British Journal of Mathematics & Computer Science
5(5): 595-605, 2015, Article no.BJMCS.2015.044
ISSN: 2231-0851

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Traveling Wave Solutions of the Simplified MCH Equation via Exp$(-\Phi(\xi))$-expansion Method

Harun-Or-Roshid[1*], Md. Nur Alam[1], M. Ali Akbar[2] and Rafiqul Islam[1]

[1]*Department of Mathematics, Pabna University of Science and Technology, Bangladesh.*
[2]*Department of Applied Mathematics, University of Rajshahi, Bangladesh.*

Article Information

DOI: 10.9734/BJMCS/2015/10800
Editor(s):
(1) Carlo Bianca, Department of Mathematical Sciences, Polytechnic University of Turin, Torino, Italy.
Reviewers:
(1) Anonymous, Zagazig University, Zagazig, Egypt.
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Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=729&id=6&aid=7002

Original Research Article

Received: 10 April 2014
Accepted: 10 May 2014
Published: 18 November 2014

Abstract

The $\exp(-\Phi(\xi))$-expansion method is an efficient method for obtaining exact traveling wave solutions of nonlinear evolution equations. In this paper, the $\exp(-\Phi(\xi))$-expansion method is applied to construct exact traveling wave solutions of the simplified MCH equation. The traveling wave solutions are expressed in terms of the hyperbolic functions, the trigonometric functions and the rational functions. It is shown that the method is straightforward and effective mathematical tool for solving nonlinear evolution equations in mathematical physics and engineering.

Keywords: The $\exp(-\Phi(\xi))$-expansion method, the simplified MCH equation, traveling wave solutions, solitary wave solutions.

PACS: 05.45.Yv, 02.30.Jr, 02.30.Ik.

1 Introduction

Many complex real world problems in nature are due to nonlinear phenomena. Nonlinear processes are one of the biggest challenges and not easy to control because the nonlinear

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**Corresponding author: harunorroshidmd@gmail.com;*

characteristic of the system abruptly changes due to some small changes of valid parameters including time. Thus the issue becomes more complicated and hence needs ultimate solution. Therefore, the studies of exact solutions of nonlinear evolution equations (NLEEs) play a crucial role to understand the internal mechanism of nonlinear phenomena. Advance nonlinear techniques are significant to solve inherent nonlinear problems, particularly those involving differential equations, dynamical systems and related areas. In recent years, both the mathematicians and physicists have made significant improvement in finding the exact solutions of NLEEs. They establish many effective and powerful methods to handle the NLEEs. For example, the Jacobi elliptic function expansion method [1,2], the Backlund transformation method [3], the F-expansion method [4, 5], the Darboux transformation method [6], the inverse scattering transform [7], the Adomian decomposition method [8, 9], the complex hyperbolic function method [10, 11], the homogeneous balance method [12-14], the (G'/G)-expansion method [15-26], the modified simple equation method [27], the auxiliary equation method[28,29], the exp-functions method [30], the $\exp(-\varphi(\eta))$-expansion method [31] and so on. Many researchers have studied CH and MCH equations applying different methods [32-36] in recent years because of its importance of applications in several areas of interest.

The objective of this article is to apply the $\exp(-\varphi(\xi))$-expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the simplified MCH equation.

The rest of the paper is organized as follows: In Section 2, we give the description of the $\exp(-\varphi(\xi))$-expansion method. In Section 3, we apply this method to the simplified MCH equation and graphical representations of the solutions. Conclusions are given in the last section.

2 Description of the $\exp(-\Phi(\xi))$-expansion Method

Let us consider a general nonlinear PDE in the form

$$F(u,u_t,u_x,u_{xx},u_{tt},u_{tx},\ldots), \tag{1}$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in $u(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. In the following, we give the main steps of this method:

Step 1: We combine the real variables x and t by a complex variable ξ

$$u(x,t) = u(\xi),\ \ \xi = x \pm V\,t, \tag{2}$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $u = u(\xi)$:

$$\Re(u,u',u'',u''',\cdots), \tag{3}$$

where $\Re$ is a polynomial of u and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ.

Step 2. Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\xi)=\sum_{i=0}^{N}\alpha_i(\exp(-\Phi(\xi)))^{i}, \tag{4}$$

where α_i $(0\le i\le N)$ are constants to be determined, such that $\alpha_N \ne 0$ and $\Phi=\Phi(\xi)$ satisfies the following ordinary differential equation:

$$\Phi'(\xi)=\exp(-\Phi(\xi))+\mu\exp(\Phi(\xi))+\lambda, \tag{5}$$

Eq. (5) gives the following solutions:

Family 1: When $\mu\ne 0,\ \lambda^2-4\mu>0,$

$$\Phi(\eta)=\ln\left(\frac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\eta+E)\right)-\lambda}{2\mu}\right) \tag{6}$$

Family 2: When $\mu\ne 0,\ \lambda^2-4\mu<0,$

$$\Phi(\eta)=\ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\eta+E)\right)-\lambda}{2\mu}\right) \tag{7}$$

Family 3: When $\mu=0,\ \lambda\ne 0,$ and $\lambda^2-4\mu>0,$

$$\Phi(\xi)=-\ln\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right) \tag{8}$$

Family 4: When $\mu\ne 0,\ \lambda\ne 0,$ and $\lambda^2-4\mu=0,$

$$\Phi(\xi)=\ln\left(-\frac{2(\lambda(\xi+E)+2)}{\lambda^2(\xi+E)}\right) \tag{9}$$

Family 5: When $\mu=0,\ \lambda=0,$ and $\lambda^2-4\mu=0,$

$$\Phi(\xi)=\ln(\xi+E) \tag{10}$$

$\alpha_N,\cdots\cdots,V,\lambda,\mu$ are constants to be determined latter, $\alpha_N\neq 0,$ the positive integer N can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 3: We substitute Eq. (4) into Eq. (3) and then we account the function $\exp(-\Phi(\xi))$. As a result of this substitution, we get a polynomial of $\exp(-\Phi(\xi))$. We equate all the coefficients of same power of $\exp(-\Phi(\xi))$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find $\alpha_N,\cdots\cdots,V,\lambda,\mu$. Substituting the values of $\alpha_N,\cdots\cdots,V,\lambda,\mu$ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

3 The Simplified MCH Equation

Now we will bring to bear the $\exp(-\varphi(\eta))$-expansion method to find exact solutions and then the solitary wave solutions of the simplified MCH equation in the form,

$$u_t+2ku_x-u_{xxt}+\beta u^2u_x=0.\quad \text{where}\quad k\in\Re,\ \ \beta>0. \tag{11}$$

Details of CH and MCH equations can be found in references [32-36].

Now, we use the traveling wave transformation Eq. (2) into Eq. (11), which yields

$$-V\,u'+2ku'+Vu'''+\beta u^2\,u'=0. \tag{12}$$

where the superscripts stand for the derivatives with respect to ξ.

Integrating Eq. (12) once with respect to ξ yields:

$$(2k-V)u+Vu''+\frac{\beta}{3}u^3+P=0. \tag{13}$$

where P is an integral constant that could be determined later.

Taking the homogeneous balance between u^3 and u'' in Eq. (13), we obtain $N=1$. Therefore, the solution of Eq. (13) is of the form

$$u(\eta)=\alpha_0+\alpha_1(\exp(-\Phi(\xi))), \tag{14}$$

where α_0, α_1 are constants to be determined such that $\alpha_N \neq 0$, while λ, μ are arbitrary constants.

Substituting Eq. (14) into Eq. (13) and then equating the coefficients of $\exp(-\Phi(\xi))$ to zero, we get

$$2V\alpha_1 + \frac{1}{3}\beta\alpha_1^{\ 3} = 0, \tag{15}$$

$$\beta\alpha_0\alpha_1^{\ 2} + 3V\alpha_1\lambda = 0, \tag{16}$$

$$-V\alpha_1 + V\alpha_1\lambda^2 + 2V\alpha_1\mu + 2k\alpha_1 + \beta\alpha_0^{\ 2}\alpha_1 = 0, \tag{17}$$

$$2k\alpha_0 + \frac{1}{3}\beta\alpha_0^{\ 3} - V\alpha_0 + P + V\alpha_1\mu\lambda = 0, \tag{18}$$

Solving the Eq. (15)-Eq. (18) yields

$$P = 0,\ V = \frac{4k}{2+\lambda^2-4\mu},\ \alpha_0 = \pm\sqrt{\frac{6k}{(-2\beta-\beta\lambda^2+4\beta\mu)}},\ \ \alpha_1 = \pm 2\sqrt{\frac{6k}{(-2\beta-\beta\lambda^2+4\beta\mu)}}.$$

where $\lambda, \mu\ \beta, k$ are arbitrary constants.

Now substituting the values of V, α_o, α_1 into Eq. (14) yields

$$u(\xi) = \pm\sqrt{\frac{6k}{(-2\beta-\beta\lambda^2+4\beta\mu)}}(1+2(\exp(-\Phi(\xi))), \tag{19}$$

where $\xi = x - (\frac{4k}{2+\lambda^2-4\mu})t$.

Now substituting Eq. (6) - Eq. (10) into Eq. (19) respectively, we get the following five traveling wave solutions of the simplified MCH equation.

When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$u_1(\xi) = \pm\sqrt{\frac{6k}{(-2\beta-\beta\lambda^2+4\beta\mu)}}(1 - \frac{4\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E))+\lambda}).$$

where $\xi = x - (\dfrac{4k}{2+\lambda^2-4\mu})t$ and E is an arbitrary constant.

When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$u_2(\xi) = \pm\sqrt{\frac{6k}{(-2\beta - \beta\lambda^2 + 4\beta\mu)}}(1 + \frac{4\mu}{\sqrt{4\mu-\lambda^2}\,\tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)) - \lambda}).$$

where $\xi = x - (\dfrac{4k}{2+\lambda^2-4\mu})t$ and E is an arbitrary constant.

When $\mu = 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0,$

$$u_3(\xi) = \pm\sqrt{\frac{6k}{(-2\beta - \beta\lambda^2)}}(1 + \frac{2\lambda}{\exp(\lambda(\xi+E)) - 1}).$$

where $\xi = x - (\dfrac{4k}{2+\lambda^2})t$ and E is an arbitrary constant.

When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$u_4(\xi) = \pm\sqrt{\frac{3k}{-\beta}}\left(1 - \frac{\lambda^2(\xi+E)}{(\lambda(\xi+E)) + 2)}\right).$$

where $\xi = x - 2kt$ and E is an arbitrary constant.

When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$

$$u_5(\xi) = \pm\sqrt{\frac{3k}{-\beta}}\left(1 + \frac{2}{(\xi+E)}\right).$$

where $\xi = x - 2kt$ and E is an arbitrary constant.

4 Graphical Representation of the Solutions

The graphical illustrations of the solutions are given below in the figures with the aid of Maple.

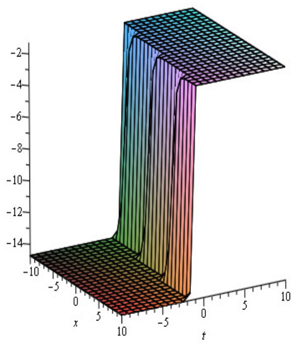

Fig. 1. Kink wave solution $u_1(\xi)$ when $\beta=1,\ k=2,\ \mu=1,\ \lambda=3,\ E=1$ and $-10\le x,t\le 10$

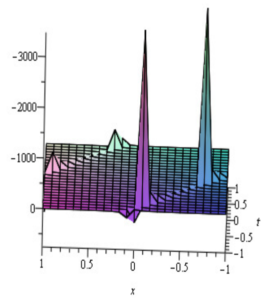

Fig. 2. Periodic solution $u_2(\xi)$ when $\beta=1,\ k=2,\ \mu=3,\ \lambda=1,$ $E=1$ and $-1\le x,t\le 1$

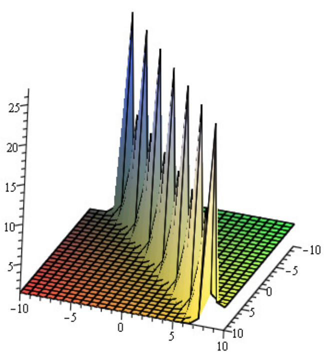

Fig. 3. Singular soliton solution $u_3(\xi)$ when $\beta=1,\ k=2,\ \mu=0,\ \lambda=2,\ E=1$ and $-10\le x,t\le 10$

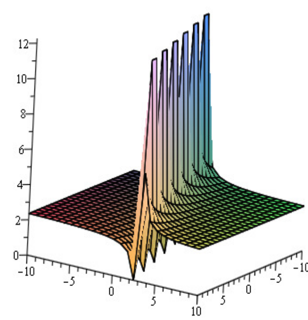

Fig. 4. Singular Kink wave solution $u_4(\eta)$ when $\beta=1,\ k=2,\ \mu=1,$ $\lambda=2,\ E=1$ and $-10\le x,t\le 10$

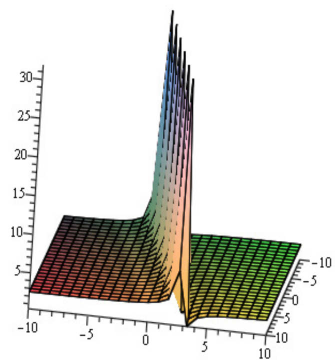

Fig. 5. Singular Kink wave solution $u_5(\eta)$ **when** $\beta=1,\ k=2,\ \mu=0,\ \lambda=0,\ E=1$ **and** $-10\le x,t\le 10$

5 Conclusion

In this paper, the $\exp(-\Phi(\xi))$ -expansion method is applied successfully for solving the simplified MCH equation. The procedure is simple, direct and constructive without the help of a computer algebra system. The results show that this method is efficient in finding the exact traveling wave solutions of nonlinear differential equations.

Competing Interests

Authors have declared that no competing interests exist.

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