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Is the Great Moderation Ending?
——UK and US Evidence

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Abstract

The Great Moderation, the significant decline in the variability of economic activity, provides a most re-markable feature of the macroeconomic landscape in the last twenty years. A number of papers document the beginning of the Great Moderation in the US and the UK. In this paper, we use the Markov regime-switching models to document the end of the Great Moderation. The Great Moderation in the US and the UK begin at different point in time. The explanations for the Great Moderation fall into generally three different catego-ries—good monetary policy, improved inventory management, or good luck. The end of the Great Modera-tion, however, occurs at approximately the same time in both the US and the UK. It seems unlikely that good monetary policy would turn into bad policy or that better inventory management would turn into worse management. Rather, the likely explanation comes from bad luck. Two likely culprits exist—energy-price and housing-price shocks.

Keywords: Great Moderation, Regime Switching, SWARCH

1. Introduction

Time-series patterns of real output growth, like many other economic and financial time series, exhibit periods of high volatility followed by periods of low volatility. Generalized autoregressive conditional heteroskedastic-ity (GARCH) models, based on the seminal works of [1] and [2], accommodate this phenomenon by explicitly modeling the tendency for more large (small) changes in the underlying time series to follow large (small) changes, thus permitting estimation of the observed vola-tility clustering. Problems in estimating GARCH models, however, arise if the underlying volatility process ex-periences structural breaks, especially shifts in the over-all level of volatility. The empirical literature shows that the sum of the estimated GARCH coefficients nearly equals, or even exceeds, one, implying a non-stationary variance process (*i.e.*, integrated GARCH or IGARCH process). According to [3], this high volatility persistence of shocks in single regime GARCH models may reflect structural changes in the variance process. For example, if high, but constant (homoskedastic), variance

for some time switches to a low, but constant, variance, then combining such high and low homoskedastic vola-tility periods produces spurious overall volatility persis-tence. That is, a GARCH model does not differentiate between homoskedastic volatility sub-periods, but iden-tifies high persistence and heteroskedasticity across the full sample. As such, disregarding regime changes leads to a misspecified GARCH model. The misspecified GARCH model systematically overstated the persistence of volatility shocks (see [4,5]).

Commonly, researchers deal with such structural breaks by introducing dummy variables for given sub-periods reflecting the change in the level of volatility. For example, Reference [6] develops a test based on the modified iterated cumulated sums of squares (ICSS) al-gorithm (see [7]) and analyzes real GDP growth rates for six OECD countries (Canada, Germany, Italy, Japan, the United Kingdom, and the United States) from 1960 to 2006 and find a number of structural breaks in the data.[1] The modified ICSS algorithm, however, suffers from an

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[1]In early work, Reference [8] introduces a similar methodology for considering the Great Moderation in the US.

important limitation. To wit, it identifies exogenously a series of structural breaks in the volatility of a time series, but assumes that the volatility remains constant between the two break points. Yet, the analysis uses these break points in a model that explicitly recognizes the random nature of volatility.

In a series of influential papers References [9] and [10] propose a Markov-switching technique to analyze non-stationary time series and to model structural breaks endogenously. This approach introduces a particularly appealing feature in that it allows the dating of low versus high volatility regimes and, therefore, avoids any ad hoc partitioning of the sample path.

We apply this methodology to analyze, once again, the Great Moderation with a new twist. That is, since the emergence of the Great Moderation, does the low volatility persistence remain unchanged until the present? Recent large-scale events such as worldwide inflationary pressures and the sub-prime lending crisis may provide a warning that the good times may soon end. The Markov-switching approach can usefully indicate when output growth volatility undergoes shifts from high to low and back again, despite the fact that the forcing variable causing the regime shifts remains unobservable or unknown. We find preliminary evidence that signals the end of the Great Moderation in the UK and the US. The next section reviews the existing literature on the Great Moderation. Section 3 identifies our data and spells out our econometric methodology. Section 4 reports the results of our econometric analysis and interprets the findings. Section 5 concludes.

2. Economic Background: the Great Moderation

The Great Moderation emerged as an important topic amongst macroeconomists, especially since the seemingly coordinated decline in volatility of real GDP growth across numerous developed countries. For example, References [11-14] identify a rather dramatic reduction in US real GDP growth rate volatility in the early 1980s. Other authors, such as [15-17], consider the G7 countries and Australia, also finding a structural break in the volatility of the output growth rate. The breaks, however, occur at different times in different countries. Similarly, Reference [18] examines a sample of 20 OECD countries and demonstrates a considerable decline in the volatility of real output growth around the developed world, while Reference [19] considers a sample of 25 developed and less-developed countries and finds at least one break in all but 9 countries and at most two breaks in 6 of the 25 countries, concluding that shifts in the volatility of the real GDP growth rate occur in many instances. Furthermore, for the identified 22 breaks, only one occurs the 1970s, 12, in the 1980s, and 9, in the 1990s.

Several important questions emerge from these findings. First, what caused the decline in volatility? Analysts offer several hypotheses, including better macroeconomic policies, structural change, or good luck. For example, [17,20] and [21] attribute the Great Moderation to good luck. Conversely, [22] and [23] argue that a substantial portion of the Great Moderation reflects better monetary policy. The distinction proves important. Good luck can turn into bad luck, whereas, presumably, good policy does not become bad policy. Thus, a return to bad luck could throw the economy into the high volatility regime, once again.

In [16] the three commonly proposed explanations of the Great Moderation—good monetary policy, improved inventory management, and good luck are discussed at length. Good monetary policy indirectly affects the volatility of real GDP growth by providing a more stable economic environment with lower inflation and lower inflation volatility. Improved inventory management provides an improved buffer between production and sales, whereby the same volatility of sales can exist with lower volatility of production. Good luck associates with lower volatility of random shocks to the macroeconomy, such as crude oil price shocks. The conclusion drawn by [16] is that for the G-7 and Australia the evidence supports the roles good monetary policy and improved inventory management, and not good luck in the Great Moderation.[2]

Second, how does one model the decline in volatility?

1) Researchers frequently adopt a GARCH modeling strategy to capture the movement in volatility. Much of this research assumes a stable GARCH process governing conditional growth volatility. The neglect of structural breaks in the variance of output leads to higher persistence in the conditional volatility.

2) Economic growth involves long-run phenomena, where for longer sample periods, structural changes in volatility will occur with a higher probability. According to [31] and [32], the long-run variance dynamics may include regime shifts, but within a regime it may follow a GARCH process. Others, such as [11,15,33], and [16] apply this approach of Markov switching heteroskedasticity with two states to examine the volatility in the growth rate of real GDP. The GARCH modeling approach provides an alternative to deal with this issue, but relaxing the implicit assumption of a constant variance process.

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[2]A related literature considers time-varying or Markov-switching structural VAR models of the macroeconomy, largely of the US, concluding that the Great Moderation reflects good luck (e.g., [24-27]). Other authors conclude that the Great Moderation reflects good policy, using sticky-price dynamic stochastic general equilibrium (DSGE) models (e.g., [28,29]). However, according to [30], structural VAR models may not provide information on the issue, as these models falsely conclude that good luck and not good policy can explain the Great moderation.

3) Reference [6] argues that the extant methods of modeling the time-series properties of the volatility of the real GDP growth rate contain misspecifications associated with structural shifts.[3] They address such misspecifications by introducing structural shifts in the volatility process, finding that the persistence found in GARCH models falls dramatically and even disappears in some cases. They conclude their paper by stating, "The true test of the cause of the Great Moderation may only await the passage of time. The current run up in oil prices may provide the acid test." Our findings of the end of the Great Moderation required only 5 and 3 additional quarters of date for the US and the UK. More importantly, the different methodology of regime switching models uncovered the result.

3. Model Specification

We conduct the empirical analysis of the dynamics of the real GDP growth rate for the UK and the US by estimating a series of univariate autoregressive non-linear Markov-switching models with two regimes. The general Markov-switching model (e.g., [9,10], and [39]) involves multiple structures that can describe the time-series behavior in different regimes and, thus, capture more complex, dynamic patterns. The model is non-linear, and assumes that the parameters of the underlying process of an observed time series depend on an unobservable (latent) state variable, describing the regimes. Non-linearities arise if processes experience discrete shifts in regimes. By sanctioning switching between regimes, where the dynamic behavior of the time series differs markedly, we can accommodate more complex dynamic patterns.

We consider five specifications of the process of output growth. To begin, we specify three models that involve AR models of order 1 and a two-state Markov-switching process. In the first specification, we assume

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[3]According to [34], structural changes may confound persistence estimation in GARCH models. That is, the integrated GARCH (IGARCH) discussed in [35] may result from instability of the constant term of the conditional variance, that is, nonstationarity of the unconditional variance. Neglecting such changes can generate spuriously measured persistence with the sum of the estimated autoregressive parameters of the conditional variance heavily biased towards one. Additionally, Reference [4] provides confirming evidence that not accounting for discrete shifts in unconditional variance, the misspecification of the GARCH model can bias upward GARCH estimates of persistence in variance. Including dummy variables to account for such shifts diminishes the degree of GARCH persistence. According to [36] the IGARCH model makes sense when non-stationary data reflect changes in the unconditional variance and Reference [37] shows that in the presence of neglected parameter change-points, even a single deterministic change-point, GARCH inappropriately measures volatility persistence. More recently, Reference [38] argues that the changes in the variance could arise from changes in the mean, demonstrating that the estimated persistence parameter in the GARCH(1, 1) model contains upward bias when researchers ignore structural changes in the mean.

that the process of output growth depends on two underlying regimes, with constant mean and constant variance in both regimes. In this specification, both the mean, the autoregressive parameter, and the variance depend on the state, that is, conditioned on the state S_t such that

$$y_t = \begin{cases} a_{01} + a_{11}y_{t-1} + \sigma_1 u_{1t}, & \text{if } S_t = 1 \\ a_{02} + a_{12}y_{t-1} + \sigma_2 u_{2t}, & \text{if } S_t = 2 \end{cases} \quad (1)$$

where S_t denotes the unobserved regime of the system. The series S_t, $t = 1, 2, \ldots, T$ provides information about the regime the economy currently occupies at date t. If we knew S_t before estimating the model, we could apply a dummy variable approach. In the Markov-switching approach, however, we assume that we do not observe S_t, and we estimate the evolution of the regimes endogenously from the data. Furthermore, we assume that a Markov process governs the transition between the two states (*i.e.*, the probability of residing in a particular state in period t depends only on the state in period t-1). With the transition probabilities p and q, we summarize the process with the following transition matrix:

$$P = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix}$$

where the transition probabilities are defined as follows: with $P(S_t = 1 | S_{t-1} = 1) = p$, $P(S_t = 2 | S_{t-1} = 1) = 1-p$, $P(S_t = 2 | S_{t-1} = 2) = q$, and $P(S_t = 1 | S_{t-1} = 2) = 1-q$. Assuming conditional normality for each regime, the conditional distribution of y_t is expressed as a mixture of distributions:

$$y_t | \Delta_{t-1} \begin{cases} N(a_{01} + a_{11}y_{t-1}, \sigma_1) \text{ with probability } \pi_t \\ N(a_{02} + a_{12}y_{t-1}, \sigma_2) \text{ with probability } 1-\pi_t \end{cases} \quad (2)$$

where $\pi_t = P\left[S_t = 1 | \Delta_{t-1} \right]$ is the conditional probability of being in regime 1 and Δ_{t-1} is the information set at time t-1. This information set includes two parts. First, I_{t-1} denotes the information set ($y_{t-1}, y_{t-2}, \ldots$) that econometricians know. Second, Γ_{t-1} equals the regime path ($S_{t-1}, S_{t-2}, \ldots$) that the econometrician does not observe.

A Gaussian mixture of distribution can provide a flexible approximation to a wide class of distributions and can well-approximate highly non-Gaussian unconditional distributions [5]. Importantly, Reference [40] notes that this model can generate persistence in the conditional variance process (aggregated over the regimes) defined as $\sigma_t^2 = E\left[y_t^2 | \Delta_{t-1} \right] - E\left[y_t | \Delta_{t-1} \right]^2$:

$$\sigma_t^2 = \pi_t \left[(a_{01} + a_{11}y_{t-1})^2 - (\sigma_1)^2 \right] + (1 - \pi_t)$$
$$\left[(a_{02} + a_{12}y_{t-1})^2 - (\sigma_2)^2 \right] \tag{3}$$
$$- \left[\pi_t (a_{01} + a_{11}y_{t-1}) + (1 - \pi_t)(a_{01} + a_{11}y_{t-1}) \right]^2$$

Assume, for example, that y_t depends on two regimes, a low-variability and a high-variability regime. Then, according to (3), if the two regimes are persistent, this model can sufficiently capture the persistence in volatility of the two regimes. Conversely, a single-regime GARCH model cannot capture the persistence that differs between regimes. Consequently, the GARCH model will imply overall strong volatility persistence even for homoskedastic variances within each regime. In [41], the constant-within-regime variance is found to sufficiently account for most time-volatility of variability.

Our second specification nests in specification (1) and assumes that the mean and the autoregressive dynamics depend on the state, but that the variance proves state independent: That is,

$$y_t = \begin{cases} a_{01} + a_{11}y_{t-1} + \sigma_1 u_{1t} & \text{if } S_t = 1 \\ a_{02} + a_{12}y_{t-1} + \sigma_1 u_{2t} & \text{if } S_t = 2 \end{cases} \tag{4}$$

Our third specification also nests in specification (1) and assumes that the mean and the autoregressive dynamics prove state independent, but that the variance depends on the state. That is,

$$y_t = \begin{cases} a_{01} + a_{11}y_{t-1} + \sigma_1 u_{1t} & \text{if } S_t = 1 \\ a_{01} + a_{11}y_{t-1} + \sigma_2 u_{2t} & \text{if } S_t = 2 \end{cases} \tag{5}$$

For comparison purposes, we also consider our fourth specification, where the rate of output growth (y_t) comes from a single Gaussian distribution with mean $a_{01} + a_{11}y_{t-1}$ and variance σ^2. That is,

$$y_t = a_{01} + a_{11}y_{t-1} + u_{1t} \tag{6}$$

This fourth specification sets the null hypothesis of no regime switch against which we test the alternative regime switches described in the three alternative hypotheses described in specification (1), (4), and (5). A problem arises in Markov switching models, however, when we test the null hypothesis of single regime against the alternative of two regimes. Under the null hypothesis, we cannot identify the states. This violates the key assumption that justifies the use of standard likelihood ratio (LR) tests. In this paper, we employ the non-standard LR bound test proposed by [42]. The method applies empirical process theory to derive an upper bound for type I error of a modified LR statistic under the null, assuming that we know the nuisance parameters under the alternative. Let L_1 equal the log-likelihood under

the alternative and L_0 equal the log-likelihood under the null, where q parameters exist only under the alternative. Define the standard likelihood ratio test as $M = 2(L_1 - L_0)$. Then, assuming a single-leaked likelihood ratio, an upper bound for the significance of M equals the following:

$$P\left[\chi_q^2 > M\right] + 2(M/2)^{\frac{q}{2}} \exp(M/2)\left(\Gamma\left(\frac{q}{2}\right)\right)^{-1} \tag{7}$$

where $\Gamma(.)$ is the gamma function.

In the presence of structural breaks, however, it is well-known that ADF test possesses low power. Does stationarity also become regime dependent? In other terms, do high and low volatility regimes exhibit different stationarity properties? Local, regime-dependent stationarity differs from global, regime-independent stationarity. Thus, as an alternative test of our regime switching specifications, we can use the Markov-switching approach to generalize the ADF regression to account for two distinct Markov-switching regimes. Following the approach proposed by [43], the MS-ADF test equals the following specification:

$$\Delta y_t = a(S_t) + b(S_t)y_{t-1} + \sum_{i=1}^q \gamma_i(S_t)\Delta y_{t-i} + u_t \tag{8}$$

where u_t equals a $N(0, \sigma^2(S_t))$ distribution and S_t equals the unobservable latent variable that follows a first-order Markov process with constant transition probability from regime i to j. When $b(S_t) < 0$ for a certain regime, y_t is locally stationary. Alternatively, when $b(S_t) = 0$, then y_t is locally nonstationary, or locally I(1). Clearly, when $a(S_t)$, $b(S_t)$, and $\gamma_i(S_t)$ do not depend on the regime so that $a(S_t) = a$, $b(S_t) = b$, and $\gamma_i(S_t) = \gamma_i$ and the error term u_t does not display regime-dependent heteroskedasticity so that $\sigma^2(S_t) = \sigma^2$, (8) becomes the standard ADF regression.

Finally, contrary to [44], we consider the possibility that volatility dynamics may still exist after accounting for variance regimes. In [31] a modification of the usual ARCH model is proposed that allows for changes in regimes, combining the idea of autoregressive conditional heteroskedasticity and the Markov-switching model (SWARCH). In the SWARCH model, different ARCH processes govern the variance within both regimes. Thus, the model contains two channels of volatility persistence, namely persistence due to shocks and persistence due to regime switching in the parameters of the variance process. This makes regime-switching ARCH more flexible regarding the estimation of the volatility persistence of output growth compared to the standard, single-regime

ARCH model as well to those models that switch regimes with constant variance within each regime. More specifically, in our fifth specification, we postulate a SWARCH (2,1,2) model with two states, an AR(1) specification for y_t, and a disturbance following an ARCH(2) as follows:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \big| I_{t-1} \sim N(0, h_t) \text{ and} \quad (9)$$

$$\frac{h_t}{\gamma_{S_t}} = b_0 + b_1 \frac{\varepsilon_{t-1}^2}{\gamma_{S_{t-1}}} + b_2 \frac{\varepsilon_{t-2}^2}{\gamma_{S_{t-2}}},$$

where γ_{S_t} equals a constant variance factor that scales the ARCH process, $S_t = 1$ denotes the low volatility regime, and $S_t = 2$ denotes the high volatility regime. Since one of the constant variance factors parameters is unidentified, we arbitrarily normalize γ_1 to 1. Hence, the move from one state to the other represents a change in the scale of the ARCH volatility process. An important feature of (9) is that we equate the parameters of the output growth equation across regimes, while the variances depend on the state and differ across regimes. This assumption simplifies the estimation and allows us to focus solely on time-variation in the conditional variance process.

4. Data and Empirical Findings

This paper employs quarterly data on real GDP for the US and the UK obtained from the International Financial Statistics of the International Monetary Fund. We construct real GDP by dividing Gross Domestic Product (GDP) in billions of national currency by the GDP Deflator (2000 = 100). Both series are seasonally adjusted. We compute the rate of growth of real GDP, y_t, as the logarithmic difference in percentage terms of seasonally adjusted quarterly real GDP. The sample period equals

1957:02 to 2007:04 for the US and 1957:02 to 2007:02 for the UK.

Figure 1 plots the data and **Table 1** reports the unconditional moments of the data together with the Jarque-Bera test of normality. Over the sample period, on average, US real GDP grew at a higher rate than the UK, but the UK experienced slightly more volatility. Both series, however, display significant leptokurtosis and non-normality.

We estimate all models by maximum likelihood (ML) using RATS 7.0 modules. The parameters estimates reported for the switching constant-variance models come from using the BFGS [41,45-47] algorithm, while the results for the switching ARCH variance models come from using the BHHH [48] algorithm, as in the latter case we encountered problems of convergence using the BFGS algorithm. Reported standard errors are heteroskedasticity consistent. In [31] and [39] the iterative ML estimation methods are discussed in detail.

4.1. Switching-Mean, Switching-Variance Model

Table 2 summarizes the results of the ML estimation of our first specification, the switching in mean and variance model (1), where we draw the rates of growth of real GDP from normal distributions that differ in mean and variance. In the US, state 2 exhibits a variance about two times as large as the variance in state 1. In the UK, instead, state 2 exhibits a variance about four times as large as the variance in state 1. In both cases, the estimated variances prove statistically significant at the 1-percent level. In the US, the mean rates of growth of real GDP in state 2 only slightly exceed those in state 1. This reflects the "narrowing gap" [11] between the mean growth rates over the business cycle. Further, in the US, both autoregressive coefficients in state 1 and state 2 are significant; while in the UK, only the autoregressive coefficient in state 1 is significant. These results suggest that the dynamics of the UK business cycle may differ from that of the US.

Table 1. Summary statistics.

	US	*UK*
Mean	0.8002	0.6169
Variance	0.8048	0.9748
Skewness	−0.3702	0.3127
	(0.0325)	(0.0723)
Kurtosis (*Excess*)	1.6812	3.8208
	(0.0000)	(0.0000)
Jarque-Bera	28.5470	125.5398
	(0.0000)	(0.0000)
No. of Observations	203	201

Note: *p*-values appear in parenthesis under statistics, where appropriate.

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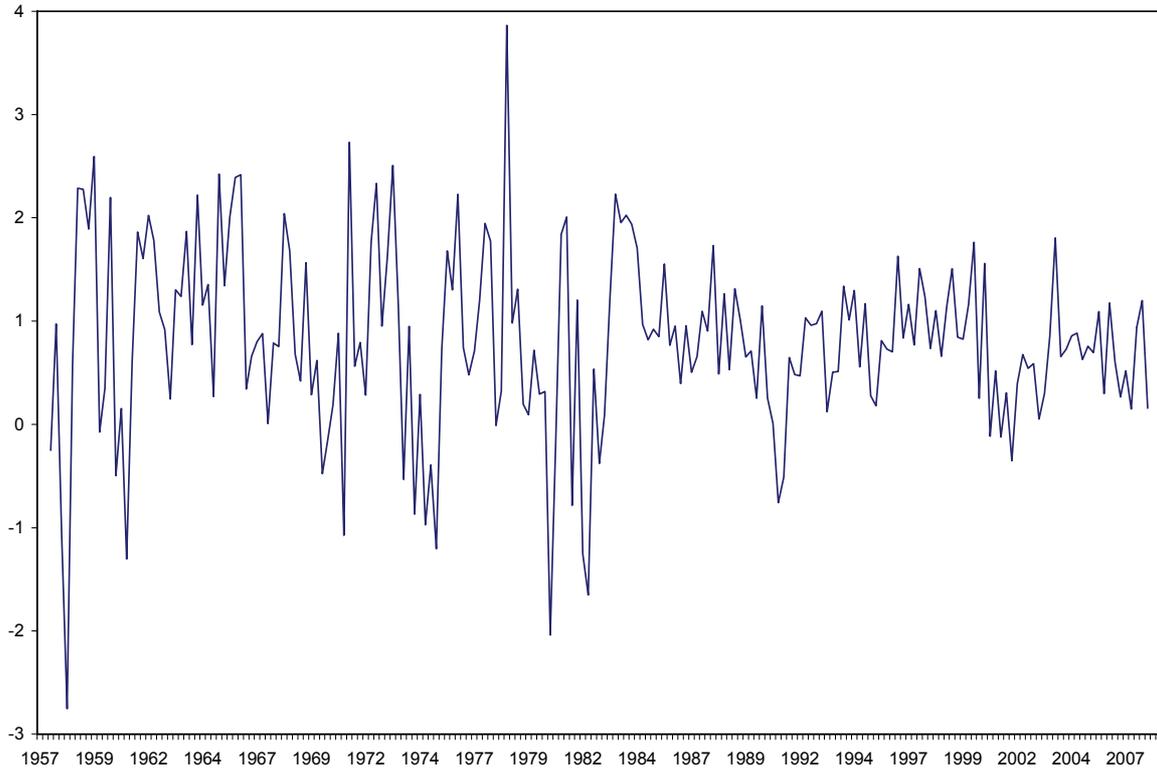
(a) US (1957:02 to 2007:04)
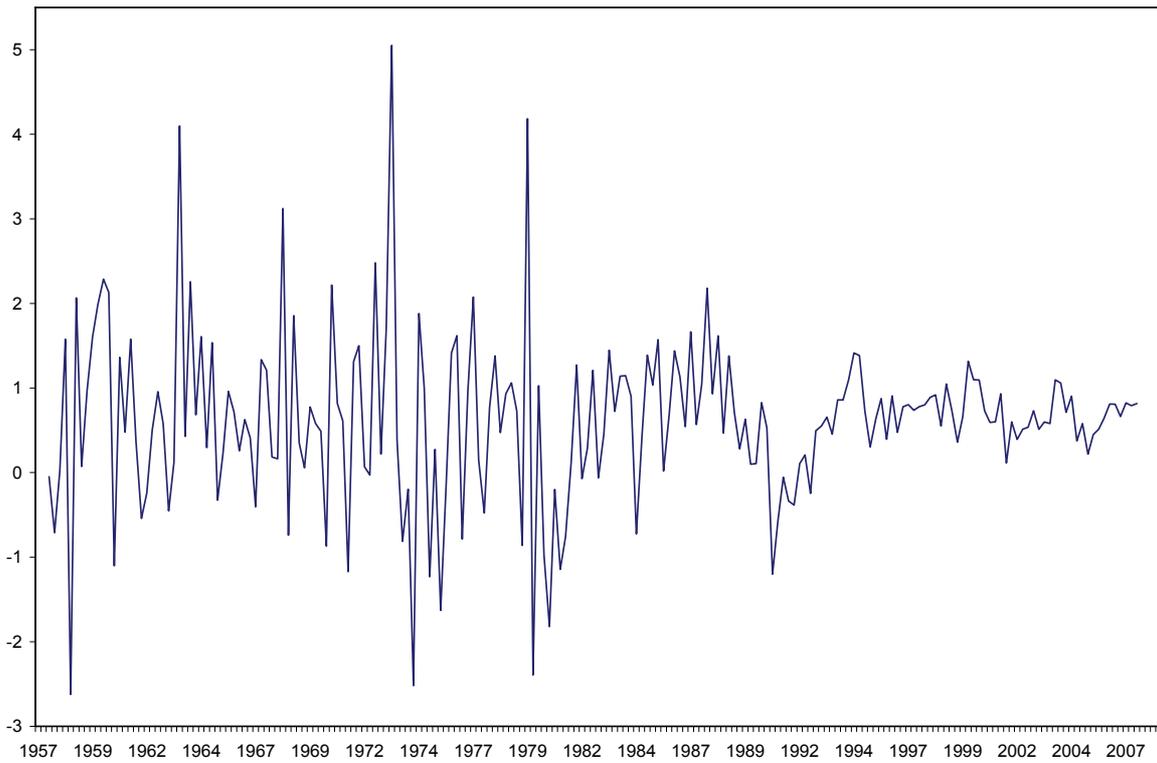
(b) UK (1957:02 to 2007:02)

Figure 1. Real GDP growth rates.

Table 2. Parameter estimates and related statistics for switching-variance, switching-mean model.

	US		UK	
Parameter	*Estimate*	*t-statistic*	*Estimate*	*t-statistic*
a_{01}	0.5719*	5.9323	0.2722*	4.5452
a_{02}	0.6046*	4.4723	0.6809*	6.2420
a_{11}	0.2362**	2.0627	0.5964*	7.5544
a_{12}	0.2956*	3.1029	−0.1077	−1.2289
σ_1	0.4780*	12.7832	0.2758*	11.5809
σ_2	1.0825*	14.3776	1.1716*	15.1208
P	0.9941*	131.4765	0.9932*	110.9160
Q	0.9945*	144.9275	0.9953*	175.3622
Log-likelihood	−230.4212		−225.2013	
AIC	472.8424		262.4026	
SIC	524.5416		513.9824	
HQ	480.8735		470.4112	
Diagnostic Tests	*Statistic*	*p-value*	*Statistic*	*p-value*
$Q_1(4)$	6.542256	0.1621	3.1652	0.5306
$Q_1(8)$	10.542728	0.2290	10.3391	0.2420
$Q_2(4)$	2.613116	0.6245	2.9133	0.5724
$Q_2(8)$	6.443422	0.5977	5.0920	0.7477
Skewness	−0.361758	0.0372	−0.0530	0.7610
Kurtosis (excess)	0.598052	0.0881	1.3582	0.0001
Jarque-Bera	7.416269	0.0245 m²	15.4679	0.0004

Note: The *AIC, SIC*, and *HQ* equal Akaike, Schwartz-Bayesian, and Hannan-Quinn information criterion. The $Q_1(k)$ and $Q_2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to *k* lags.
* denotes 1% significance level.
** denotes 5% significance level.

Table 2 also reports the results of a series of diagnostic tests. $Q_1(4)$ and $Q_1(8)$ equal the Ljung-Box statistics for the joint significance of autocorrelations of standardized residuals for the first 4 and 8 lags, respectively, and $Q_2(4)$ and $Q_2(8)$ equal the Ljung-Box statistics for the joint significance of autocorrelations of squared standardized residuals for the same number of lags. Under the null hypothesis of zero autocorrelation, each statistic is distributed as a chi-square variable with 4 and 8 degrees of freedom, respectively. The Ljung-Box statistics indicate that the regime switching model can successfully capture the serial correlation in the conditional mean and variance of the US and UK rates of real GDP growth and show no evidence of non-linear dependencies or omitted ARCH effects. This finding is particularly interesting because growth rates of real GDP show strong ARCH effects, as widely documented [49-51]. Further, the regime-switching model reduces the excess kurtosis of standardized residuals relative to the excess kurtosis present in the actual data, although some degree of leptokurtosis remains in the UK results.[4]

The high persistence of the regimes, where the transition probabilities *p* and *q* lie close to 1, proves an important feature of the estimation. That is, these high prob-

abilities indicate that if the economy begins in either state 1 or state 2, it will likely remain in that state.

Figures 2 and **3** provide a visual interpretation of the results, showing how the probability of being in either state 1 or state 2 evolves over the sample.

We base our inference on the full sample and the estimated ML parameters. We calculate these "smoothed" probabilities, $\Pr[S_t = 1 \mid \Delta_T]$ and in contrast $\Pr[S_t = 2 \mid \Delta_T]$ for each quarter based on the knowledge of the complete sample of data, in contrast to the "ex ante" probabilities, $\Pr[S_t = 1 \mid \Delta_t]$ and $\Pr[S_t = 2 \mid \Delta_t]$, which we calculate for each quarter based on information available up to date *t*. The "smoothed" probabilities provide a relatively objective method of dating major shifts in conditional volatility. In Hamilton [10] a direct method is proposed for dating regime switches, whereby an observation belongs to a given state if the corresponding smoothed probability exceeds 0.5. The "smoothed" probability in **Figures 2** and **3** strongly indicate the presence of two regimes. Both for the US and the UK, the probabilities remain extremely close to one or zero, indicating that the non-linear filter that generates the "smoothed" probabilities does reflect an underlying switching process rather than simply fitting parameters in an ad hoc manner.

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[4]Using a different methodology, Reference [6] finds similar results.
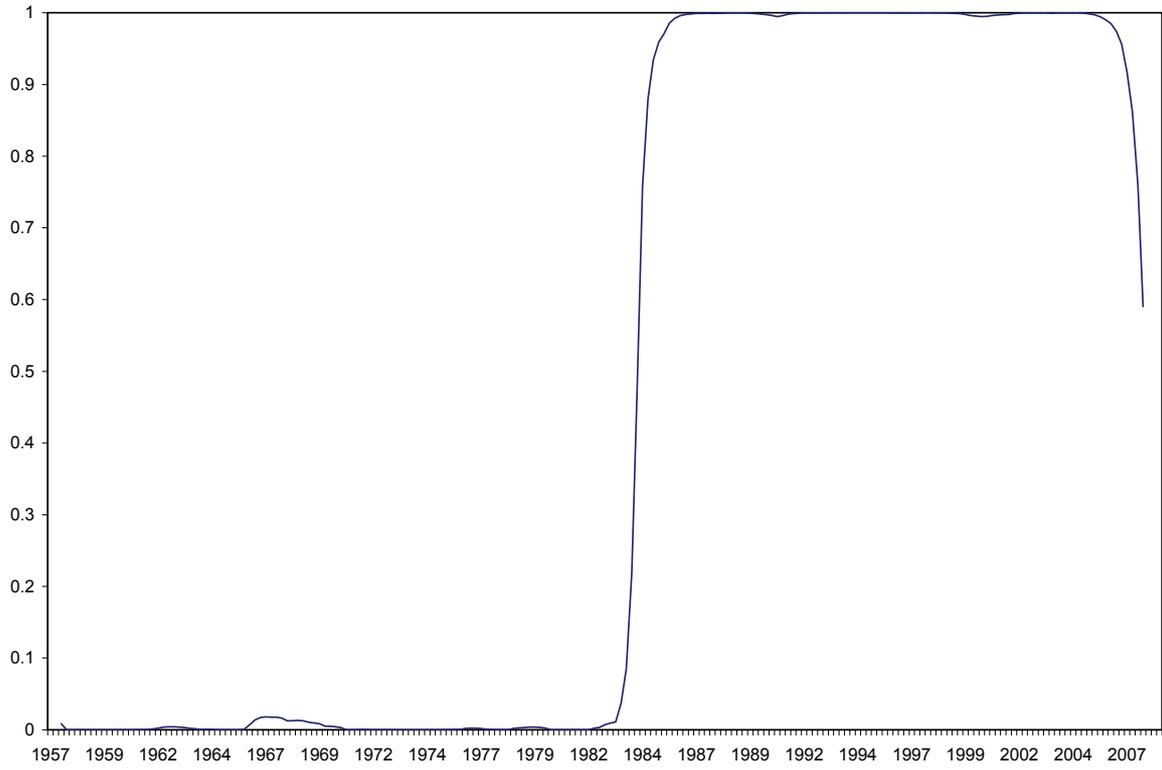
(a) US
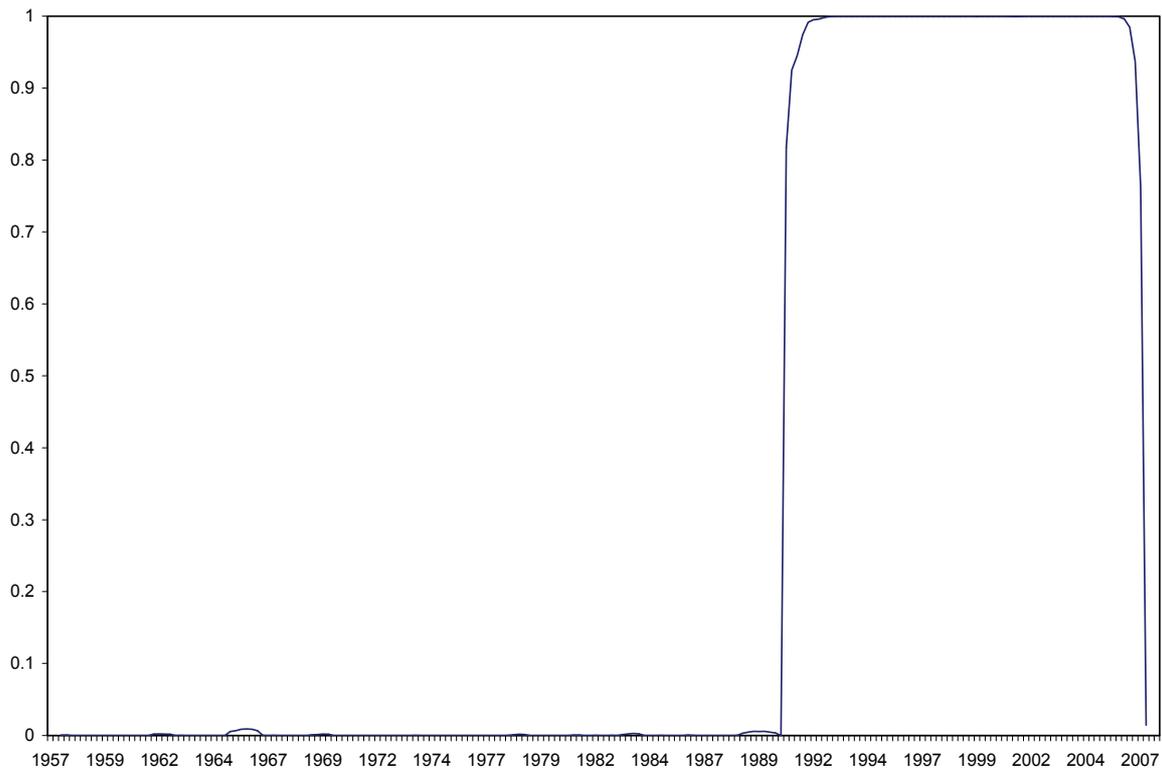
(b) UK

Figure 2. Smoothed probability of low volatility in state 1 (switching-mean and -variance model).
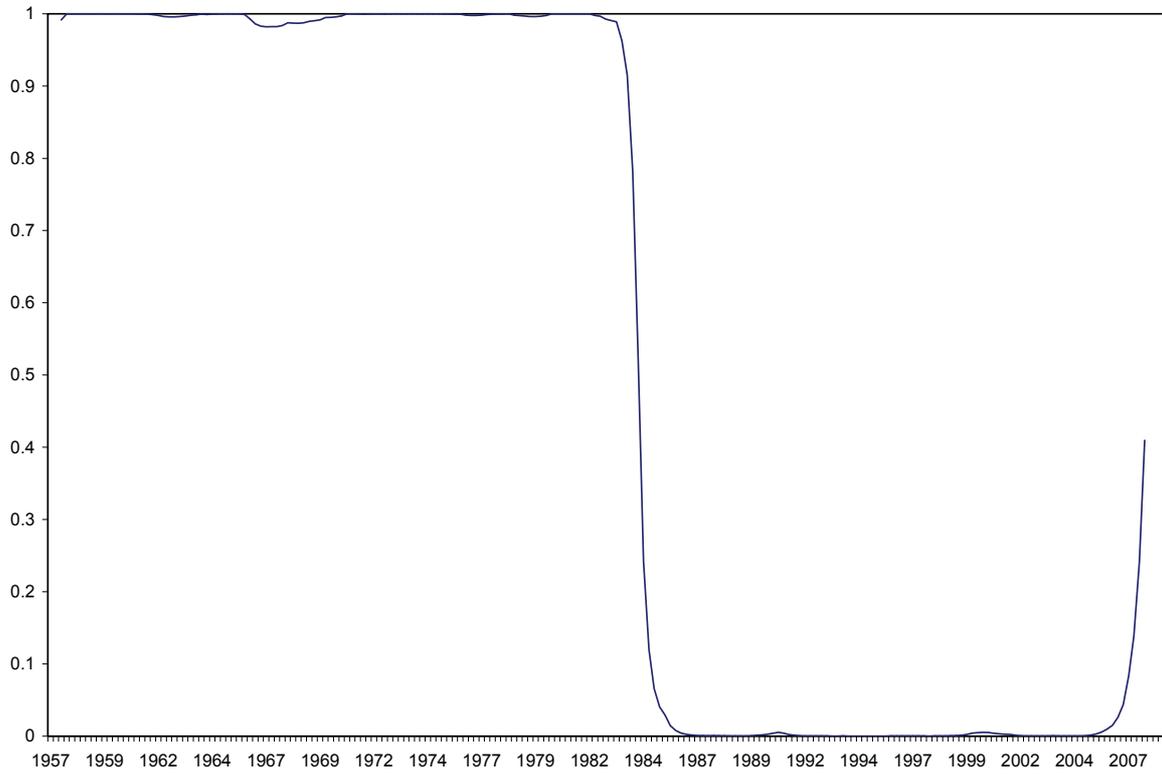
(a) US
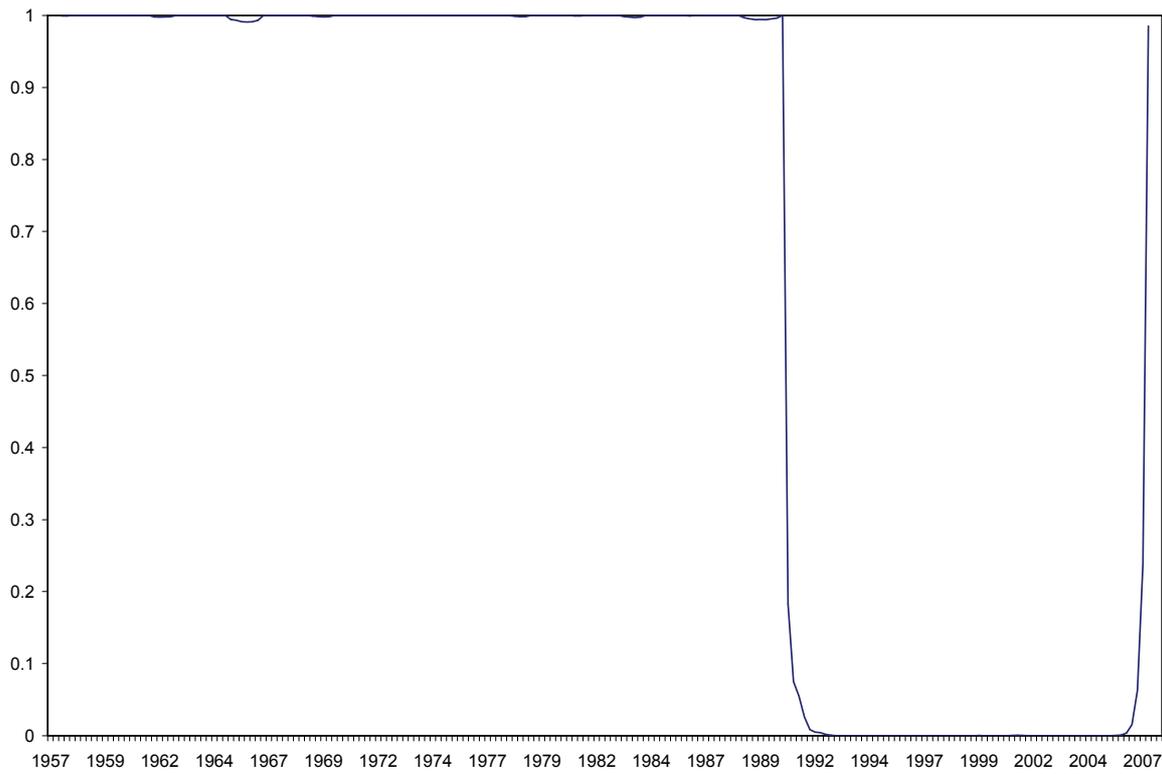
(b) UK

Figure 3. Smoothed probability of high volatility in state 2 (switching-mean and -variance model).

More specifically, these figures document the presence of two significant structural breaks both in the US and the UK economic growth process. In the US case, the first structural break occurs in 1984:03 and the second takes place in 2007:04. On the other hand, in the UK, the first structural break occurs in 1990:04 and the second in 2007:02. These two dates prove important in determining the length and duration of the Great Moderation in the two countries.

Prior to 1984:03 in the US, the probability of state 1 lies numerically extremely close to zero. This means that from the beginning of the sample through 1983:03, the US rate of growth of real GDP experiences high volatility. Beginning in 1984:03, however, the probability of the low-volatility state 1 switches from 0.08 in 1983:04 to 0.21 in 1984:01, to 0.47 in 1984:02, and to 0.75 in 1984:03. From 1984:04 to 2006:04 this probability remains above 0.95, the period that coincides with the Great Moderation. Beginning with 2007:01, however, signs begin to suggest that the Great Moderation may come to an end (see **Figures 2** and **3**). The probability of the low-volatility state 1 starts to decline, in a fast and swift manner. In 2007:01, the probability of state 1 falls from nearly one to 0.91. This probability declines further to 0.86 in 2007:02, then to 0.75 in 2007:03 and finally to 0.59 in 2007:04. While technically still greater than 0.5, this evidence points to the beginning of the end of the Great Moderation era in the US.

The evidence favoring the ending of the Great Moderation appears stronger in the UK case. In 1990:04, the probability of state 1 increases to 0.81 from 0.000001 in the previous period and remains close to 0.99 until 2006:04, at which time the first slight decline occurs, from 0.98 in 2006:03 to 0.93 in 2006:04. This probability declines dramatically in the next two quarters, to 0.76 in 2007:01 and 0.01 in 2007:02, the end of the sample period for the UK.

4.2. Constant-Mean, Constant-Variance Model

Table 3 reports the estimates of the linear AR(1) single-regime constant-variance model, our fourth specification (6), and related diagnostic statistics. Clearly, the model does a poor job of modeling the volatility of both the US and the UK growth rates of real GDP. The distribution of the standardized residuals exhibits heavy leptokurticity and displays a significant departure from normality. Furthermore, significant evidence emerges of second-moment nonlinear dependencies in the standardized residuals. As noted by [39], the single-regime model effectively averages the variance over the sample period so that the model does a poor job of describing the data in either regime. This, in turn, induces positive serial correlation in the standardized squared residuals, as it overstates the variance in the low-variance regime and understates the variance in the high-variance regime.

Table 3. Parameter estimates and related statistics for single-regime, constant-variance model.

	US		UK	
Parameter	*Estimate*	*t-statistic*	*Estimate*	*t-statistic*
a_{01}	0.5736*	7.9804	0.6591*	8.4099
a_{11}	0.2885*	4.9006	−0.0631	−1.1316
σ	0.7324*	13.2973	0.9687*	16.6574
Log-likelihood	−255.1804		−280.6104	
AIC	516.3608		567.2208	
SIC	542.2104		593.0107	
HQ	530.3919		581.2294	
Diagnostic Tests	*Statistic*	*p-value*	*Statistic*	*p-value*
Q(4)	2.24512	0.6908	6.3642	0.1735
Q(8)	8.1799	0.4161	15.3956	0.0519
Q2(4)	13.7143	0.0083	17.3848	0.0016
Q2(8)	28.3926	0.0004	20.7653	0.0078
Skewness	−0.2495	0.1508	0.3084	0.0772
Kurtosis (*excess*)	1.6488	0.0000	3.7844	0.0000
Jarque-Bera	24.9777	0.0000	122.5144	0.0000

Note: The *AIC, SIC,* and *HQ* equal Akaike, Schwartz-Bayesian, and Hannan-Quinn information criterion. The $Q_1(k)$ and $Q_2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to *k* lags.
* denotes 1% significance level.
** denotes 5% significance level.

As previously noted, the test of the null hypothesis of a single-regime constant-variance model against the alternative of a regime-switching model is not straightforward. Under the null, only one regime exists in fact that governs the rate of growth of real GDP. Thus, we cannot identify the regime staying probabilities p and q. This makes the asymptotic distribution of the usual tests (likelihood ratio, Wald and Lagrange multiplier) no longer chi-square [42,52,53]. To interpret the likelihood ratio statistics, we appeal to the methods in [15]. Testing the null of single regime against the alternative of a switching regime implies that r = 3, where r equals the number of restrictions (*i.e.*, $a_{01} = a_{02}$, $a_{11} = a_{12}$, and $\sigma_1 = \sigma_2$). From (7), we can calculate that the 0.05 (0.01) upper bound requires a value of 12.94 (16.91), rather than the conventional chi-square value of 7.81 (11.30). Values exceeding this upper bound suggest rejecting the null hypothesis. The LR test statistics for the US equals 49.51 and for the UK, 110.81. These numbers imply that we reject the null in both cases, even after invoking the upper bound in [42]. Thus, these results provide strong evidence in favor of the two-state regime-switching specification for the growth rates of real GDP of the US and the UK.

4.3. Switching-Mean, Constant-Variance Model

Table 4 reports the ML estimates of the switching-mean, constant-variance model, our second specification (4), (*i.e.*, $a_{01} \neq a_{02}$, $a_{11} \neq a_{12}$, but $\sigma_1 = \sigma_2$). The large difference in mean growth rates between the two regimes provides the most conspicuous feature of the estimates. The estimates of the transition probabilities imply that the probability of remaining in the low volatility state 1 remains extremely high for both the US and the UK. The situation differs for state 2. The probability in the US that state 2 will persist for more than one quarter equals only 0.1757, while the probability in the UK that state 2 will persist for more than one quarter equals a value about four times as high.

Figures 4 and **5** show how the "smoothed" probability of residing in either state 1 or state 2 evolves over the sample. The evidence in **Figure 4** indicates that when the probability of residing in the low volatility state 1 deviates from 1, it does so for a short period of time. The figure reflects this in the sharp spikes at irregular intervals, especially during the mid and late seventies, the early eighties, and the early nineties. The switching-mean model improves over the single-regime, con-

Table 4. Parameter estimates and related statistics for switching-mean, constant-variance model.

Parameter	US		UK	
	Estimate	*t-statistic*	*Estimate*	*t-statistic*
a_{01}	0.7249*	9.4073	0.8914*	11.2831
a_{02}	−1.3946*	−3.9970	−0.7775*	−3.1722
a_{11}	0.2330*	3.6135	−0.1406**	−2.2785
a_{12}	0.5542**	2.2063	−0.4749*	−3.2266
σ	0.7303*	12.1771	0.8294*	18.3436
p	0.9519*	30.8059	0.9658*	58.9268
q	0.1757*	5.7510	0.7436**	2.3222
Log-likelihood	−249.2448		−271.9547	
AIC	508.4896		553.9094	
SIC	551.5722		596.8925	
HQ	518.5207		563.9180	
Diagnostic Tests	*Statistic*	*p-value*	*Statistic*	*p-value*
$Q(4)$	2.7166	0.6063	2.6249	0.6224
$Q(8)$	8.6743	0.3705	14.3417	0.0733
$Q^2(4)$	14.6559	0.0055	13.7299	0.0082
$Q^2(8)$	30.6716	0.0002	19.5367	0.0122
Skewness	−0.2116	0.2227	0.4929	0.0047
Kurtosis (*Excess*)	1.5593	0.0000	3.9960	0.0000
Jarque-Bera	21.9747	0.0000	141.1692	0.0000

Note: The *AIC, SIC*, and *HQ* equal Akaike, Schwartz-Bayesian, and Hannan-Quinn information criterion. The $Q_1(k)$ and $Q_2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to k lags.
* denotes 1% significance level.
** denotes 5% significance level.
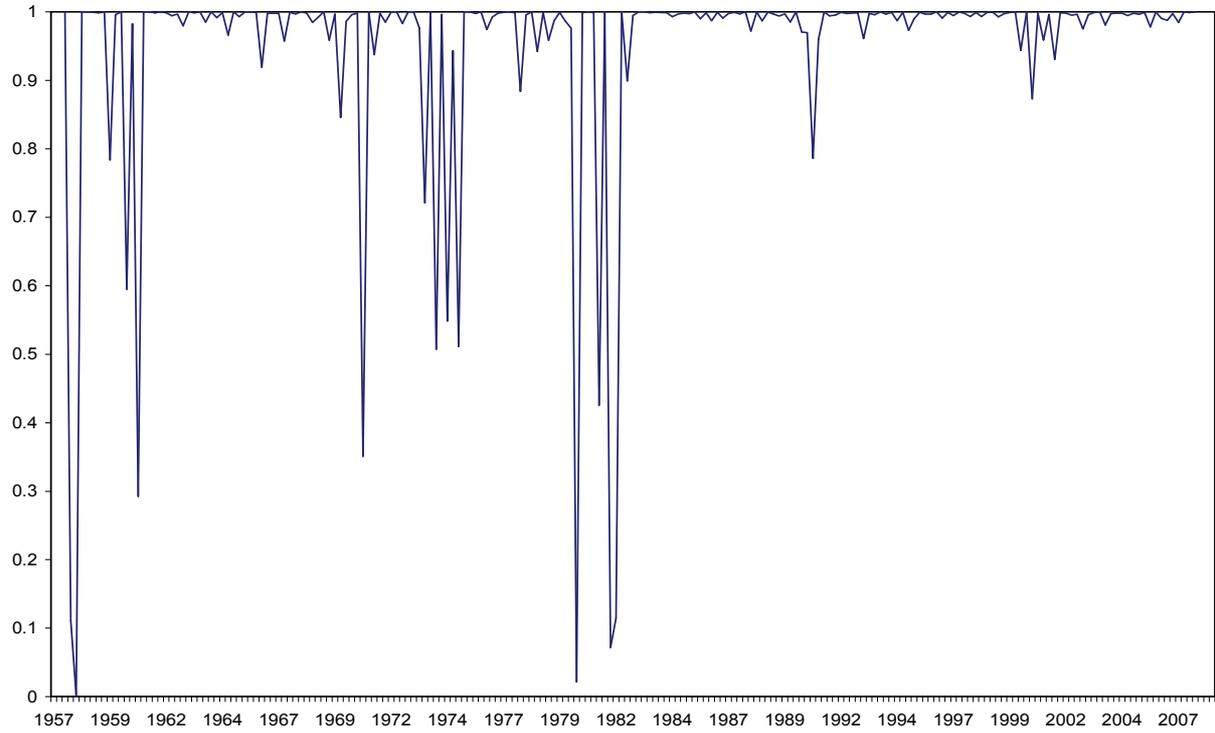
(a) US
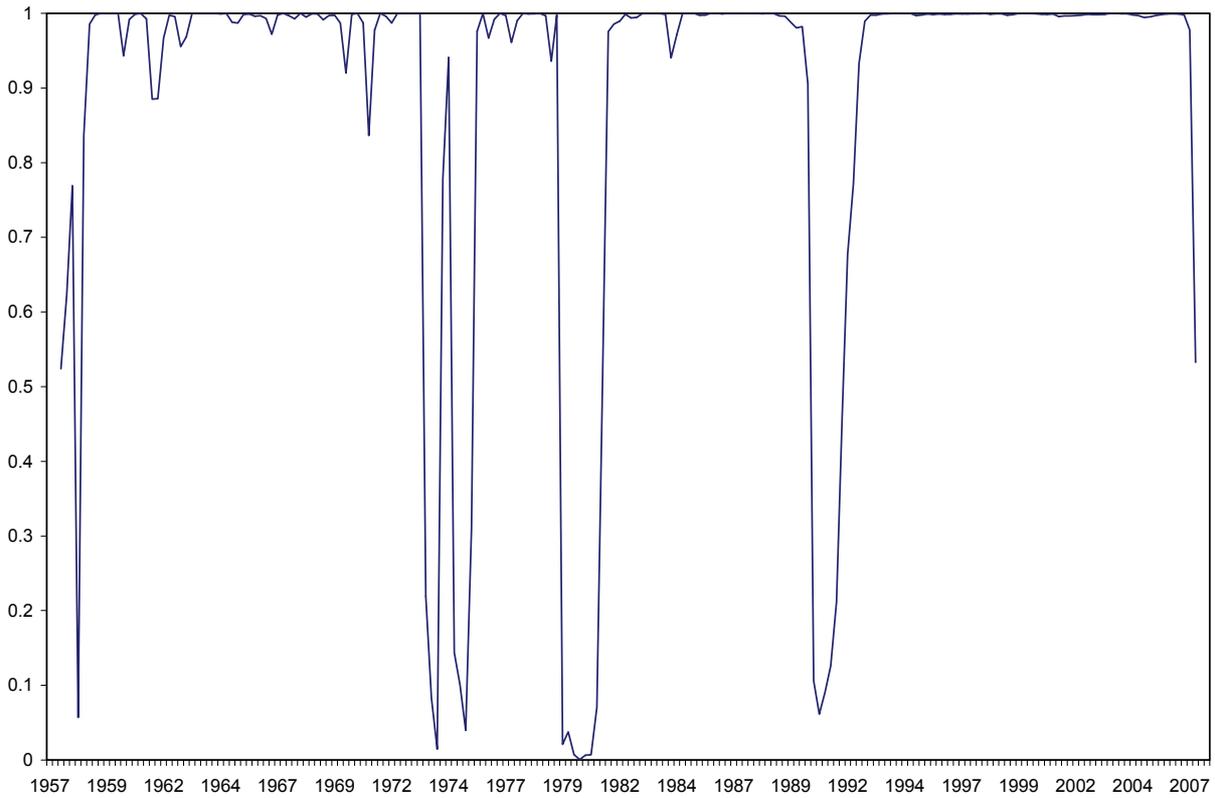
(b) UK

Figure 4. Smoothed probability of state 1 (switching-mean, constant-variance model).
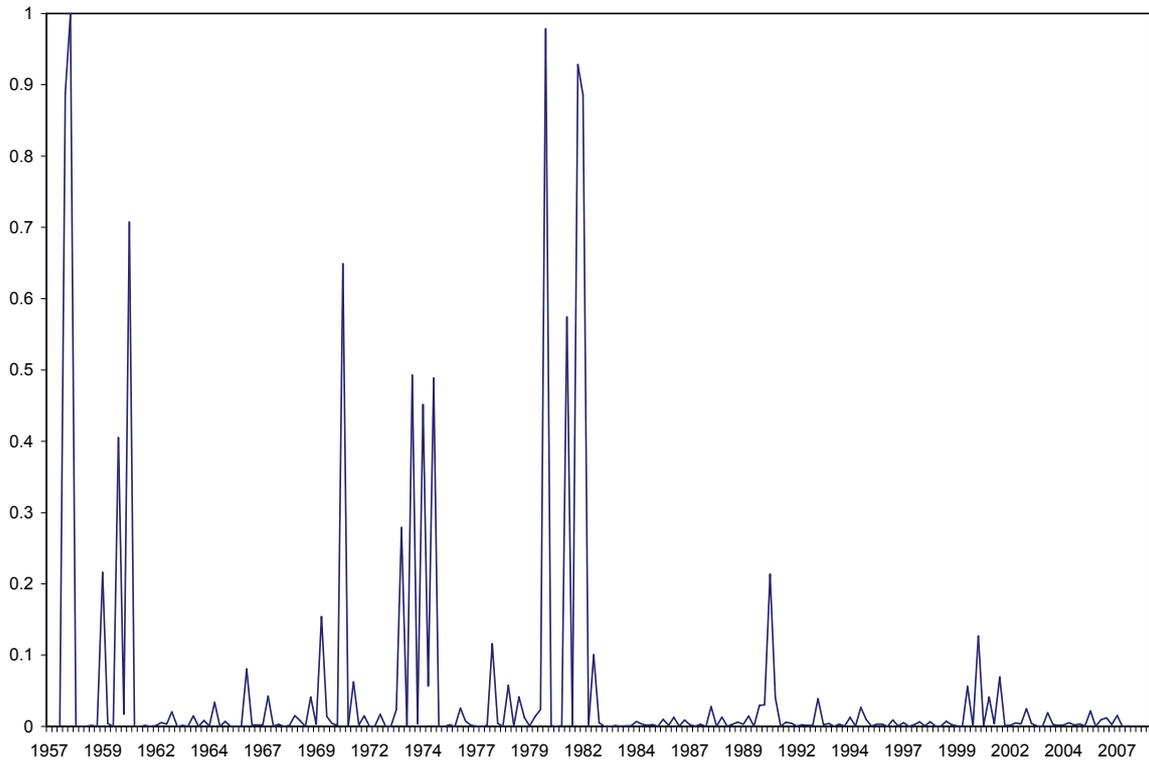
(a) US
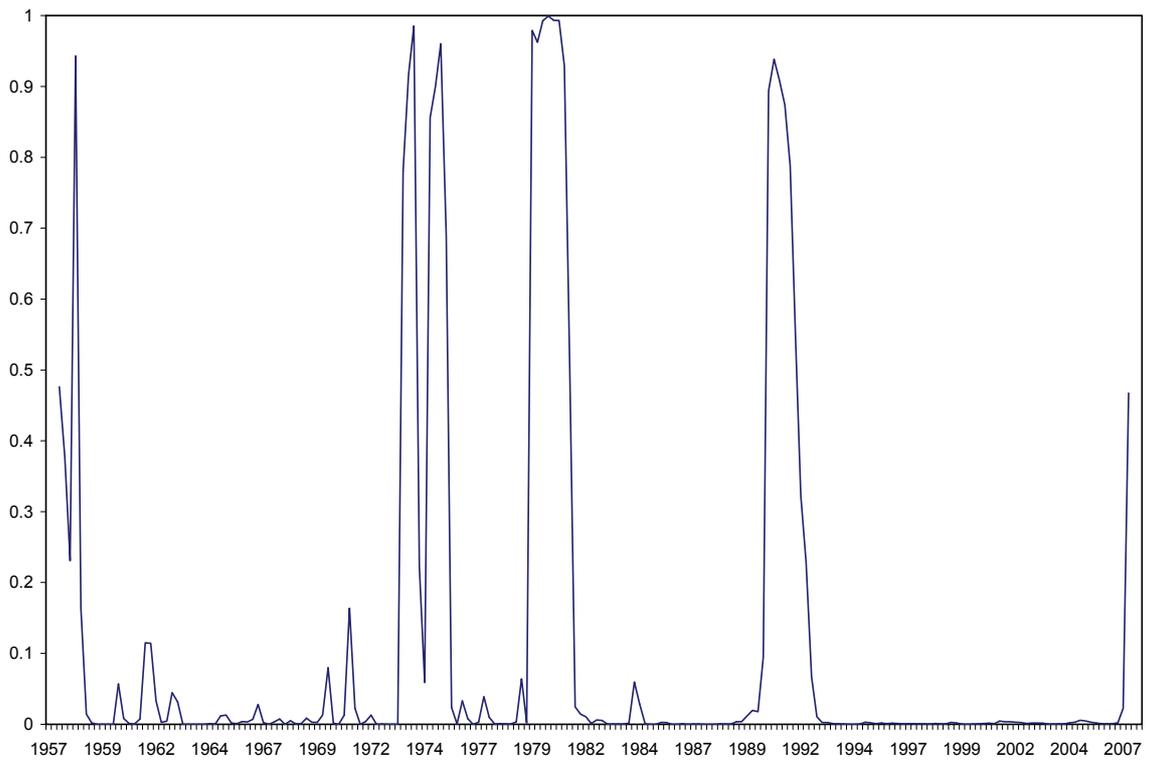
(b) UK

Figure 5. Smoothed probability of state 2 (switching-mean, constant-variance model).

stant-variance model. The log-likelihood function increases slightly in the US and the UK from –255.1804 to –249.2448 and from –280.6104 to –271.9547, respectively. Furthermore, the switching-mean model captures a divergent pattern displayed by the autoregressive dynamics of output growth as the autoregressive coefficient in high-volatility state 2 is twice as large as in low-volatility state 1. This result has important economic implications as it suggests that the autoregressive dynamics of output growth varies along the business cycle. The model remains distinctly inadequate, however, as still evidence exists of second-moment dependencies, leptokurticity, and non-normality in the standardized residuals. We can easily test the null hypothesis of the switching-mean, constant-variance model against the alternative of the switching-mean and -variance model. That is, the LR test statistic, chi-square distributed with one degree of freedom under the null, equals 37.64 for the US and 93.50 for the UK, proving significant at usual levels. We, thus, reject the restricted switching-mean, constant variance model in favor of the unrestricted switching-mean and -variance model.

4.4. Switching-Variance, Constant-Mean Model

Table 5 reports the ML estimates of the switching-variance, constant-mean model, our third specification (5), (*i.e.*, $a_{01} = a_{02}$, $a_{11} = a_{12}$, but $\sigma_1 \neq \sigma_2$). The esti-

mates of σ_1 and σ_2 show that in the US, the variance of output growth is about two times as high in high-volatility state 2 as in low-volatility state 1, while in the UK, it is about four times as high in state 2 as in state 1. The estimates of the transition probabilities show that both states imply extreme persistence. This contrasts with the results of the specification with switching-mean, constant-variance model, where the transition probability of state 2 did not indicate persistence.

Figures 6 and **7** illustrate the smoothed probabilities of states 1 and 2. The graphs prove quite dissimilar to the graphs in **Figures 2** and **3**. An extended period of high volatility exists followed by a period of low volatility. Based upon Hamilton's dating method, the period of low volatility starts in 1984:02 for the US, as the smoothed probability of low-volatility state 1 increases to 0.61, a value which, for the first time, exceeds 0.5. Conversely, for the UK the period of low volatility starts later, in 1992:03, as the smoothed probability of state 1 increases to 0.74 for the first time since the beginning of the sample. The peculiar feature of the Figures, however, does not rest with the dating of the beginning of the Great Moderation, which received much attention in the applied econometric literature. Rather, it rests with the dating of the end of that period. A detailed scrutiny of the path of the probability of low-volatility state 1 indicates that in the US, the probability of state 1 declines beginning in 2007:02. More specifically, the probability of

Table 5. Parameter estimates and related statistics for switching-variance, constant-mean model.

	US		UK	
Parameter	Estimate	t-statistic	Estimate	t-statistic
a_{01}	0.5578*	6.2811	0.6567*	8.6642
a_{11}	0.2772*	3.2523	0.0729	0.8703
σ_1	0.4811*	12.8593	0.2602*	9.6612
σ_2	1.0863*	14.4597	1.1786*	16.4466
P	0.9941*	128.6160	0.9923*	90.8772
Q	0.9945*	147.1974	0.9951*	169.8925
Log-likelihood	–230.6990		–232.3390	
AIC	469.3980		472.6780	
SIC	503.8641		507.0645	
HQ	481.4291		484.6866	
Diagnostic Tests	Statistic	p-value	Statistic	p-value
Q(4)	7.0479	0.1334	4.1217	0.3898
Q(8)	10.7297	0.2175	9.5028	0.3017
Q^2(4)	2.5105	0.6427	2.3297	0.6754
Q^2(8)	6.8100	0.5573	3.8522	0.8702
Skewness	–0.3336	0.0546	–0.2493	0.1530
Kurtosis (excess)	0.4833	0.1682	1.7175	0.0000
Jarque-Bera	5.7146	0.0574	26.6573	0.0000

Note: The *AIC, SIC*, and *HQ* equal Akaike, Schwartz-Bayesian, and Hannan-Quinn information criterion. The $Q_1(k)$ and $Q_2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to *k* lags.
* denotes 1% significance level.
** denotes 5% significance level.
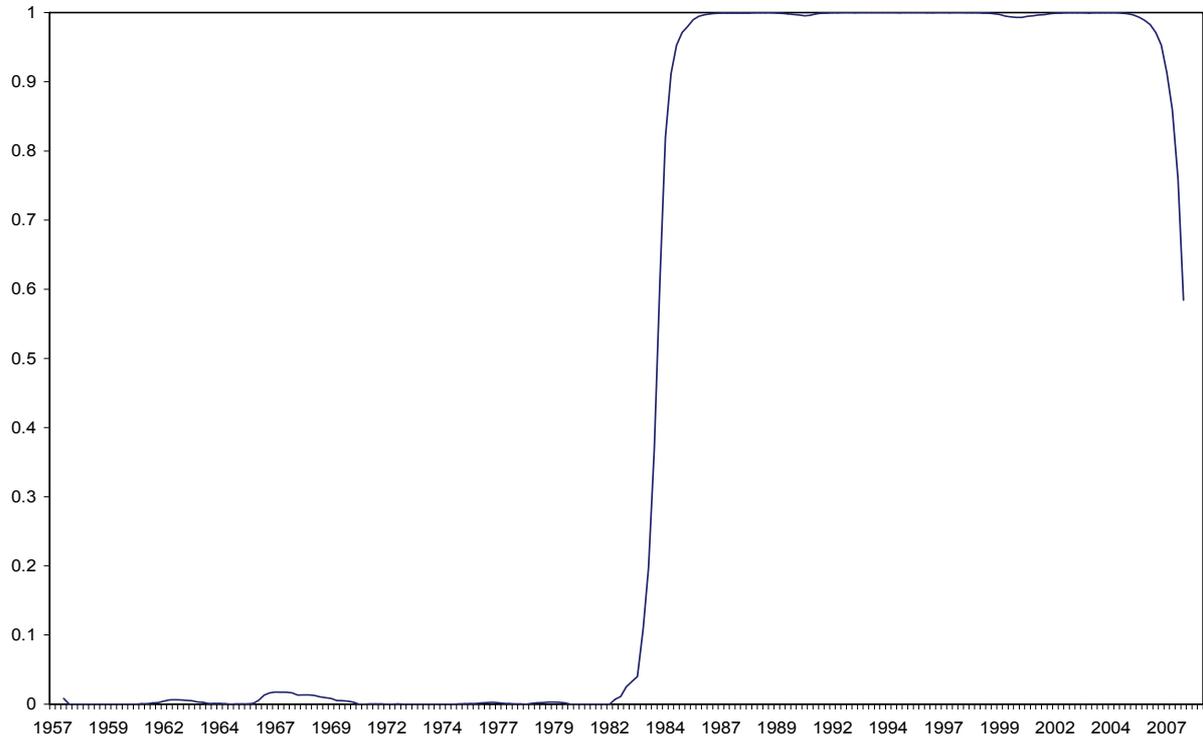
(a) US
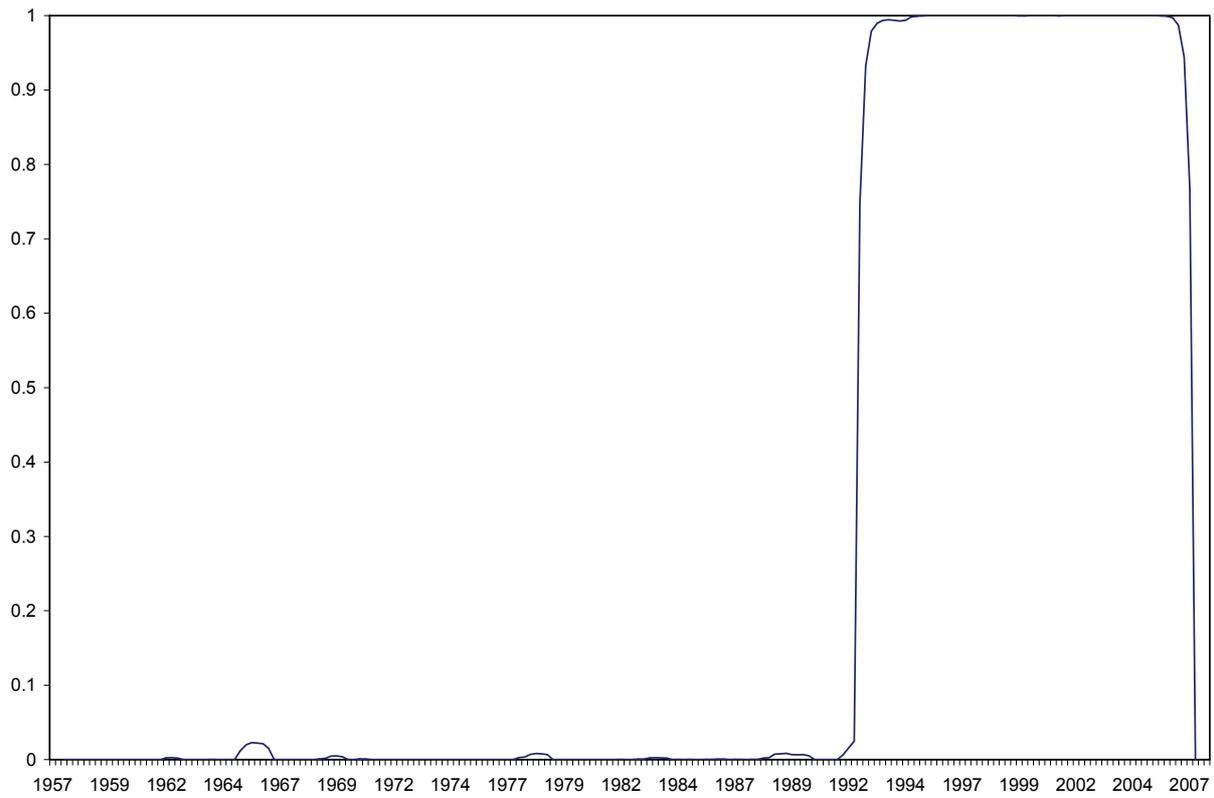
(b) UK

Figure 6. Smoothed probability of state 1 (switching-variance, constant-mean model).
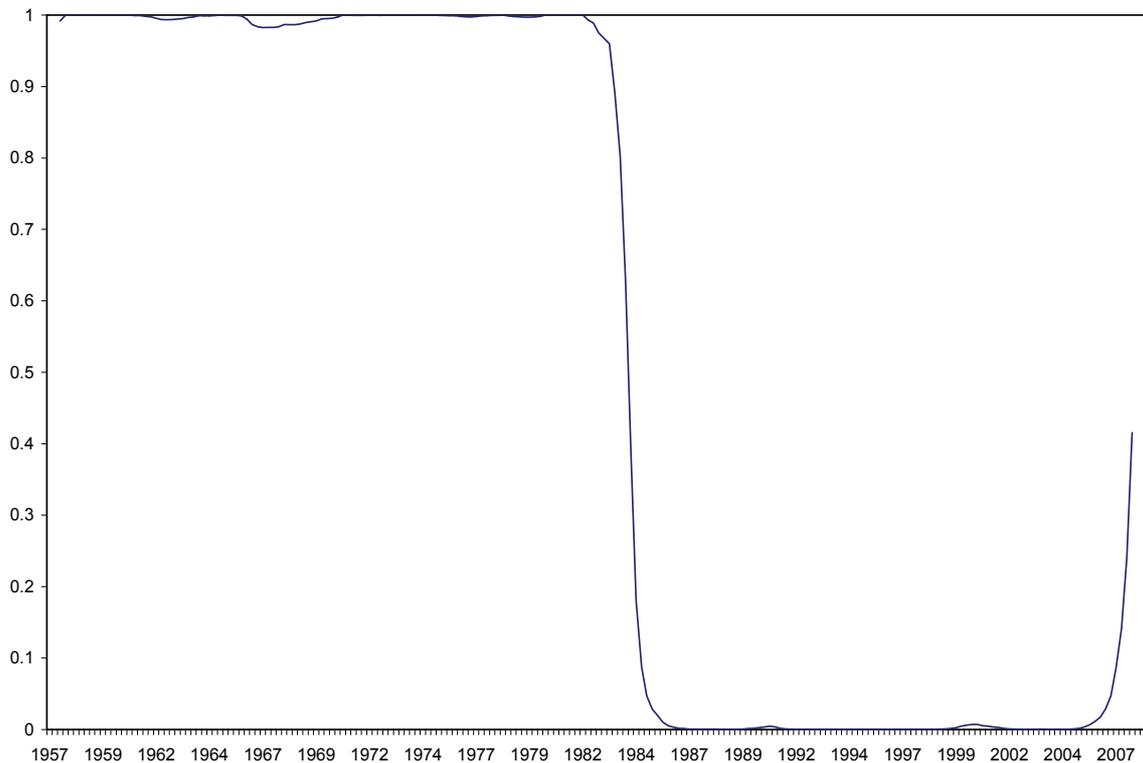
(a) US
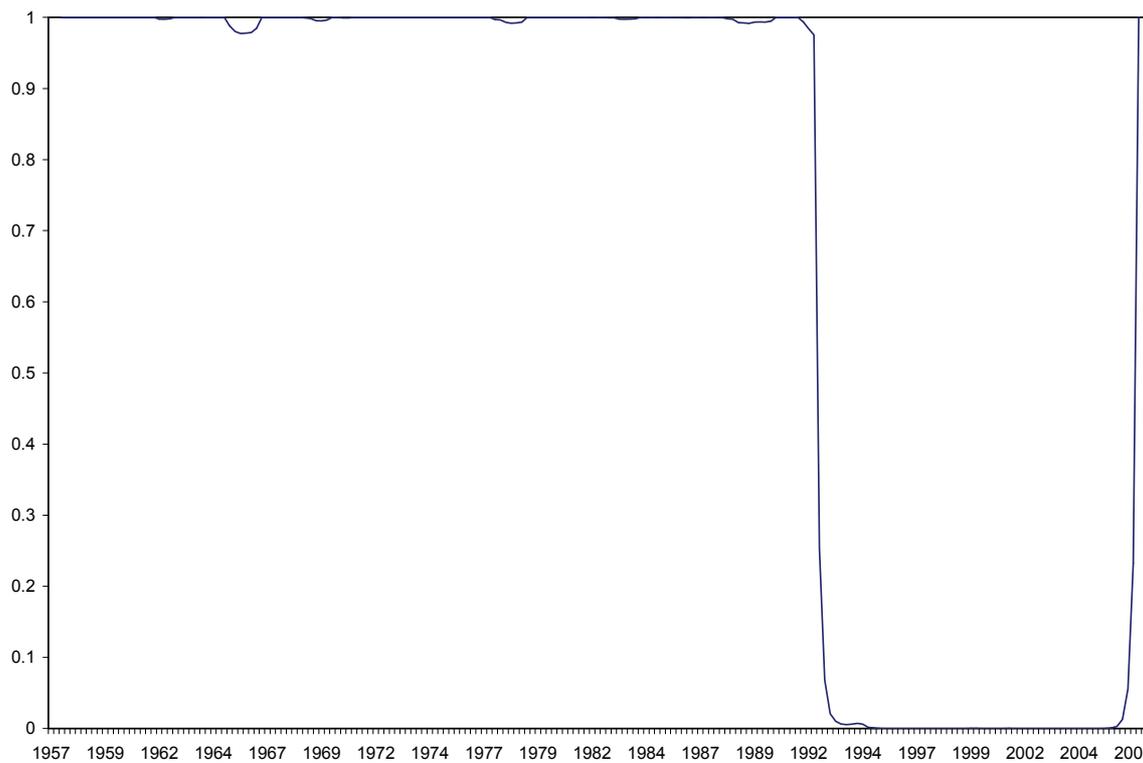
(b) UK

Figure 7. Smoothed probability of state 2 (switching-variance, constant-mean model).

low volatility goes from 0.91 in 2007:01 to 0.85 in 2007:02, to 0.75 in 2007:03, and to 0.58 in 2007:04. In the UK, the evidence that "Great Moderation" ended appears even more striking. The probability of low variability in 2006:04 equals 0.94, but in 2007:01 it drops to 0.76, and in 2007:02 to 0.00.

Unlike the switching-mean, constant-variance model, we cannot reject the restricted switching-variance, constant-mean model in the US case in favor of the unrestricted switching-mean and -variance model. The LR test statistic, chi-square distributed with two degree of freedom under the null, equals 0.5556, which is not significant. In the UK, however, the LR test statistic equals 14.2754, which is significant at usual levels. Thus, we can reject the switching-mean, constant- variance model for both the US and the UK in favor of the switching-mean and -variance model, but we can only reject the switching-variance, constant-mean only for the UK.

The results of our analysis suggest that the growth of real GDP for the US and the UK exhibit Markov-switching behavior. Based on the evidence of a two-state Markov-switching dynamics, the issue, however, arises with respect to the stationarity of the two growth-rate series. According to the single-regime standard ADF test statistics, the two series prove stationary. The ADF statistics (with intercept and 0 lags on the differences) equal −10.53258 and −15.00517, respectively, for the US and the UK.

4.5. Regime-Switching Stationarity Tests

Table 6 reports the estimation results for the switching regime ADF test (8) for $q = 0$ (*i.e.*, a switching regime DF test). Strong evidence emerges to support locally stationary output growth in both the US and the UK. The estimates of b_{11} and b_{12} both prove negative in the high and low volatility regimes, and the associated t-values far exceed in absolute value the Dickey-Fuller statistics. Note, however, that these t-values do not follow the Dickey-Fuller distribution. In [43] Monte-Carlo methods are used to calculate the p-values for the t-statistics. We do not pursue this approach for two reasons. First, we strongly reject the single-regime ADF in favor of Markov switching ADF. The maximized values of likelihood function for the single regime ADF equals −255.18 and −280.61 for the US and the UK, respectively. Consequently, the LR test statistic equals 51.68 for the US, while for the UK, it equals 114.48. Thus, we can clearly reject the null in both cases even after invoking Davies' upper bound. Second, both regimes prove locally stationary, vastly different from the results obtained by [32]. Furthermore, our main interest lies in dating the two regimes. From this viewpoint, the results of the Markov switching ADF regressions corroborate the dating evidence on the Great Moderation previously obtained. **Figures 8** and **9** plot the smoothed probabilities.

Table 6. Parameter estimates and related statistics for the markov-switching unit-root model.

Parameter	US		UK	
	Estimate	t-statistic	Estimate	t-statistic
a_{01}	0.5715*	6.6226	0.2724*	4.7169
a_{02}	0.5975*	4.5554	0.6951*	6.1209
b_{11}	−0.7633*	−7.1754	−0.4037*	−5.0967
b_{12}	−0.7008*	−7.5390	−1.1131*	−13.0996
σ_1	0.4781*	12.4593	0.2758*	12.2513
σ_2	1.0867*	14.0693	1.1697*	17.9006
p	0.9941*	138.5665	0.9932*	115.0833
q	0.9945*	174.948	0.9952*	164.9087
Log-likelihood	−229.3400		−223.3798	
AIC	470.6801		458.7598	
SIC	522.3196		510.2792	
HQ	478.6999		466.7569	
Diagnostic Tests	Statistic	p-value	Statistic	p-value
Q(4)	2.2451	0.6908	6.3642	0.1735
Q(8)	8.1799	0.4161	15.3955	0.0519
Q^2(4)	13.7143	0.0083	17.3847	0.0016
Q^2(8)	28.3926	0.0004	20.7653	0.0078
Skewness	3.9664	0.0000	0.3083	0.0772
Kurtosis (excess)	19.7141	0.0000	3.7843	0.0000
Jarque-Bera	3800.7748	0.0000	122.5144	0.0000

Note: The *AIC, SIC*, and *HQ* equal Akaike, Schwartz-Bayesian, and Hannan-Quinn information criterion. The $Q_1(k)$ and $Q_2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to k lags.
* denotes 1% significance level.
** denotes 5% significance level.

(a) US

(b) UK
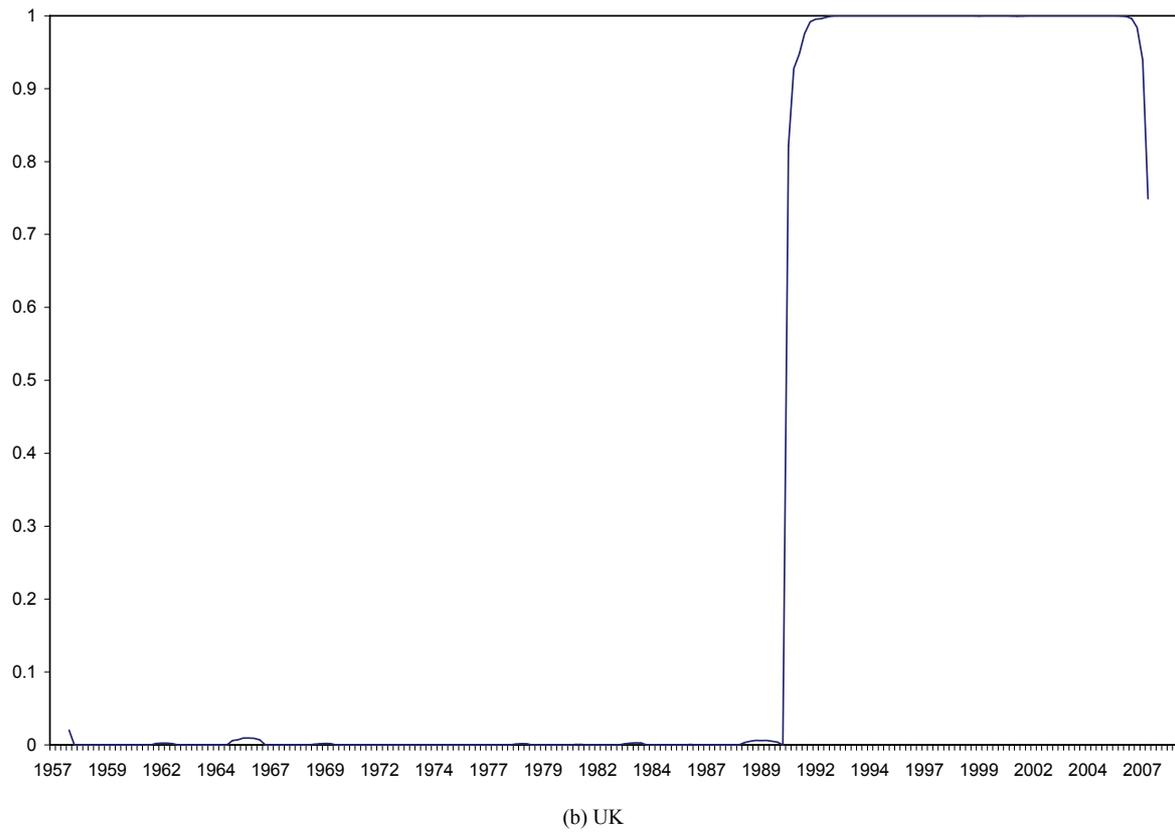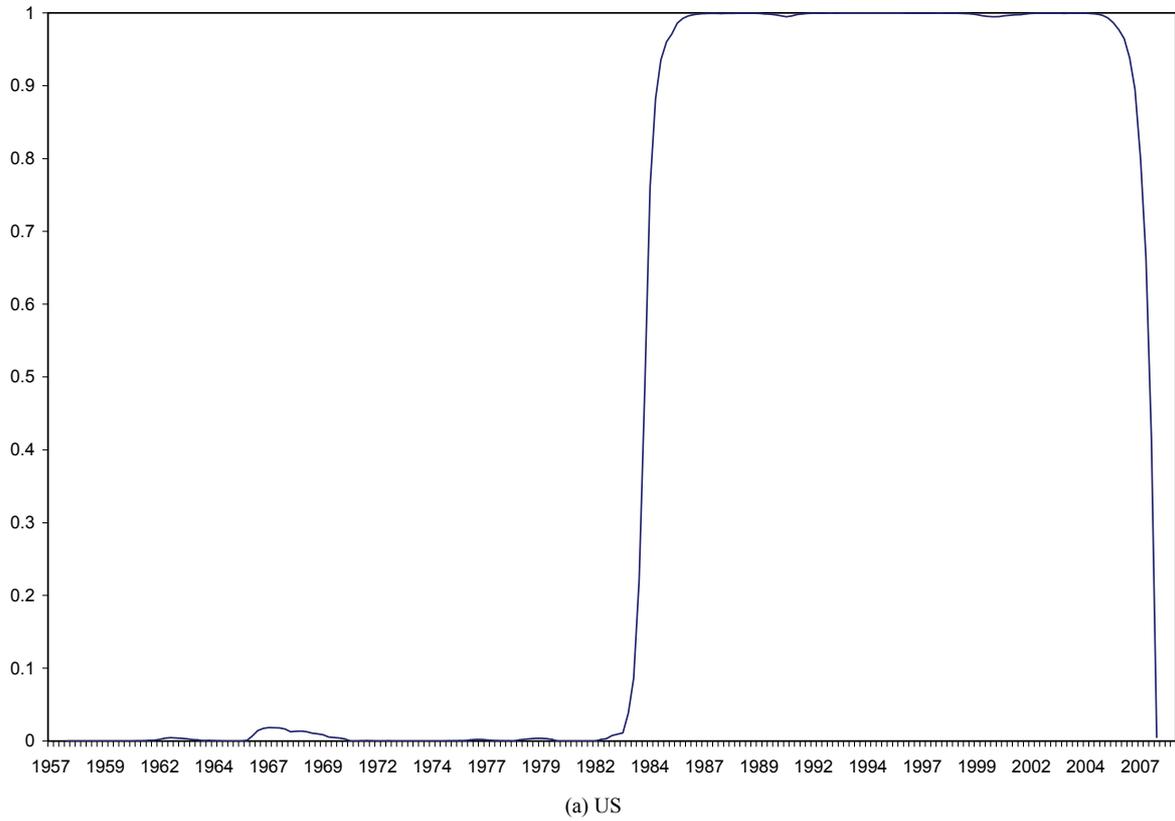
Figure 8. Smoothed probability of state 1 (switching-ADF model).

ME
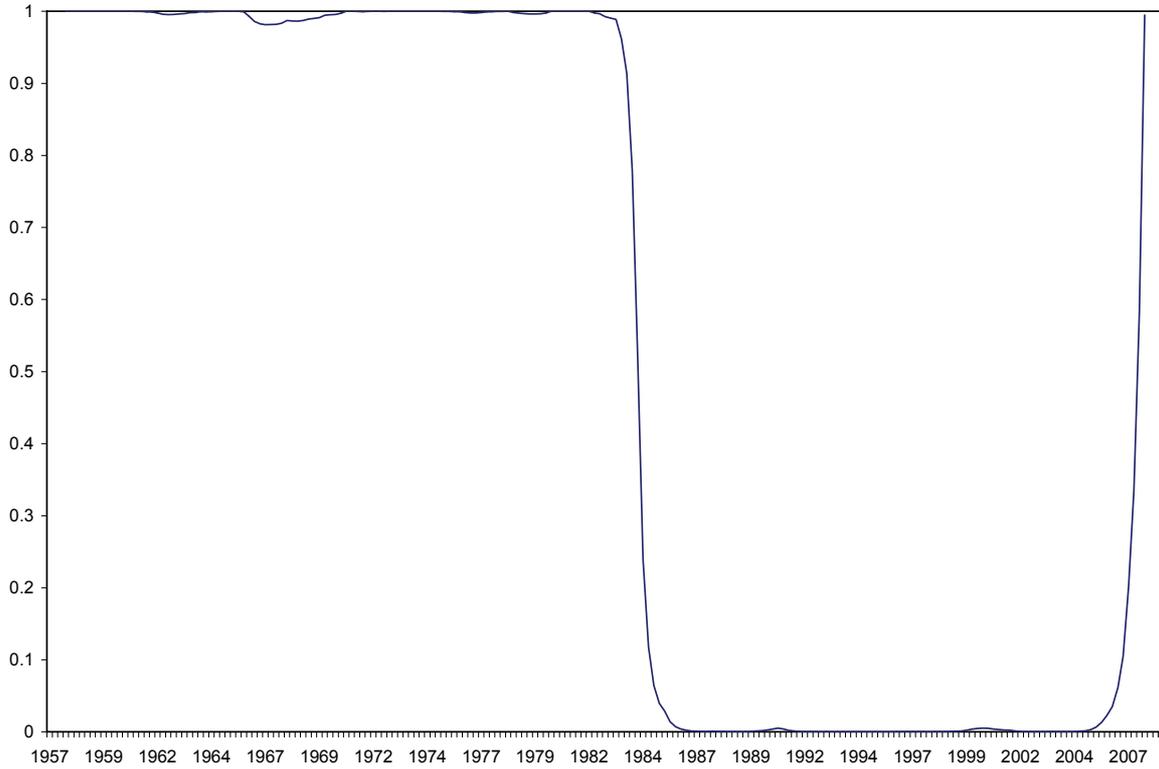
(a) US
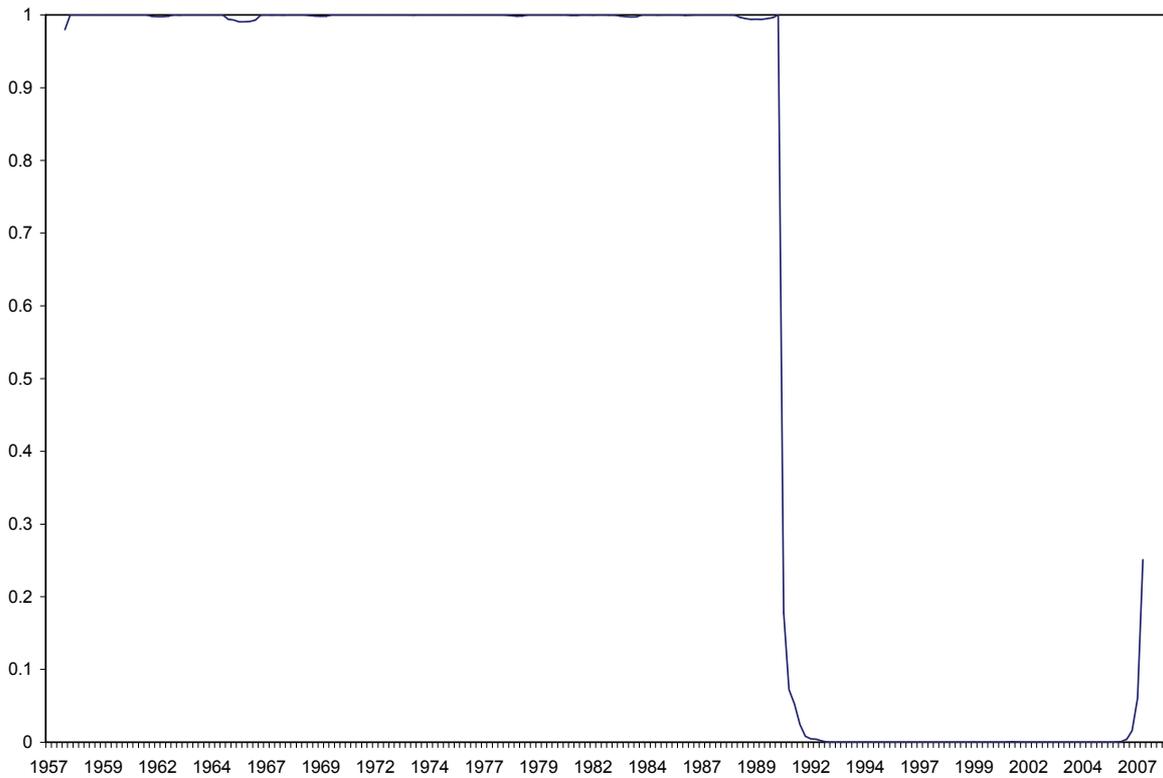
(b) UK

Figure 9. Smoothed probability of state 2 (switching-ADF model).

They also show ample evidence for regime changes in the real GDP growth rate. Such changing-persistence behavior would not emerge from the standard unit-root tests, which assume persistence remains constant through the sample sub-periods.

The dates of the beginning and ending of the Great Moderation nearly match those obtained using the Markov-switching models. Based upon Hamilton's dating method, the period of low volatility starts for the US in 1984:03, as the smoothed probability of state 1 increases to 0.76 and ends in 2007:03 as the smoothed probability of low variability decreases to 0.41. This decline is immediately followed in 2007:04 by a further sharp decrease to 0.0052. For the UK, instead, the dates of the beginning and ending of the Great Moderation are slightly different from the ones detected by the Markov-switching model. The Markov switching ADF regression places the beginning of the Great Moderation on the last quarter of 1990 rather than the third quarter of 1992. The Markov-switching ADF regression does not date the end of the Great Moderation in the UK, but hints at it, as the probability of low variability declines from 0.94 in 2007:01 to 0.74 in 2007:02.

4.6. Autoregressive Conditional Heteroskedastic Variance Markov Regime-Switching Model

We now relax the assumption of constant variance within each regime and allow the conditional variances to follow a switching ARCH (2) process-SWARCH(2), our fifth specification (9). We use the AIC criterion to choose the SWARCH (2) structure. **Table 7** reports the estimates for the single-regime version of the model. The autoregressive parameters nearly match those reported for the constant variance regime-switching model. The conditional-variance parameters prove statistically significant, as expected. For the US, however, the sum of the ARCH estimates $b_1 + b_2$ falls significantly below unity, which satisfies the stationarity assumption. Conversely, for the UK, a Wald test supports the violation of the stationarity assumption, whereby the conditional variance follows an integrated ARCH and $b_1 + b_2 = 1$. The Wald test statistic, distributed chi-square(1) under the null, equals 0.066, which proves insignificant at any usual level (*p*-value = 0.7971).

Table 8 reports estimates of the regime-switching AR (1)-ARCH (2) model. Results remain virtually unchanged for higher ARCH (3) or lower ARCH (1) lags of the ARCH process. The striking feature of the results suggests that although the states remain highly persistent, the underlying fundamental ARCH (2) process does not. That is, the volatility effects as revealed by the switching ARCH estimates do not exhibit high persistence. This reflects the estimates of the decay parameter, $\lambda = b_1 + b_2$ of the ARCH processes. The volatility effects for

Table 7. Parameter estimates and related statistics for the single-regime, AR (1)-ARCH (2) model.

	US		UK	
Parameter	*Estimate*	*t-statistic*	*Estimate*	*t-statistic*
a_0	0.5969*	7.1186	0.5854*	6.7926
a_1	0.3307*	4.7919	0.1393	1.2009
b_0	0.2955*	4.8300	0.2858*	4.3675
b_1	0.2249*	2.6940	0.5764*	3.2228
b_2	0.4765*	3.2284	0.4802**	2.2408
Log-likelihood	−240.7973		−261.8643	
AIC	491.5946		533.7286	
SIC	534.6772		576.7117	
HQ	501.6257		543.7372	
Diagnostic Tests	*Statistic*	*p-value*	*Statistic*	*p-value*
$Q_1(4)$	8.2489	0.0829	7.4829	0.1125
$Q_1(8)$	12.6372	0.1250	18.1080	0.0204
$Q_2(4)$	5.1937	0.2680	3.4514	0.4853
$Q_2(8)$	14.3546	0.0730	11.2561	0.1876
Skewness	−0.1916	0.2696	0.0484	0.7813
Kurtosis (excess)	1.3780	0.0000	3.6587	0.0000
Jarque-Bera	17.2194	0.0001	111.6327	0.0000

Note: The *AIC, SIC*, and *HQ* equal Akaike, Schwartz-Bayesian, and Hannan-Quinn information criterion. The $Q_1(k)$ and $Q_2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to *k* lags.
* denotes 1% significance level.
** denotes 5% significance level.

Table 8. Parameter estimates and related statistics for the markov regime-switching AR (1)-ARCH (2) model.

	US		UK	
Parameter	*Estimate*	*t-statistic*	*Estimate*	*t-statistic*
a_0	0.5663*	7.0196	0.6320*	10.0572
a_1	0.3050*	3.9860	0.0960	1.1875
b_0	0.1775*	3.8577	0.0433*	2.9659
b_1	0.0741	0.8184	0.2324	1.8457
b_2	0.1666	1.1977	0.1504	1.3099
p	0.9942*	68.7670	0.9919*	33.9063
q	0.9945*	108.7333	0.9948*	226.9010
γ_2	5.2573*	3.5984	22.3155*	3.3458
Log-likelihood	–228.8731		–228.7725	
AIC	469.7462		469.5451	
SIC	521.4454		521.1248	
HQ	477.7773		477.5536	
Diagnostic Tests	*Statistic*	*p-value*	*Statistic*	*p-value*
$Q_1(4)$	7.3931	0.1165	4.4542	0.3480
$Q_1(8)$	10.5687	0.2274	11.3292	0.1837
$Q_2(4)$	1.2823	0.8644	2.4926	0.6460
$Q_2(8)$	7.9561	0.4378	3.9427	0.8622
Skewness	–0.1409	0.4167	0.1413	0.4179
Kurtosis (excess)	0.2589	0.4602	2.0608	0.0000
Jarque-Bera	1.2338	0.5396	36.0588	0.0000

Note: The *AIC, SIC*, and *HQ* equal Akaike, Schwartz-Bayesian, and Hannan-Quinn information criterion. The $Q_1(k)$ and $Q_2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to k lags.
* denotes 1% significance level.
** denotes 5% significance level.

the US switching ARCH model die out in about 3 quarters ($\lambda^3 = 0.0139$), while those of the single-regime ARCH model persist for more than three years ($\lambda^{12} = 0.0141$). Conversely, the volatility effects for the UK switching ARCH model die out in about 4 quarters ($\lambda^4 = 0.0214$).

We note, however, that the ARCH terms in the single-regime model prove highly significant while in the switching-regime model, they lose their significance. In the switching-ARCH model of (9), changes in the regime do not affect the dynamics of the process, just the scale [31,54,55], which reflects the γ_2 parameter. The estimates of this parameter indicate that for the UK, the conditional variance in the high volatility state exceeds the low-volatility state by more than 22 times. For the US, instead, this ratio equals about 5. The residual diagnostics clearly indicate that no evidence exists of second-moment nonlinear dependencies in the standardized residuals.

In fact, the autoregressive coefficients for the ARCH(2) models in both regimes prove insignificantly different from zero. This suggests a homoskedastic error process, which matches the findings of [6]. They report that the GARCH and ARCH processes disappear once dummy variables capture the shift from high to low-volatility regimes.

A LR test rejects the single-regime constant-variance model in favor of the single-regime ARCH model. The LR test statistic, distributed as chi-squared with two degrees of freedom under the null, equals 28.7662 in the US and 37.4922 in the UK, which proves significant at any usual level. The regime-switching AR(1)-ARCH(2) model yields significantly higher log likelihood values than the single-regime AR(1)-ARCH(2). So, we unambiguously reject the null of no Markov switching by the Davies upper-bound test. The LR test statistics, distributed as chi-squared with one degree of freedom under the null, equal 23.8484 and 66.1836 for the US and the UK, respectively. These values, even after invoking Davies's upper-bound adjustment, prove highly significant. Thus, while the application of the single-regime ARCH model leads to nearly non-stationary variance processes, the use of the Markov-switching ARCH model substantially improves the results.

The results of the SWARCH model further confirm the previous dates of the beginning and end of the Great Moderation. The smoothed probabilities for the low- and high-volatility regimes (states 1 and 2, respectively) follow very closely the results found without the ARCH component. **Figures 10** and **11** illustrate this point. Based on Hamilton's dating method, the switching-ARCH model captures reasonably well the period of the Great Moderation. The low-volatility regime starts in the US in 1984:02, as the smoothed probability increases to
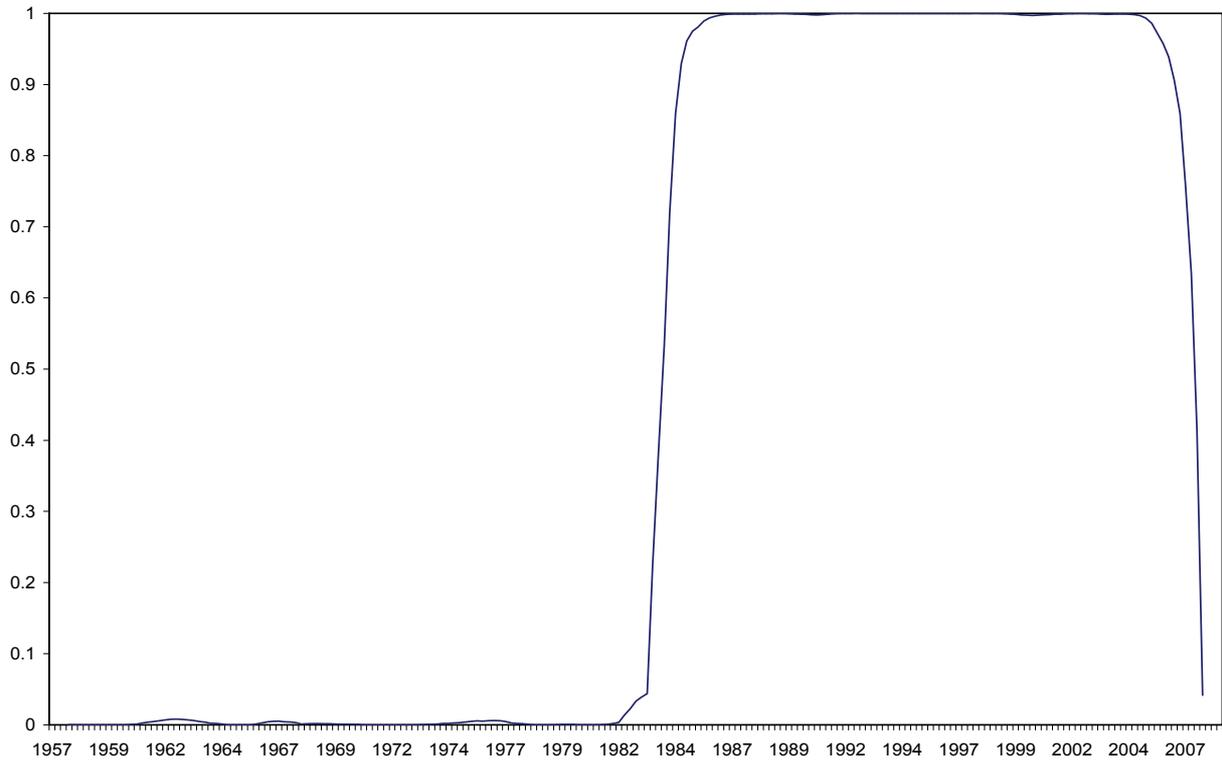
(a) US
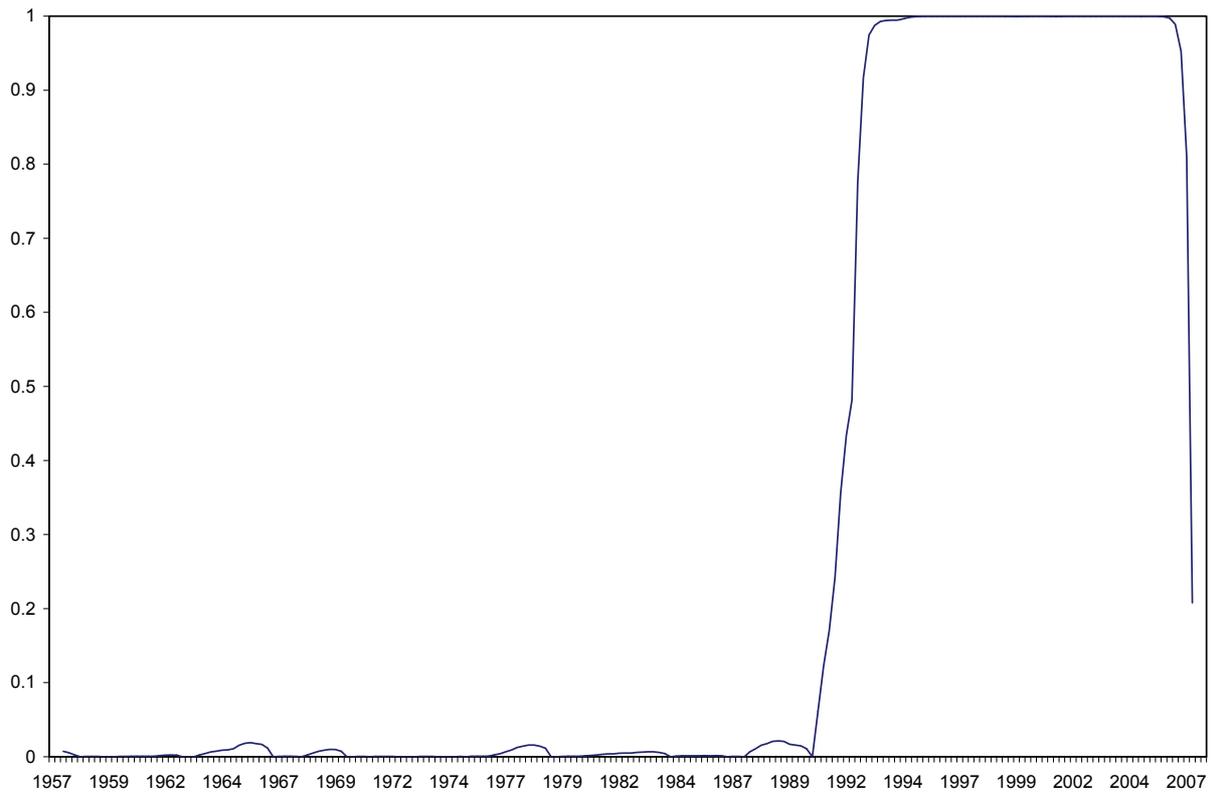
(b) UK

Figure 10. Smoothed probability of state 1 (switching-ARCH model).
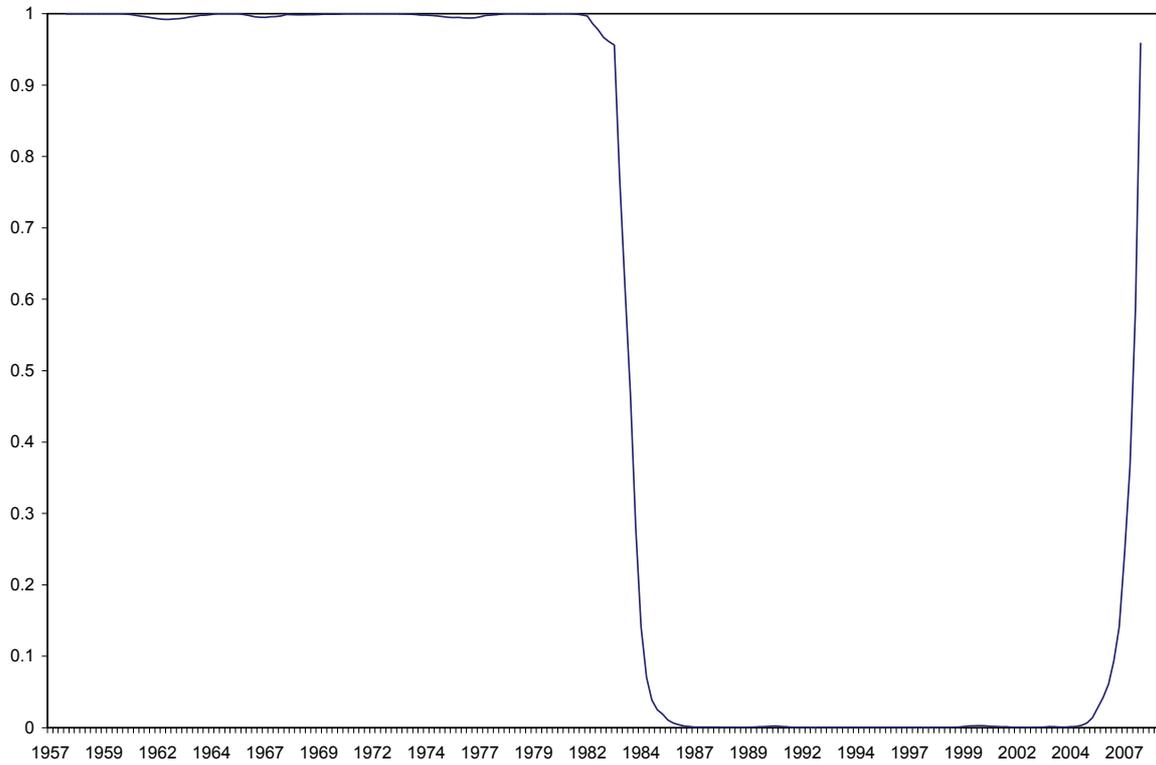
(a) US
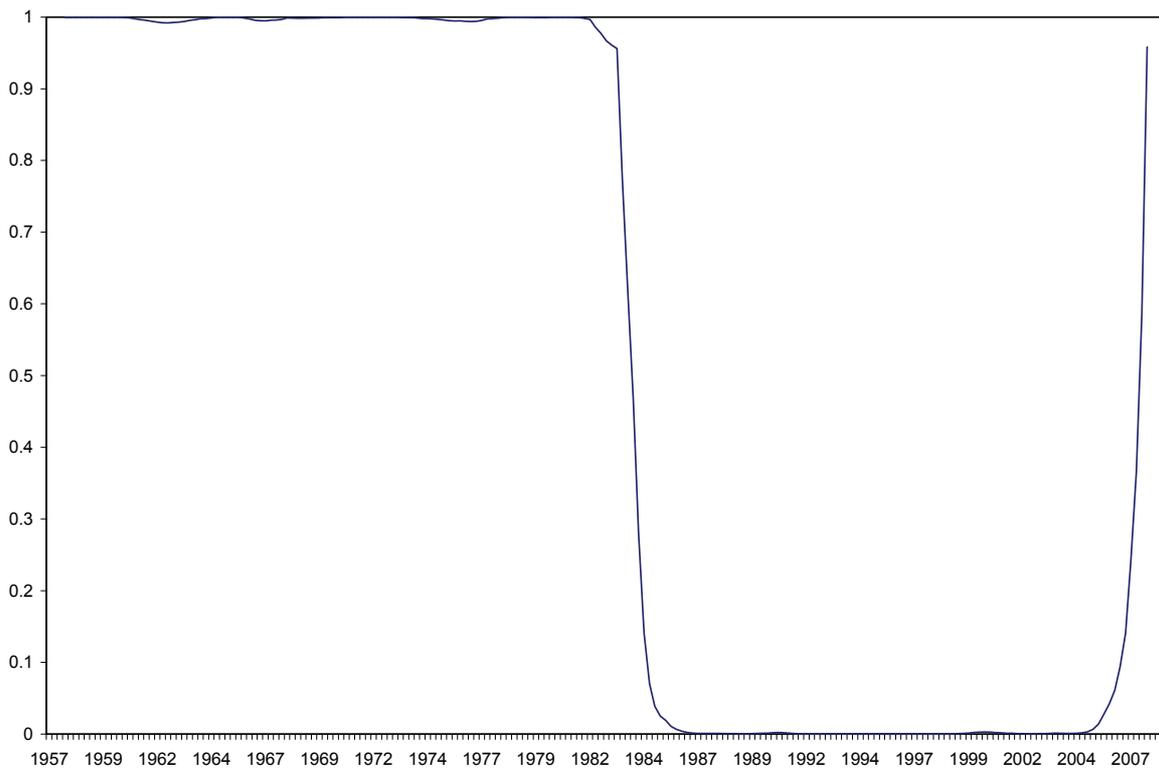
(b) UK

Figure 11. Smoothed probability of state 2 (switching-ARCH model).

0.72, and ends in 2007:03, as the smoothed probability of low variability decreases to 0.41. This decline is immediately followed in 2007:04 by a further sharp decrease to 0.0414. Similarly, for the UK, the low-volatility regime starts in 1992:03, as the smoothed probability rises to 0.77 and ends in 2007:02 as the smoothed probability of low-variability declines to 0.20.

5. Conclusions

The Great Moderation, the significant decline in the variability of economic activity, provides a most remarkable feature of the macroeconomic landscape in the last twenty years. A number of papers document the beginning of the Great Moderation in the US and the UK (e.g., [11-17]). In this paper, we use the Markov regime-switching models of [10] and [31] to document the end of the Great Moderation. The analysis uses quarterly rates of growth of real GDP from 1957:02 to 2007:04 for the US and from 1957:02 to 2007:02 for the UK. Our results place the end of the Great Moderation in 2007.

The Great Moderation in the US and the UK begin at different point in time. In the US the Great Moderation starts in 1983. In the UK, instead, it begins almost 10 years later.[5] The explanations for the Great Moderation fall into generally three different categories—good monetary policy, improved inventory management, or good luck. According to [16], a combination of good monetary policy and better inventory management led to the Great Moderation.

The end of the Great Moderation, however, occurs at approximately the same time in both the US and the UK. The end of the Great Moderation may reflect different reasons, and one may conjecture about reasons for the end. It seems unlikely that good monetary policy would

turn into bad policy or that better inventory management would turn into worse management. Rather, the likely explanation comes from bad luck. Two likely culprits exist—energy price and housing price shocks.[6] We leave this conjecture about the end of the Great Moderation for future research as more data become available with which to address the question.

Relating directly to the comments in the prior paragraph, Reference [56] compares the current sub-prime crisis in the US to 18 bank-centered financial crises. Striking similarities exist between the current US situation and those of the 18 financial crises examined, including the run up and collapse of housing and equity prices, the current level of the current account deficit to GDP, the pattern of changes in real GDP per capita growth, and the rise in the public debt's share of GDP. They also state that a similar situation exists in the UK. In sum, the US situation, and the situation in the UK, provide "stunning quantitative and qualitative parallels across a number of standard financial crisis indicators."

Besides the Great Moderation issue, another reason exists to investigate regime changes in the volatility of economic activity. The well-known autoregressive conditionally heteroskedastic models, based on the seminal work by [1] and [2], play an important role in the estimation of volatilities. Problems associated with estimating such models, however, may arise if the underlying volatility process incorporates structural breaks, especially shifts in the overall level of volatility.[7] In this paper, we show that the variance process is (almost) non-stationary. The high persistence that we find in single-regime models may merely reflect the disregarding the problem of regime changes (*i.e.*, the high persistence may simply occur because of a misspecified model). We find persistence. The persistence, however, does not reside in the shocks, but rather in the regimes.

We must confess in conclusion that we did not expect our finding of the possible end to the Great Moderation. That finding came as a complete surprise. Is it true? Time will tell. Before concluding, we offer some caveats about our finding. First, the reliability of our data series probably deteriorates at the end of the sample, where data revisions may still occur. Such data revisions could reverse our finding. Second, if the Great Moderation largely reflects better monetary policy, then will not the central banks engage in the appropriate actions that will lead to a false signal? That is, will monetary policy makers neutralize those factors that signal a return to the high volatility regime? Third, the added worldwide demand coming from China, India, and other countries may constitute an added dose of "bad luck," especially when combined with the energy and housing market shocks. In sum, we conclude that the empirical evidence signals the end of the Great Moderation. Nonetheless, we still carry some reservations about our finding.

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[5]Our findings on the beginning of the Great Moderation, using different methodologies, match those reported in [20]. The methodology employed by [6], however, cannot identify the end of the Great Moderation, except with the passage of time.

[6]The reasons why the effects of oil price shocks differ so much between the 1970s and the 2000s are considered by [13], using data through 2005: 4. According to [13] four different factors help to explain the differences -"(a) good luck (*i.e.*, lack of concurrent adverse shocks), (b) smaller share of oil in production, (c) more flexible labor markets, and (d) improvements in monetary policy." (p. 1). We note that since 2005:4, the oil price shock worsened dramatically and the housing market crisis in the US and the UK appeared, another concurrent adverse shock.

[7]In this regard, our findings confirm those of [6], who use a different methodology. They find that introducing dummy variables to capture the regime switches in the volatility of real GDP growth eliminates the GARCH and ARCH processes for the volatility processes in each sub-period. **Table 8** reports similar results in that the autoregressive coefficients in the ARCH (2) processes in the Markov regime-switching AR (1)-ARCH (2) model prove insignificantly different from zero. In other words, a homoskedastic error process exists for the high-and low-volatility regimes.

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6. References

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