–19]. In 2012, Balin et al. [20] developed multicriteria decision making model for energy sources. Huang et al. [21] worked on the application used in multicriteria decision making technique in the field of environmental science. TOPSIS is a most useful method. According to certain studies, the TOPSIS technique exhibits a monotonically increasing or decreasing preference for each criterion [22, 23]. Compensatory approaches, like TOPSIS, are widely used in numerous fields of multicriteria decision-making due to the potential of criteria modeling. Some researchers [24, 25] worked on the significance of TOPSIS approach in MADM problem. Pehlivan and Yalçın [26] utilized the TOPSIS approach in a neutrosophic environment to identify sustainable suppliers in a low market chain in 2022. Distinct techniques [27, 28, 30] utilize different versions of neutrosophic sets in decision making challenges, such as single valued neutrosophic sets, single valued neutrosophic type2 fuzzy sets, and type2 neutrosophic model. Jun et al. [29] discovered the cubic set in 2012. Jun et al. [30] studied the concept of cubic subalgebras/ideals in BCK/BCI-algebras and their characteristics. Jun et al. [31] have also presented the neutrosophic cubic set notion (NCS). Gulistan and Khan [32] show the extension of neutrosophic cubic set via complex fuzzy set with application. Some researchers [33, 34] have used the different version of fuzzy sets in the decision-making environment.

Since neutrosophic set provides higher uncertainty and ambiguity than intuitionistic fuzzy set, interval valued fuzzy set and fuzzy set. To further analyze uncertainty, we therefore require a neutrosophic octahedron set. Compared to intuitionistic octahedron sets and octahedron sets, neutrosophic octahedron sets also reduce information loss about ambiguity and uncertainty. So, neutrosophic octahedron set covers broader area as compare to intuitionistic fuzzy set, fuzzy set, and interval valued fuzzy set.

1.1. Contribution of the Study. The following is a list of the planned study's contributions.

(1) Interval number, intuitionistic number, octahedron number, neutrosophic set, and octahedron set are some of the core notions discussed in the literature

(2) This work conceptualises the construction of a NOS with set theoretic operation

(3) In a neutrosophic octahedron environment, the TOPSIS method is proposed

(4) The paper is summarised, along with its scope and future research prospects

1.2. Organization of the Study. The following is a diagram illustrating the study's structure: Section 2: recall some useful information from the previous research. The construction of the NOS is described in Section 3 as a novel mathematical instrument for solving the problem of uncertainty. Introduce the internal and external NOSs, as well as their union and intersection. The NOS's operational features are addressed. Also, the practical element of the suggested structure is developed in this section. Section 4 describes the TOPSIS approach in the context of a NOS as a decision-making problem, and Section 5 describes the comparison, while Section 6 summarises the conclusion and future directions.

2. Materials and Methods

This section of the document reviews the available literature to give some basic materials and methods for a clear understanding of the planned work.

Definition 1 (see [4]). A intuitionistic neutrosophic set is the structure of the form $A = (x, T(x), I(x), F(x))$ such that $T(x) \wedge I(x) \leq 0.5$, $T(x) \wedge F(x) \leq 0.5$, $F(x) \wedge I(x) \leq 0.5$, with $0 \leq T(x) + I(x) + F(x) \leq 2$, for all $x \in X$.

Definition 2 (see [5]). Denote members of $[I] \times (I \oplus I) \times I$ as

$$\tilde{\tilde{x}} = <\tilde{x}, \bar{\lambda}, x> \ = \ < [x-, x-], \left(\nu^{\in}, x^{\notin}\right), x>, \tag{1}$$

and it is called octahedron number.

Definition 3 (see [5]). Let X be the collection of some elements and let $A^O = [A^-, A^+] \in I]^X$, $B^O = (B^{\in}, B^{\notin}) \in (I \oplus I)^X$, and $\lambda^O \in I^X$. Then, the triplet $O = \langle A^O, B^O, \lambda^O \rangle$ is called an octahedron set in X. The mapping $O : X \longrightarrow I] \times (I \oplus I) \times I$ is known as octahedron.

Definition 4 (see [8]). Let X be the collection of some elements. A neutrsophic set in X is a structure of the type $A = \{x\,; T(x), I(x), F(x) | x \in X\}$, which is characterised by truth-membership (t-MM) T, indeterminacy-membership (i-MM) I, and falsity-membership(f-MM) F, in such a way that $0 \leq T(x) + I(x) + F(l\text{times}) \leq 3$.

3. Neutrosophic Octahedron Sets with Basic Operations

In this section, we introduce new notion of NOS with some interesting properties and basic operations. Also the score function, neutrosophic octahedron weighted average operator, and neutrosophic octahedron order the weighted average operator are discussed.

Definition 5. Let X be the collection of some elements. A structure of the form $A=(A_1, A_2, A_3)$, where $A_1 : X \longrightarrow I]$ denotes the interval valued neutrsophic set, $A_2 : X \longrightarrow (I \oplus I)$ denotes the intuitionistic neutrosophic set, $A_3 : X \longrightarrow I$ denotes the neutrsophic set, is called the neutrosophic octahedron set (NOS) with $A : X \longrightarrow I] \times (I \oplus I) \times I$.

Example 1. Let $X=\{\dot{x}, \ddot{x}, \dddot{x}\}$ be a nonempty set and $A=(A_1, A_2, A_3)$: $X \longrightarrow I] \times I \oplus I] \times I$ be the mapping given by

$$A(\dot{x})=\left\langle \begin{array}{c} A_1(\dot{x})=([0.2,0.4],[0.3,0.5],[0.3,0.5]), \\ A_2(\dot{x})=(0.8,0.2,0.4), \\ A_3(\dot{x})=(0.6,0.8,0.4) \end{array} \right\rangle, \quad (2)$$

$$A(\ddot{x})=\left\langle \begin{array}{c} A_1(\ddot{x})=([0.3,0.4],[0.4,0.5],[0.4,0.6]), \\ A_2(\ddot{x})=(0.8,0.2,0.4), \\ A_3(\ddot{x})=(0.5,0.7,0.6) \end{array} \right\rangle, \quad (3)$$

$$A(\dddot{x})=\left\langle \begin{array}{c} A_1(\dddot{x})=([0.1,0.3],[0.4,0.6],[0.4,0.5]), \\ A_2(\dddot{x})=(0.8,0.2,0.4), \\ A_3(\dddot{x})=(0.4,0.6,0.5) \end{array} \right\rangle. \quad (4)$$

Then, $A=(A_1, A_2, A_3)$ is NOS.

Definition 6. Let X be the collection of some elements. A structure of the form $A=(A_1, A_2, A_3)$, where $A_1=\{[A_T^-, A_T^+], [A_I^-, A_I^+], [A_F^-, A_F^+]\} \in I]$, $A_2=(A_{T^*}, A_{I^*}, A_{F^*}) \in (I \oplus I)^X$, $A_3=\{A_T, A_I, A_F\} \in I$, is called the NOS in X, with the mapping, $A : X \longrightarrow I] \times (I \oplus I) \times I$. We consider following special NOSs:

$$\left\langle \hat{0}, \check{0}, 0 \right\rangle = 0, \quad (5)$$

$$\left\langle \hat{0}, \check{0}, 1 \right\rangle, \left\langle \hat{0}, \check{1}, 0 \right\rangle, \left\langle 0, 0, 1 \right\rangle, \quad (6)$$

$$\left\langle \hat{0}, \check{1}, 1 \right\rangle, \left\langle \hat{1}, 0, 1 \right\rangle, \left\langle \hat{1}, \check{1}, 0 \right\rangle, \quad (7)$$

$$\left\langle \hat{1}, \check{1}, 1 \right\rangle = 1. \quad (8)$$

In the above case, 0 (resp., 1) is called a neutrosophic octahedron empty (resp., neutrosophic octahedron whole set) in X.

Remark 7.

(1) Every NOS is an Octahedron set

The set of all NOS of X is denoted by $N^O(X)$.

Definition 8. Let X be the collection of some elements and let $A=(A_1, A_2, A_3)$, and $B=(B_1, B_2, B_3) \in N^O(X)$. Then, we can define the order relations between A and B as follows:

(i) Equality

$$A=B \text{ if and only if } A_1=B_1, \quad A_2=B_2, \quad A_3=B_3, \quad (9)$$

(ii) Type 1-order

$$A \subset_1 B \text{ if and only if } A_1 \subset B_1, \quad A_2 \subset B_2, \quad A_3 \leq B_3, \quad (10)$$

(iii) Type 2-order

$$A \subset_2 B \text{ if and only if } A_1 \subset B_1, \quad A_2 \subset B_2, \quad A_3 \geq B_3, \quad (11)$$

(iv) Type 3-order

$$A \subset_3 B \text{ if and only if } A_1 \subset B_1, \quad A_2 \supset B_2, \quad A_3 \leq B_3, \quad (12)$$

(v) Type 4-order

$$A \subset_3 B \text{ if and only if } A_1 \subset B_1, \quad A_2 \supset B_2, \quad A_3 \geq B_3. \quad (13)$$

Definition 9. Let X denote a universe of discourse and (A_j) $j \in \bar{J}=\langle A_{1j}, A_{2j}, A_{3j} \rangle j \in \bar{J}$ denote a family of neutrosophic octahedron sets in X. Then, for $(A_j) j \in J (i=1,2,3,4)$, the type i-union $\cup^i$ and type i-intersection $\cap^i$ are defined as follows:

(i) Type i-union

$$\cup^1_{j \in \bar{J}} A=\left(\cup_{j \in \bar{J}} A_{1j}, \cup_{j \in \bar{J}} A_{2j}, \cup_{j \in \bar{J}} A_{3j}\right), \quad (14)$$

$$\cup^2_{j\in\bar{J}} A = \left(\cup_{j\in\bar{J}} A_{1j}, \cup_{j\in\bar{J}} A_{2j}, \cap_{j\in\bar{J}} A_{3j}\right), \quad (15)$$

$$\cup^3_{j\in\bar{J}} A = \left(\cup_{j\in\bar{J}} A_{1j}, \cap_{j\in\bar{J}} A_{2j}, \cup_{j\in\bar{J}} A_{3j}\right), \quad (16)$$

$$\cup^4_{j\in\bar{J}} A = \left(\cup_{j\in\bar{J}} A_{1j}, \cap_{j\in\bar{J}} A_{2j}, \cap_{j\in\bar{J}} A_{3j}\right). \quad (17)$$

(ii) Type i-intersection

$$\cap^2_{j\in\bar{J}} A = \left(\cap_{j\in\bar{J}} A_{1j}, \cap_{j\in\bar{J}} A_{2j}, \cup_{j\in\bar{J}} A_{3j}\right), \quad (18)$$

$$\cap^3_{j\in\bar{J}} A = \left(\cap_{j\in\bar{J}} A_{1j}, \cup_{j\in\bar{J}} A_{2j}, \cap_{j\in\bar{J}} A_{3j}\right), \quad (19)$$

$$\cap^4_{j\in\bar{J}} A = \left(\cap \cap_{j\in\bar{J}} A_{1j}, \cup_{j\in J} A_{2j}, \cup_{j\in\bar{J}} A_{3j}\right). \quad (20)$$

Proposition 10. *Let X be the collection of some elements and let $A = (A_1, A_2, A_3)$, $B = (B_1, B_2, B_3)$, $C = (C_1, C_2, B_3)$, and $\bar{\propto} = (\bar{\propto}_1, \bar{\propto}_2, \bar{\propto}_3)$ be neutrosophic octrahedron sets. Then, for each $i = 1, 2, 3, 4$.*

(i) If $A \subset_i B$ and $B \subset_i C$ then $A \subset_i C$

(ii) If $A \subset_i B$ and $A \subset_i C$ then $A \subset_i B \cap C$

(iii) If $A \subset_i B$ and $C \subset_i B$ then $A \cup C \subset_i B$

(iv) If $A \subset_i B$ and $C \subset_i \bar{\propto}$ then $A \cup C \subset_i B \cup \bar{\propto}$ and $A \cap C \subset_i B \cap \bar{\propto}$

Definition 11. Let X be the collection of some elements and let $A = (A_1, A_2, A_3)$ be a neutrosophic octahedron set in X. Then, the complement A^c,[] and $\diamond$ of A are defined as follows:

(i) $A^C = (A_1, A_2, A_3)$

(ii) $[]A = (A_1, []A_2, A_3)$

(iii) $\diamond A = (A_1, \diamond A_2, A_3)$

From Definition 6, we can easily see that the following holds:

$$\hat{0}^c = 1, 1^c = 0, \quad (21)$$

$$\left\langle \hat{0}, \check{0}, 1 \right\rangle^c = \langle \hat{1}, \check{1}, 0 \rangle, \langle \hat{1}, \check{1}, 0 \rangle^c = \left\langle \hat{0}, \check{0}, 1 \right\rangle, \quad (22)$$

$$\left\langle \hat{0}, \check{1}, 0 \right\rangle^c = \left\langle \hat{1}, \check{0}, 1 \right\rangle, \left\langle \hat{1}, \check{0}, 1 \right\rangle^c = \left\langle \hat{0}, \check{1}, 0 \right\rangle, \quad (23)$$

$$\left\langle \hat{1}, \check{0}, 0 \right\rangle^c = \langle 0, \hat{1}, \check{1} \rangle, \langle 0, \hat{1}, \check{1} \rangle^c = \left\langle \hat{1}, \check{0}, 0 \right\rangle, \quad (24)$$

$$\left\langle \hat{0}, \check{1}, 1 \right\rangle^c = \left\langle \hat{1}, \check{0}, 0 \right\rangle, \left\langle \hat{1}, \check{0}, 0 \right\rangle^c = \left\langle \hat{0}, \check{1}, 1 \right\rangle, \quad (25)$$

$$\left\langle \hat{1}, \check{0}, 1 \right\rangle^c = \left\langle \hat{0}, \check{1}, 0 \right\rangle, \left\langle \hat{0}, \check{1}, 0 \right\rangle^c = \left\langle \hat{1}, \check{0}, 1 \right\rangle, \quad (26)$$

$$\langle \hat{1}, \check{1}, 0 \rangle^c = \left\langle \hat{0}, \check{0}, 1 \right\rangle, \left\langle \hat{0}, \check{0}, 1 \right\rangle^c = \langle \hat{1}, \check{1}, 0 \rangle. \quad (27)$$

Remark 12. The union, intersection, and complement of NOS does not hold in general, i.e., $A \cup A^c = 1$ and $A \cap A^c = 0$.

Proposition 13. *Let X be the collection of some elements and let $A = (A_1, A_2, A_3)$, and $B = (B_1, B_2, B_3)$ be two neutrosophic octrahedron sets in X. If $A \subset_i B$, then $B^C \subset_i A^C$, for each $i = 1, 2, 3$.*

Proposition 14. *Let $A \in N^O(X)$ and let $(A_J)_j \in J \subset N^O(X)$. Then*

(i) $(A^C)^C = A$

(ii) For each $i = 1, 2, 3$

$$\left(\bigcup_{j\in J}^{i} A_j\right)^C = \bigcap_{j\in J}^{i} A_j^C, \quad (28)$$

$$\left(\bigcap_{j\in J}^{i} A_j\right)^C = \bigcup_{j\in J}^{i} A_j^C. \quad (29)$$

Proposition 15. *Let X be the collection of some elements and let $A = (A_1, A_2, A_3)$, and $B = (B_1, B_2, B_3)$ be two neutrosophic octrahedron sets in X. If $A \subset B$, then $N^N_{CB} \subset N^N_{CA}$ for each $i = 1, 2, 3, 4$.*

Definition 16. Let X be the collection of some elements and let $A = (A_1, A_2, A_3) \in N^O(X)$, then, A is called an internal and external neutrosopic octahedron set if the following are satisfied:

A truth-internal NOS (briefly, INOS) in X, for each $x \in X$,

$$A_{2TA}(x), A_{3TA}(x) \in A_1 = ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]). \quad (30)$$

An indeterminacy-internal NOS (briefly, INOS) in X, for each $x \in X$,

$$A_{2IA}(x), A_{3IA}(x) \in A_1 = ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]). \quad (31)$$

A falsity-internal NOS (briefly, INOS) in X, for each $x \in X$,

$$A_{2FA}(x), A_{3FA}(x) \in A_1 = ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]). \quad (32)$$

A truth-external NOS (briefly, $\notin$ −ENOS) in X, for each

Figure 1: Source of solar energy.

Figure 2: Source of wind energy.

$x \in X$,

$$A_{2TA}(x), A_{3TA}(x) \notin A_1 = ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]). \tag{33}$$

An indeterminacy-external NOS (briefly, INOS) in X, for each $x \in X$,

$$A_{2IA}(x), A_{3IA}(x) \notin A_1 = ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]). \tag{34}$$

A falsity-external NOS (briefly, INOS) in X, for each $x \in X$,

$$A_{2FA}(x), A_{3FA}(x) \notin A_1 = ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]). \tag{35}$$

Proposition 17. *Let X be the collection of some elements and*

Figure 3: Source of geothermal energy.

Figure 4: Source of hydropower energy.

let $A = (A_1, A_2, A_3) \in N^O(X)$. If A is not external NOSs, then, there is $x \in X$ such that

$$A_2(x) \in ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]), \tag{36}$$

or

$$1 - A_2(x) \in ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]), \tag{37}$$

$$A_3(x) \in ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]). \tag{38}$$

Proposition 18. *Let X be the collection of some elements and let $A = (A_1, A_2, A_3) \in A(X)$. if A is both internal and external NOSs, then, there is $x \in X$,*

$$A_2(x), 1 - A_2(x), A_3(x) \in U(A_1) \cup L(A_1), ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}]), \tag{39}$$

Figure 5: Ranking.

where $U(A_1) = \{A^+_{TA}, A^+_{IA}, A^+_{FA} : x \in X\}$ *and* $L(A_2) = \{A^-_{TA}, A^-_{IA}, A^-_{FA} : x \in X\}$.

Proposition 19. *Let X be the collection of some elements and let* $A = (A_1, A_2, A_3) \in A(X)$. *if A is an internal (resp., external) NOSs, then,* $\urcorner A$ *(complement) is external (resp., internal).*

Example 2. Let $A = \langle A_1, A_2, A_3\rangle$ be a NOS in X given by for each $x \in X$,

$$A(x) = \left\langle \begin{array}{c} \left(\left[\frac{x}{4}, \frac{1+x}{2}\right], \left[\frac{x}{6}, \frac{1+x}{4}\right], \left[\frac{x}{8}, \frac{1+x}{6}\right]\right), \\ \left(\left[\frac{x}{3}, \frac{1+x}{5}\right], \left[\frac{x}{5}, \frac{1+x}{7}\right], \left[\frac{\tilde{a}}{7}, \frac{1+x}{9}\right]\right), \left(\frac{x}{2}, \frac{x}{4}, \frac{x}{6}\right) \end{array} \right\rangle. \tag{40}$$

$A_2(x), A_3(x) \in A_1$ for each $x \in X$ is then easily calculated, but $A_2(x) \notin ([A^-_{TA}, A^+_{TA}], [A^-_{IA}, A^+_{IA}], [A^-_{FA}, A^+_{FA}])$, for each $x \in X$ such that $x > 3/7$. Thus, A is an $\in-$INOS but not a $\notin -$INOS in X.

Proposition 20. *Let X be the collection of some elements and let* $A = (A_1, A_2, A_3)$, *and* $B = (B_1, B_2, B_3)$ *be two neutrosophic octrahedron sets in X. Suppose A and B are internal for each* $x \in X$.

Definition 21. The sum between two NOSs $A = (A_1, A_2, A_3)$, and $B = (B_1, B_2, B_3)$ is defined as

$$A \oplus B = (A_1 + B_1 - A_1.B_1, A_2 + B_2 - A_2.B_2, A_3 + B_3 - A_3.B_3). \tag{41}$$

Definition 22. The product between two NOSs $A = (A_1, A_2, A_3)$, and $B = (B_1, B_2, B_3)$ is defined as $A \otimes B = (A_1.B_1, A_2.B_2, A_3.B_3)$.

Definition 23. Scalar multiplication with a neutrosophic octahedron set of a Scalar $\lambda A = (A_1, A_2, A_3)$, is defined as λ A.

Theorem 24. *Let* $A = (A_1, A_2, A_3)$, $B = (B_1, B_2, B_3)$ *and* $C = (C_1, C_2, C_3)$ *be three NOSs of A, where A be a collection of NOSs. Then* $(A, \oplus)$ *is a commutative monoid.*

Proof.

(1) Let $A, B \in A$. Then, we have

$$A \oplus B = \langle (A_1 + B_1 - A_1.B_1, A_2 + B_2 - A_2.B_2, A_3 + B_3 - A_3.B_3)\rangle, \tag{42}$$

which is clearly in A.

(2) Let $A, B, C \in A$. Then, we prove $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

$$\begin{aligned} &\left(N_1^{NC} \oplus N_2^{NC}\right) \oplus N_3^{NC} \\ &\quad = \langle (A_1, A_2, A_3) \oplus (B_1, B_2, B_3)\rangle \oplus \langle C_1, C_2, C_3\rangle \\ &\quad = \langle (A_1 + B_1 - A_1.B_1, A_2 + B_2 - A_2.B_2, A_3 + B_3 - A_3.B_3)\rangle \\ &\qquad \oplus \langle C_1, C_2, C_3\rangle = (A_1, A_2, A_3) \\ &\qquad \oplus \langle (B_1 + C_1 - B_1.C_1, B_2 + C_2 - B_2.C_2, B_3 + C_3 - B_3.C_3)\rangle \\ &\quad = A \oplus (B \oplus C). \end{aligned} \tag{43}$$

(3) Let $A, B \in A$. Then, we have

$$
\begin{aligned}
A \oplus B &= \langle (A_1, A_2, A_3) \oplus (B_1, B_2, B_3) \rangle \\
&= \langle (A_1 + B_1 - A_1.B_1, A_2 + B_2 - A_2.B_2, A_3 + B_3 - A_3.B_3) \rangle \\
&= \langle (B_1 + A_1 - B_1.A_1, B_2 + A_2 - B_2.A_2, B_3 + A_3 - B_3.A_3) \rangle \\
&= B \oplus A.
\end{aligned} \tag{44}
$$

Hence, $(A, \oplus)$ is a commutative semigroup. □

Theorem 25. *Let $A = (A_1, A_2, A_3)$, and $B = (B_1, B_2, B_3)$ be any two NOSs. Then, the following holds*

(1) $\ddot{\lambda}(A \oplus B) = \ddot{\lambda}A \oplus \ddot{\lambda}B$

(2) $(\ddot{\lambda}_1 + \ddot{\lambda}_2)A = \ddot{\lambda}_1 A + \ddot{\lambda}_2 A$, where $\ddot{\lambda}$ is any scalar

Proof.

(1) Let A, B be two NOSs and $\ddot{k}_0$ be any constant. Then, we have

$$\ddot{\lambda}_0(A \oplus B) = \ddot{\lambda}_0(\langle A_1, A_2, A_3 \rangle \oplus \langle B_1, B_2, B_3 \rangle)$$

$$= \ddot{\lambda}_0 \langle (A_1 + B_1 - A_1.B_1, A_2 + B_2 - A_2.B_2, A_3 + B_3 - A_3.B_3) \rangle$$

$$= \left\langle \begin{array}{c} \left(-1 - (-1 - (A_1 + B_1 - A_1.B_1))^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - (A_2 + B_2 - A_2.B_2))^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - (A_3 + B_3 - A_3.B_3))^{\ddot{\lambda}_0}\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(-1 - (-1 - A_1 - B_1 + A_1.B_1)^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - A_2 - B_2 + A_2.B_2)^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - A_3 - B_3 + A_3.B_3)^{\ddot{\lambda}_0}\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(-1 - (-1 - A_1 - B_1(1 - A_1.B_1))^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - A_2 - B_2(1 - A_2.B_2))^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - A_3 - B_3(1 - A_3.B_3))^{\ddot{\lambda}_0}\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(-1 - (-1 - A_1 - B_1(1 - B_1))^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - A_2 - B_2(1 - B_2))^{\ddot{\lambda}_0}\right), \\ \left(-1 - (-1 - A_3 - B_3(1 - B_3))^{\ddot{\lambda}_0},\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(-1 - ((-1 - A_1)(1 - B_1))^{\ddot{\lambda}_0}\right), \\ \left(-1 - ((-1 - A_2 I)(1 - B_2))^{\ddot{\lambda}_0}\right), \\ \left(-1 - ((-1 - A_3)(1 - B_3))^{\ddot{\lambda}_0}\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(\begin{array}{c} -1 - (-1 - A_1)^{\ddot{\lambda}_0} - (1 + B_2)^{\ddot{\lambda}_0} + 1 + (-1 - A_1)^{\ddot{\lambda}_0} \\ +(1 + B_1)^{\ddot{\lambda}_0} - (-1 - A_1)^{\ddot{\lambda}_0}(1 + B_1)^{\ddot{\lambda}_0}, \end{array} \right), \\ \left(\begin{array}{c} -1 - (-1 - A_2)^{\ddot{\lambda}_0} - (1 + B_2)^{\ddot{\lambda}_0} + 1 + (-1 - A_2)^{\ddot{\lambda}_0} \\ +(1 + B_2)^{\ddot{\lambda}_0} - (-1 - A_2)^{\ddot{\lambda}_0}(1 + B_2)^{\ddot{\lambda}_0}, \end{array} \right), \\ \left(\begin{array}{c} -1 - (-1 - A_3)^{\ddot{\lambda}_0} - (1 + B_3)^{\ddot{\lambda}_0} + 1 + (-1 - A_3)^{\ddot{\lambda}_0} \\ +(1 + B_3)^{\ddot{\lambda}_0} - (-1 - A_3)^{\ddot{\lambda}_0}(1 + B_3)^{\ddot{\lambda}_0}, \end{array} \right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(\begin{array}{c} 2 - (-1 - A_1)^{\ddot{\lambda}_0} - (1 + B_1)^{\ddot{\lambda}_0} - \left(-1 - (1 + B_1)^{\ddot{\lambda}_0} \right. \\ \left. -(-1 - A_1)^{\ddot{\lambda}_0}\right) + (-1 - A_1)^{\ddot{\lambda}_0}(1 + B_1)^{\ddot{\lambda}_0}, \end{array} \right), \\ \left(\begin{array}{c} 2 - (-1 - A_2)^{\ddot{\lambda}_0} - (1 + B_2)^{\ddot{\lambda}_0} - (-1 - (1 + B_2)^{\ddot{\lambda}_0} \\ \left. -(-1 - A_2)^{\ddot{\lambda}_0}\right) + (-1 - A_2)^{\ddot{\lambda}_0}(1 + B_2)^{\ddot{\lambda}_0}, \end{array} \right), \\ \left(\begin{array}{c} 2 - (-1 - A_3)^{\ddot{\lambda}_0} - (1 + B_3)^{\ddot{\lambda}_0} - (-1 - (1 + B_3)^{\ddot{\lambda}_0} \\ \left. -(-1 - A_3)^{\ddot{\lambda}_0}\right) + -(-1 - A_1)^{\ddot{\lambda}_0}\Big) + (-1 - A_3)^{\ddot{\lambda}_0}(1 + B_3)^{\ddot{\lambda}_0}, \end{array} \right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(\begin{array}{c} -1 - (-1 - A_1)^{\ddot{\lambda}_0} + 1 - (1 - B_1)^{\ddot{\lambda}_0} \\ -\left(1 - (-1 - A_1)^{\ddot{\lambda}_0}\right)\left(1 - (1 - B_1)^{\ddot{\lambda}_0}\right), \end{array} \right), \\ \left(\begin{array}{c} -1 - (-1 - A_2)^{\ddot{\lambda}_0} + 1 - (1 - B_2)^{\ddot{\lambda}_0} \\ -\left(1 - (-1 - A_2)^{\ddot{\lambda}_0}\right)\left(1 - (1 - B_2)^{\ddot{\lambda}_0}\right), \end{array} \right), \\ \left(\begin{array}{c} -1 - (-1 - A_3)^{\ddot{\lambda}_0} + 1 - (1 - B_3)^{\ddot{\lambda}_0} \\ -\left(1 - (-1 - A_3)^{\ddot{\lambda}_0}\right)\left(1 - (1 - B_3)^{\ddot{\lambda}_0}\right), \end{array} \right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} -1 - (-1 - A_1)^{\ddot{\lambda}_0}, \\ -1 - (-1 - A_2)^{\ddot{\lambda}_0}, \\ -1 - (-1 - A_3)^{\ddot{\lambda}_0} \end{array} \right\rangle \oplus \left\langle \begin{array}{c} -1 - (-1 - B_1)^{\ddot{\lambda}_0}, \\ -1 - (-1 - B_2)^{\ddot{\lambda}_0}, \\ -1 - (-1 - B_3)^{\ddot{\lambda}_0} \end{array} \right\rangle,$$

$$= \ddot{\lambda}_0 \langle A_1, A_2, A_3 \rangle \oplus \ddot{\lambda}_0 \langle B_1, B_2, B_3 \rangle \tag{45}$$

we have $\ddot{\lambda}_0(A \oplus B) = \lambda A \oplus \ddot{\lambda}_0 B$.

(2) Let $\Omega \in A$ and $\ddot{\lambda}_1, \ddot{\lambda}_2$ be any constant. Then, we have

$$\left(\ddot{\lambda}_1+\ddot{\lambda}_2\right)A=\left(\ddot{\lambda}_1+\ddot{\lambda}_2\right)\langle A_1,A_2,A_3\rangle$$

$$=\left\langle\begin{array}{c}\left(-1-(-1-A_1)^{\ddot{\lambda}_1+\ddot{\lambda}_2}\right),\\ \left(-1-(-1-A_2)^{\ddot{\lambda}_1+\ddot{\lambda}_2}\right),\\ \left(-1-(-1-A_3)^{\ddot{\lambda}_1+\ddot{\lambda}_2}\right)\end{array}\right\rangle$$

$$=\left\langle\begin{array}{c}\begin{pmatrix}-2-(1-A_1)^{\ddot{\lambda}_1}-(1-A_1)^{\ddot{\lambda}_2}+(1-A_1)^{\ddot{\lambda}_1}\\ -1+(1-A_1)^{\ddot{\lambda}_2}-(1-A_1)^{\ddot{\lambda}_1+\ddot{\lambda}_2},\end{pmatrix},\\ \begin{pmatrix}-2-(1-A_2)^{\ddot{\lambda}_1}-(1-A_2)^{\ddot{\lambda}_2}+(1-A_2)^{\ddot{\lambda}_1}\\ -1+(1-A_2)^{\ddot{\lambda}_2}-(1-A_2)^{\ddot{\lambda}_1+\ddot{\lambda}_2},\end{pmatrix},\\ \begin{pmatrix}-2-(1-A_3)^{\ddot{\lambda}_1}-(1-A_3)^{\ddot{\lambda}_2}+(1-A_3)^{\ddot{\lambda}_1}\\ -1+(1-A_3)^{\ddot{\lambda}_2}-(1-A_3)^{\ddot{\lambda}_1+\ddot{\lambda}_2},\end{pmatrix}\end{array}\right\rangle$$

$$=\left\langle\begin{array}{c}\begin{pmatrix}-1-(-1-A_1)^{\ddot{\lambda}_1}+1-(-1-A_1)^{\ddot{\lambda}_2}\\ -\left(1-(1-A_1)^{\ddot{\lambda}_1}\right)\left(-1-(1-A_1)^{\ddot{\lambda}_2}\right)\end{pmatrix},\\ \begin{pmatrix}-1-(-1-A_2)^{\ddot{\lambda}_1}+1-(-1-A_2)^{\ddot{\lambda}_2}\\ -\left(1-(1-A_2)^{\ddot{\lambda}_1}\right)\left(-1-(1-A_2)^{\ddot{\lambda}_2}\right),\end{pmatrix},\\ \begin{pmatrix}-1-(-1-A_3)^{\ddot{\lambda}_1}+1-(-1-A_3)^{\ddot{\lambda}_2}\\ -\left(1-(1-A_3)^{\ddot{\lambda}_1}\right)\left(-1-(1-A_3)^{\ddot{\lambda}_2}\right),\end{pmatrix}\end{array}\right\rangle$$

$$=\left\langle\begin{array}{c}1-(1-A_1)^{\ddot{\lambda}_1},\\ 1-(1-A_2)^{\ddot{\lambda}_1},\\ 1-(1-A_3)^{\ddot{\lambda}_1}\end{array}\right\rangle\oplus\left\langle\begin{array}{c}-1-(-1-A_1)^{\ddot{\lambda}_2},\\ -1-(-1-A_2)^{\ddot{\lambda}_2},\\ -1-(-1-A_3)^{\ddot{\lambda}_2}\end{array}\right\rangle \tag{46}$$

we have $(\ddot{\lambda}_1+\ddot{\lambda}_2)A_1=\ddot{\lambda}_1A_1+\ddot{\lambda}_2A_1$. □

Definition 26. Let $A=(A_1,A_2,A_3)$ be a NOS, and we define score function as

$$S(A)=\frac{A_1+A_2+A_3}{12}. \tag{47}$$

Definition 27. Let $A=(A_1,A_2,A_3)_t(t=1,2,\cdots m)$ be the collection of values of neutrosophic octahedron and weighted average operator is defined as $\mathrm{NOWA}:\Omega^n\longrightarrow\Omega$ by $\mathrm{NOWA}_w(A_1,A_2\cdots A_m)=\sum_{Tru=1}^{m}w_tA_1$, where $W=(w_1,w_2,\cdots,w_m)^t$ is the weight vector, such that $w_t\in 0,1]$ and $\sum_{t=1}^{m}w_t=1$.

Definition 28. Let $A=(A_1,A_2,A_3)_t(t=1,2,\cdots m)$ be the collection of values of neutrosophic octahedron and order weighted average operator as $\mathrm{NOOWA}:\Omega^n\longrightarrow\Omega$ by $\mathrm{NOOWA}_w(A_1,A_1\cdots,A_1)=\sum_{t=1}^{m}w_tA_1$, where NOOWA is order weighted average operator A_1 is the the largest, $W=(w_1,w_2\cdots,w_m)^t$ is the weight vector of $A_1(t=1,2,\cdots m)$, such that $w_t\in 0,1]$ and $\sum_{t=1}^{m}w_t=1$.

4. Energy Source Selection by TOPSIS Method

It is essential to select an energy source that has the least impact on the natural environment, and it must take into account crucial factors like as reliability, cost, and maintenance. As a result, selecting the optimal energy source is not a simple task, as this decision may be fraught with uncertainty and ambiguity. To deal with ambiguity and vagueness, Zadeh developed the fuzzy theory. In 1975, he defined interval-valued fuzzy sets as a more general class of fuzzy sets. Intuitionistic fuzzy sets, neutrosophic sets, interval neutrosophic sets, intuitionistic neutrosophic sets, neutrosophic cubic sets, neutrosophic soft sets, rough neutrosophic sets, and octahedron sets are some well-known kinds of fuzzy sets. We use neutrosophic octahedron sets to define decision making problem. The algorithms are proposed in this section. The algorithm shows the procedure of TOPSIS method based on the following terminologies. Some example of energy sources are solar energy, wind energy, geothermal energy, and hydropower energy.

Solar energy: solar power is the conversion of solar energy into thermal or electrical energy. Solar energy is the most abundant and environmentally friendly source of renewable energy available today. The source of solar energy is shown as in Figure 1.

Wind energy: wind is a type of solar energy. Winds are created by the heating of the atmosphere by the sun, the rotation of the Earth, and irregularities in its surface. The source of wind energy is shown as in Figure 2.

Geothermal energy: geothermal energy is the heat that exists in the earth's crust. Geothermal energy is derived from the Greek words geo (earth) and therm (heat). Because heat is constantly produced in the earth, geothermal energy is a renewable energy source. The source of geothermal energy is shown as in Figure 3.

Hydropower energy: the conversion of energy from running water into electricity is known as hydroelectricity. It is the oldest and largest renewable energy source in the world. The source of hydropower energy is shown as in Figure 4.

These energy sources are renewable. These resources do not pollute the environment in any way, and H = human activities have no effect on renewable resources. It is important to choose the best energy source for their country which minimum effects the environment. The important parameter of energy sources is reliability, yields, cost, and maintenance. Where U_1, U_2, U_3, and U_4 stand for solar energy, wind energg, geothermal energy, and hydropower energy. These sources are evaluated against the four parameters which are represented by $\ddot{\lambda}_1$, $\ddot{\lambda}_v$, $\ddot{\lambda}_3$, and $\ddot{\lambda}_4$ where these parameters stand for reliability, yields, cost, and maintenance.

For this purpose, we select a panel which are consist of expertise. The panel assessed the energy sources according to given criteria. The panel gives their judgements in the form of decision matrix. Suppose the decision matrix is represented by $a=[a_{i_j}]m\times n$, where a_{i_j} shows evaluation of ith alternative with respect to jth criteria.

Step 1. Standardize the decision matrix as follows:

$$
D=\begin{array}{ccccc}
 & U_1 & U_2 & U_3 & U_4 \\
\ddot{\lambda}_1 & \left\{\begin{matrix}0.1,0.2],\\0.3,0.1],\\0.1,0.2],\\(0.5,0.2,0.4),\\(0.2,0.3,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.1],\\0.1,0.1],\\0.1,0.1],\\(0.3,0.2,0.4),\\(0.3,0.5,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.2],\\0.1,0.2],\\0.1,0.2],\\(0.3,0.2,0.4),\\(0.3,0.5,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.2],\\0.1,0.1],\\0.1,0.2],\\(0.3,0.2,0.4),\\(0.3,0.5,0.4)\end{matrix}\right\}, \\
\ddot{\lambda}_2 & \left\{\begin{matrix}0.1,0.1],\\0.1,0.2],\\0.1,0.3],\\(0.5,0.2,0.3),\\(0.3,0.4,0.2)\end{matrix}\right\} & \left\{\begin{matrix}0.2,0.1],\\0.3,0.2],\\0.1,0.2],\\(0.3,0.2,0.4),\\(0.5,0.7,0.6)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.2],\\0.1,0.2],\\0.2,0.1],\\(0.5,0.2,0.4),\\(0.2,0.3,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.1],\\0.2,0.2],\\0.2,0.1],\\(0.5,0.2,0.3),\\(0.3,0.4,0.2)\end{matrix}\right\}, \\
\ddot{\lambda}_3 & \left\{\begin{matrix}0.1,0.2],\\0.1,0.2],\\0.2,0.1],\\(0.2,0.2,0.5),\\(0.5,0.4,0.3)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.2],\\0.1,0.1],\\0.2,0.2],\\(0.3,0.2,0.2),\\(0.4,0.3,0.2)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.2],\\0.1,0.2],\\0.2,0.2],\\(0.8,0.2,0.4),\\(0.6,0.8,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.2],\\0.2,0.1],\\0.2,0.2],\\(0.2,0.2,0.5),\\(0.5,0.4,0.3)\end{matrix}\right\}, \\
\ddot{\lambda}_4 & \left\{\begin{matrix}0.1,0.2],\\0.1,0.2],\\0.1,0.2],\\(0.8,0.2,0.4),\\(0.6,0.8,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.2,0.1],\\0.1,0.2],\\0.3,0.1],\\(0.8,0.2,0.4),\\(0.6,0.8,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.1,0.1],\\0.2,0.2],\\0.1,0.1],\\(0.5,0.2,0.4),\\(0.2,0.3,0.4)\end{matrix}\right\} & \left\{\begin{matrix}0.2,0.1],\\0.1,0.1],\\0.1,0.2],\\(0.2,0.2,0.5),\\(0.5,0.4,0.3)\end{matrix}\right\}.
\end{array}
\tag{48}
$$

Step 2. Construct normalized decision matrix, using the following equation:

$$
\mathring{\lambda}_{i_j}=\frac{\mathrm{u}_{i_j}}{\sqrt{\left(\sum \mathrm{u}_{i_j}^2\right)}} \quad \text{for} \quad i=1,\cdots,m\,;j=1,\cdots,n. \tag{49}
$$

$$
\begin{array}{ccccc}
 & U_1 & U_2 & U_3 & U_4 \\
\mathring{\lambda}_1 & \left\{\begin{matrix}0.25,0.29],\\0.5,0.143],\\0.2,0.25],\\\begin{pmatrix}0.461,0.5,\\0.492\end{pmatrix},\\\begin{pmatrix}0.233,0.292,\\0.596\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.17,0.1],\\0.17,0.17],\\0.143,0.17],\\\begin{pmatrix}0.314,0.5,\\0.554\end{pmatrix},\\\begin{pmatrix}0.6324,0.413,\\0.471\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.25,0.29],\\0.2,0.25],\\0.17,0.33],\\\begin{pmatrix}0.314,0.5,\\0.554\end{pmatrix},\\\begin{pmatrix}0.6324,0.413,\\0.471\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.20,0.33],\\0.17,0.20],\\0.17,0.29],\\\begin{pmatrix}0.314,0.5,\\0.554\end{pmatrix},\\\begin{pmatrix}0.6324,0.413,\\0.471\end{pmatrix}\end{matrix}\right\}, \\
\mathring{\lambda}_2 & \left\{\begin{matrix}0.25,0.143],\\0.17,0.29],\\0.2,0.38],\\\begin{pmatrix}0.461,0.5,\\0.369\end{pmatrix},\\\begin{pmatrix}0.349,0.389,\\0.298\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.33,0.1],\\0.5,0.33],\\0.143,0.33],\\\begin{pmatrix}0.314,0.5,\\0.554\end{pmatrix},\\\begin{pmatrix}0.539,0.578,\\0.707\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.25,0.29],\\0.2,0.25],\\0.33,0.17],\\\begin{pmatrix}0.461,0.5,\\0.492\end{pmatrix},\\\begin{pmatrix}0.233,0.292,\\0.596\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.20,0.17],\\0.33,0.4],\\0.33,0.143],\\\begin{pmatrix}0.461,0.5,\\0.369\end{pmatrix},\\\begin{pmatrix}0.349,0.389,\\0.298\end{pmatrix}\end{matrix}\right\}, \\
\mathring{\lambda}_3 & \left\{\begin{matrix}0.25,0.29],\\0.17,0.29],\\0.5,0.125],\\\begin{pmatrix}0.184,0.5,\\0.616\end{pmatrix},\\\begin{pmatrix}0.581,0.389,\\0.447\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.17,0.2],\\0.17,0.17],\\0.29,0.33],\\\begin{pmatrix}0.314,0.5,\\0.277\end{pmatrix},\\\begin{pmatrix}0.431,0.247,\\0.236\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.25,0.29],\\0.2,0.25],\\0.33,0.33],\\\begin{pmatrix}0.838,0.5,\\0.554\end{pmatrix},\\\begin{pmatrix}0.647,0.661,\\0.471\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.20,0.33],\\0.33,0.20],\\0.33,0.29],\\\begin{pmatrix}0.184,0.5,\\0.616\end{pmatrix},\\\begin{pmatrix}0.581,0.389,\\0.447\end{pmatrix}\end{matrix}\right\}, \\
\mathring{\lambda}_4 & \left\{\begin{matrix}0.25,0.29],\\0.17,0.29],\\0.2,0.25],\\\begin{pmatrix}0.736,0.5,\\0.493\end{pmatrix},\\\begin{pmatrix}0.698,0.779,\\0.596\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.33,0.1],\\0.17,0.33],\\0.43,0.17],\\\begin{pmatrix}0.838,0.5,\\0.554\end{pmatrix},\\\begin{pmatrix}0.647,0.661,\\0.471\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.25,0.143],\\0.4,0.25],\\0.17,0.17],\\\begin{pmatrix}0.461,0.5,\\0.492\end{pmatrix},\\\begin{pmatrix}0.233,0.292,\\0.596\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.40,0.17],\\0.17,0.20],\\0.17,0.29],\\\begin{pmatrix}0.184,0.5,\\0.616\end{pmatrix},\\\begin{pmatrix}0.581,0.389,\\0.447\end{pmatrix}\end{matrix}\right\}.
\end{array}
\tag{50}
$$

Step 3. Create the weighted normalized decision matrix using the equation below

$$
\ddot{\lambda}_{i_j}=w_j.\mathring{\lambda}_{i_j}, \tag{51}
$$

$$
\begin{array}{ccccc}
 & G_1 & G_2 & G_3 & G_4 \\
\ddot{\lambda}_1 & \left\{\begin{matrix}0.075,0.087],\\0.15,0.043],\\0.06,0.075],\\\begin{pmatrix}0.138,0.15,\\0.148\end{pmatrix},\\\begin{pmatrix}0.069,0.088,\\0.179\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.017,0.01],\\0.017,0.017],\\0.0143,0.017],\\\begin{pmatrix}0.0314,0.05,\\0.0554\end{pmatrix},\\\begin{pmatrix}0.06324,0.0413,\\0.0471\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.05,0.06],\\0.04,0.05],\\0.034,0.066],\\\begin{pmatrix}0.0628,0.1,\\0.1108\end{pmatrix},\\\begin{pmatrix}0.1265,0.083,\\0.0942\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.08,0.132],\\0.07,0.08],\\0.07,0.12],\\\begin{pmatrix}0.1256,0.2,\\0.2216\end{pmatrix},\\\begin{pmatrix}0.253,0.1652,\\0.1882\end{pmatrix}\end{matrix}\right\}, \\
\ddot{\lambda}_2 & \left\{\begin{matrix}0.075,0.043],\\0.051,0.087],\\0.06,0.114],\\\begin{pmatrix}0.138,0.15,\\0.1107\end{pmatrix},\\\begin{pmatrix}0.105,0.117,\\0.089\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.033,0.01],\\0.05,0.033],\\0.0143,0.033],\\\begin{pmatrix}0.0314,0.05,\\0.0554\end{pmatrix},\\\begin{pmatrix}0.0539,0.0578,\\0.0707\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.05,0.06],\\0.04,0.05],\\0.066,0.034],\\\begin{pmatrix}0.0922,0.1,\\0.0984\end{pmatrix},\\\begin{pmatrix}0.0466,0.0584,\\0.1192\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.08,0.07],\\0.132,0.16],\\0.132,0.06],\\\begin{pmatrix}0.1844,0.2,\\0.1476\end{pmatrix},\\\begin{pmatrix}0.1396,0.1556,\\0.1192\end{pmatrix}\end{matrix}\right\}, \\
\ddot{\lambda}_3 & \left\{\begin{matrix}0.075,0.087],\\0.051,0.087],\\0.15,0.038],\\\begin{pmatrix}0.055,0.15,\\0.185\end{pmatrix},\\\begin{pmatrix}0.174,0.117,\\0.134\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.017,0.02],\\0.017,0.017],\\0.029,0.033],\\\begin{pmatrix}0.0314,0.05,\\0.0277\end{pmatrix},\\\begin{pmatrix}0.0431,0.0247,\\0.0236\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.05,0.06],\\0.04,0.05],\\0.066,0.066],\\\begin{pmatrix}0.168,0.1,\\0.1108\end{pmatrix},\\\begin{pmatrix}0.129,0.1322,\\0.0942\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.08,0.132],\\0.132,0.08],\\0.132,0.12],\\\begin{pmatrix}0.0736,0.2,\\0.246\end{pmatrix},\\\begin{pmatrix}0.2324,0.156,\\0.179\end{pmatrix}\end{matrix}\right\}, \\
\ddot{\lambda}_4 & \left\{\begin{matrix}0.075,0.087],\\0.051,0.087],\\0.06,0.075],\\\begin{pmatrix}0.221,0.15,\\0.148\end{pmatrix},\\\begin{pmatrix}0.209,0.234,\\0.179\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.033,0.01],\\0.017,0.033],\\0.043,0.017],\\\begin{pmatrix}0.0838,0.05,\\0.0554\end{pmatrix},\\\begin{pmatrix}0.0647,0.0661,\\0.0471\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.05,0.143],\\0.08,0.05],\\0.034,0.034],\\\begin{pmatrix}0.0922,0.1,\\0.0984\end{pmatrix},\\\begin{pmatrix}0.0466,0.0584,\\0.1192\end{pmatrix}\end{matrix}\right\} & \left\{\begin{matrix}0.40,0.07],\\0.07,0.08],\\0.07,0.12],\\\begin{pmatrix}0.0736,0.2,\\0.2464\end{pmatrix},\\\begin{pmatrix}0.2324,0.156,\\0.179\end{pmatrix}\end{matrix}\right\}.
\end{array}
\tag{52}
$$

Step 4. Identify the ideal and negative ideal solutions. Ideal solution $\lambda^*=\{\lambda_1^*,\cdots,\lambda_n^*\}$, where

$$
\lambda_j^*=\left\{\max\left(\lambda_{i_j}\right)\text{if } j\in J\,;\min\left(\lambda_{i_j}\right)\text{if} j\in J'\right\}. \tag{53}
$$

Negative ideal solution

$$
\lambda'=\left\{\lambda_1',\cdots,\lambda_n'\right\}, \tag{54}
$$

where

$$
\lambda_j'=\left\{\max\left(\lambda_{i_j}\right)\text{if} j\in J\,;\min\left(\lambda_{i_j}\right)\text{if} j\in J'\right\}, \tag{55}
$$

$$\begin{array}{ll}
\lambda^* & \left\{\begin{array}{c} 0.075,0.087], \\ 0.15,0.087], \\ 0.15,0.114], \\ \begin{pmatrix} 0.221,0.15, \\ 0.185 \end{pmatrix}, \\ \begin{pmatrix} 0.209,0.234, \\ 0.179 \end{pmatrix} \end{array}\right\} \left\{\begin{array}{c} 0.033,0.02], \\ 0.05,0.033], \\ 0.029,0.033], \\ \begin{pmatrix} 0.0838,0.0578, \\ 0.0554 \end{pmatrix}, \\ \begin{pmatrix} 0.0647,0.0661, \\ 0.0707 \end{pmatrix} \end{array}\right\} \left\{\begin{array}{c} 0.05,0.143], \\ 0.08,0.05], \\ 0.066,0.066], \\ \begin{pmatrix} 0.168,0.1, \\ 0.1108 \end{pmatrix}, \\ \begin{pmatrix} 0.129,0.1322, \\ 0.1192 \end{pmatrix} \end{array}\right\} \left\{\begin{array}{c} 0.08,0.07], \\ 0.07,0.07], \\ 0.07,0.06], \\ \begin{pmatrix} 0.0736,0.2, \\ 0.1476 \end{pmatrix}, \\ \begin{pmatrix} 0.1396,0.156, \\ 0.1476 \end{pmatrix} \end{array}\right\}, \\
\lambda' & \left\{\begin{array}{c} 0.075,0.043], \\ 0.051,0.043], \\ 0.06,0.038], \\ \begin{pmatrix} 0.055,0.15, \\ 0.1107 \end{pmatrix}, \\ \begin{pmatrix} 0.069,0.088, \\ 0.089 \end{pmatrix} \end{array}\right\} \left\{\begin{array}{c} 0.017,0.01], \\ 0.017,0.017], \\ 0.0143,0.017], \\ \begin{pmatrix} 0.0314,0.033, \\ 0.0277 \end{pmatrix}, \\ \begin{pmatrix} 0.0431,0.0247, \\ 0.0236 \end{pmatrix} \end{array}\right\} \left\{\begin{array}{c} 0.06,0.05], \\ 0.04,0.05], \\ 0.034,0.034], \\ \begin{pmatrix} 0.0628,0.1, \\ 0.0984 \end{pmatrix}, \\ \begin{pmatrix} 0.0466,0.0584, \\ 0.0942 \end{pmatrix} \end{array}\right\} \left\{\begin{array}{c} 0.40,0.132], \\ 0.132,0.16], \\ 0.132,0.12], \\ \begin{pmatrix} 0.2324,0.2, \\ 0.246 \end{pmatrix}, \\ \begin{pmatrix} 0.253,0.1652, \\ 0.1882 \end{pmatrix} \end{array}\right\}.
\end{array} \tag{56}$$

Step 5. Calculate the separation measures for each alternatives, with the help of the following equations as

$$s_i^* = \sqrt{\left[\sum\left(\lambda_j^* - \lambda_{i_j}\right)^2\right]} i = 1, \cdots, m. \tag{57}$$

Separation from negative ideal alternatives is also expressed as

$$s_i' = \sqrt{\left[\sum\left(\lambda_j' - \lambda_{i_j}\right)^2\right]} i = 1, \cdots, m, \tag{58}$$

$$\ddot{s}_1^* = 0.3476, \tag{59}$$

$$\ddot{s}_2^* = 0.3531, \tag{60}$$

$$\ddot{s}_3^* = 0.3369, \tag{61}$$

$$\ddot{s}_4^* = 0.4106, \tag{62}$$

$$\ddot{s}_1' = 0.4106, \tag{63}$$

$$\ddot{s}_2' = 0.4066, \tag{64}$$

$$\ddot{s}_3' = 0.4386, \tag{65}$$

$$\ddot{s}_4' = 0.3830. \tag{66}$$

Step 6. Calculate the distance between relative closeness and ideal solution D_i^* where

$$D_i^* = \frac{s_i'}{\left(s_i^* + s_i'\right)} 0 \le D_i^* \le 1, \tag{67}$$

select the option with D_i^* closest to 1.

$$\ddot{\lambda}_1 = 0.4585, \ddot{\lambda}_2 = 0.4648, \ddot{\lambda}_3 = 0.4344, \ddot{\lambda}_4 = 0.5174, \tag{68}$$

$$\ddot{\lambda}_4 > \ddot{\lambda}_2 > \ddot{\lambda}_1 > \ddot{\lambda}_3. \tag{69}$$

The ranking order of $\ddot{\lambda}_1, \ddot{\lambda}_2, \ddot{\lambda}_3$, and $\ddot{\lambda}_4$ is shown as in Figure 5.

5. Comparison

Topsis method is a common technique to handle decision making problems. In a neutrosophic set, a group decision-making procedure was presented by Abdel et al. and Biswas et al. [35, 36]. The several iterations of the neutrosophic set were also used in decision-making issues by Zulqarnain et al. and Dey et al. [37, 38]. All of these techniques are relevant to the ongoing effort. We now contrast the suggested method with two comparable ways to analyze the benefits and drawbacks of the current model in order to demonstrate the technological achievements in this research. The primary distinction between them is that whereas Biswas focused on the hybridization of the two ideas, namely, generalized neutrosophic sets, and soft sets. Abdel examined the truth, indeterminacy, and falsity membership values. As a result, the decision data in the current model is broader. Consequently, the strategy described in this paper is more circumspect.

6. Conclusion

We proposed a new notion known as neutrosophic octahedron set in this article by combining the concepts of neutrosophic set, intuitionistic fuzzy, and octahedron set. The major goal of this concept is to resolve uncertainty in real-world situations. We also look at some basic NOS operations including union, intersection, and complement, as well as their characteristics. Define some operational features as well. We also discussed the fact that the need for energy planning has increased with the development of new energy-related technologies and energy sources. The problem of decision-making is made even more difficult by the need for collaboration between various stakeholders in order to produce effective decisions. In order to quantitatively reflect the ambiguity and imprecision of the data, neutrosophic octahedron sets are a useful tool. Finally, using our proposed method and a numerical example, we presented a decision-making process.

In the future, this structure can be extended in interval neutrosophic octahedron set and can be applied in many real-life applications such as pattern recognition, medical diagnosis, and personal selection. Moreover, one can use this concept and develop a new decision-making technique with VIKOR, ELECTRE, CODAS, and AHP under a neutrosophic octahedron environment.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there were no conflicts of interest regarding the publication of this article.

Authors' Contributions

All authors contributed equally to the preparation of this manuscript.

References

[1] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] L. A. Zadeh, “The concept of a linguistic variable and its application to approximate reasoning–I,” *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.

[3] K. Atanassov, “Intuitionistic fuzzy sets,” *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[4] K. T. Atanassov and G. Gargov, “Interval valued intuitionistic fuzzy sets,” *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.

[5] J. G. Lee, G. Senel, P. K. Lim, J. Kim, and K. Hur, “Octahedron sets,” *Annals of Fuzzy Mathematics Inform*, vol. 19, no. 3, pp. 211–238, 2020.

[6] F. Smarandache, “Neutrosophic set, a generalization of the intuitionistic fuzzy set,” *International Journal of Pure and Applied Mathematics*, vol. 24, no. 3, pp. 287–297, 2005.

[7] F. Smarandache, “Neutrosophic set,” in *A Generalization of the Intuitionistic Fuzzy Set in Proceedings of 2006 IEEE International Conference on Granular Computing*, Y.-Q. Zhang and T. Y. Lin, Eds., pp. 38–42, Georgia State University, Atlanta, 2006.

[8] F. Smarandache, *Neutrosophy: neutrosophic probability, set, and logic*, American Research Press, 1998.

[9] F. G. Lupiáñez, “On neutrosophic sets and topology,” *Procedia Computer Science*, vol. 120, pp. 975–982, 2017.

[10] H. Wang, F. Smarandache, Y.-Q. Zhang, and R. Sunderraman, *Interval neutrosophic sets and logic: theory and applications in computing: theory and applications in computing*, Hexis, Phoenix, 2005.

[11] M. Bhowmik and M. Pal, “Intuitionistic neutrosophic set,” *Journal of Information and Computing Science*, vol. 4, no. 2, pp. 142–152, 2009.

[12] P. Kumar Maji, “Neutrosophic soft set,” *Ann. Fuzzy Math. Informatics*, vol. 5, no. 1, pp. 157–168, 2013.

[13] M. Alkhazaleh and S. Uluçay, “Neutrosophic soft expert sets,” *Neutrosophic Soft Expert Sets. Appl. Math.*, vol. 6, no. 1, pp. 116–127, 2015.

[14] S. Alias, D. Mohamad, and A. Shuib, “Rough neutrosophic multisets,” *Neutrosophic Sets and Systems*, vol. 16, pp. 80–88, 2017.

[15] M. Ali and F. Smarandache, “Complex neutrosophic set,” *Neural Comput. Appl.*, vol. 28, no. 7, pp. 1817–1834, 2017.

[16] Z. Xiao, S. Xia, K. Gong, and D. Li, “The trapezoidal fuzzy soft set and its application in MCDM,” *Applied Mathematical Modelling*, vol. 36, no. 12, pp. 5844–5855, 2012.

[17] M. Deveci, “Site selection for hydrogen underground storage using interval type-2 hesitant fuzzy sets,” *International Journal of Hydrogen Energy*, vol. 43, no. 19, pp. 9353–9368, 2018.

[18] X. Geng, X. Chu, and Z. Zhang, “A new integrated design concept evaluation approach based on vague sets,” *Expert Systems with Applications*, vol. 37, no. 9, pp. 6629–6638, 2010.

[19] H. Shidpour, C. Da Cunha, and A. Bernard, “Group multi-criteria design concept evaluation using combined rough set theory and fuzzy set theory,” *Expert Systems with Applications*, vol. 64, pp. 633–644, 2016.

[20] A. Balin, P. Alcan, and H. Basligil, “Co performance comparison on CCHP systems using different fuzzy multi criteria decision making models for energy sources,” *Fuelling the Future*, vol. 2022, pp. 591–595, 2012.

[21] I. B. Huang, J. Keisler, and I. Linkov, “Multi-criteria decision analysis in environmental sciences: ten years of applications and trends,” *Science of the Total Environment*, vol. 409, no. 19, pp. 3578–3594, 2011.

[22] Z. Pavić and V. Novoselac, “Notes on TOPSIS method,” *International Journal of Research in Engineering and Science*, vol. 1, no. 2, pp. 5–12, 2013.

[23] H. D. Arora and A. Naithani, “Significance of TOPSIS approach to MADM in computing exponential divergence measures for Pythagorean Fuzzy Sets,” *Decision Making: Applications in Management and Engineering*, vol. 5, no. 1, pp. 146–163, 2022.

[24] I. Petrovic and M. Kankaras, “A hybridized IT2FS-DEMATEL-AHP-TOPSIS multicriteria decision making approach: case study of selection and evaluation of criteria for determination of air traffic control radar position,” *Decision Making: Applications in Management and Engineering.*, vol. 3, no. 1, pp. 134–152, 2020.

[25] I. M. Hezam, A. R. Mishra, R. Krishankumar, K. S. Ravichandran, S. Kar, and D. S. Pamucar, “A single-valued neutrosophic decision framework for the assessment of sustainable transport investment projects based on discrimination measure,” in *Management Decision*, Emerald Insight, 2022.

[26] N. Y. Pehlivan and N. Yalçın, “Neutrosophic TOPSIS method for sustainable supplier selection in a discount market chain,” in *Handbook of Research on Advances and Applications of Fuzzy Sets and Logic*, pp. 692–715, IGI Global, 2022.

[27] S. Özlü and F. Karaaslan, “Hybrid similarity measures of single-valued neutrosophic type-2 fuzzy sets and their application to MCDM based on TOPSIS,” *Soft Computing.*, vol. 26, no. 9, pp. 4059–4080, 2022.

[28] V. Simic, I. Gokasar, M. Deveci, and A. Karakurt, “An integrated CRITIC and MABAC based type-2 neutrosophic model for public transportation pricing system selection,” *Socio-Economic Planning Sciences*, vol. 80, p. 101157, 2022.

[29] Y. B. Jun, C. S. Kim, and K. O. Yang, “Cubic sets,” *Ann. Fuzzy Math. Inform.*, vol. 4, no. 1, pp. 83–98, 2012.

[30] Y. B. Jun, C. S. Kim, and M. S. Kang, “Cubic sub-algebras and ideals of BCK/BCI-algebras,” *Far East Journal of Mathematical Sciences*, vol. 44, pp. 239–250, 2010.

[31] Y. B. Jun, F. Smarandache, and C. S. Kim, “Neutrosophic cubic sets,” *New mathematics and natural computation*, vol. 13, no. 1, pp. 41–54, 2017.

[32] M. Gulistan and S. Khan, “Extentions of neutrosophic cubic sets via complex fuzzy sets with application,” *Complex & Intelligent Systems*, vol. 6, no. 2, pp. 309–320, 2020.

[33] D. Bozanic, D. Tešić, D. Marinković, and A. Milić, “Modeling of neuro-fuzzy system as a support in decision-making processes,” *Reports in Mechanical Engineering*, vol. 2, no. 1, pp. 222–234, 2021.

[34] D. Stanujkić and D. Karabašević, “An extension of the WASPAS method for decision-making problems with intuitionistic fuzzy numbers: a case of website evaluation,” *Operational Research in Engineering Sciences: Theory and Applications*, vol. 1, no. 1, pp. 29–39, 2019.

[35] M. Abdel-Basset, G. Manogaran, A. Gamal, and F. Smarandache, “A group decision-making framework based on neutrosophic TOPSIS approach for smart medical device selection,” *Journal of medical systems*, vol. 43, no. 2, p. 38, 2019.

[36] P. Biswas, S. Pramanik, and B. C. Giri, “Neutrosophic TOPSIS with group decision making,” in *In fuzzy multi-criteria decision-making using neutrosophic sets*, pp. 543–585, Springer, Cham, 2019.

[37] R. M. Zulqarnain, X. L. Xin, M. Saeed, F. Smarandache, and N. Ahmad, *Generalized neutrosophic TOPSIS to solve multi-criteria decision-making problems*, Infinite Study, 2020.

[38] P. P. Dey, S. Pramanik, and B. C. Giri, “Generalized neutrosophic soft multi-attribute group decision making based on TOPSIS,” *Critical Review*, vol. 11, pp. 41–55, 2015.